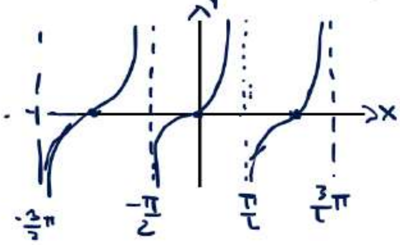
LEZIONE 4

Altre funzioni goniometriche

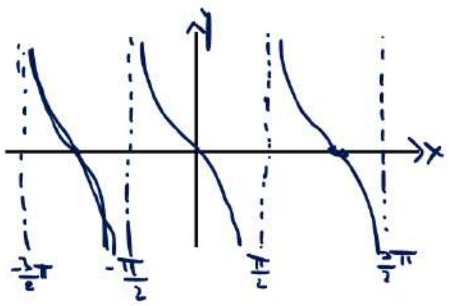


$$y = tg\,x = \frac{sen\,x}{cos\,x}$$

$$\boxed{D = \{x \in \mathbb{R} \setminus x = \frac{\pi}{2} + k\pi\}}$$

DOMINIO

$tg\,x = tg(x + k\pi)$ $\quad k \in \mathbb{Z} \quad k = \ldots, -2, -1, 0, 1, 2, \ldots$



$$\boxed{y = f(x) = \text{cotg}\, x = \frac{1}{\text{tg}\, x} = \frac{\cos x}{\text{sen}\, x}}$$

$$D = \{x \in \mathbb{R} \mid x \neq k\pi \,;\ k \in \mathbb{Z}\}$$

$$\text{cotg}\, x = \text{cotg}\,(x + k\pi) \qquad k \in \mathbb{Z}$$

$$k = \ldots, -2, -1, 0, 1, 2 \ldots$$

$$\boxed{\sec\alpha = \frac{1}{\cos\alpha}}$$

FUNZIONE SECANTE

$$\boxed{\operatorname{cosec}\alpha = \frac{1}{\operatorname{sen}\alpha}}$$

FUNZIONE COSECANTE

sen α	cosec α
cos α	sec α
tg α	cotg α

ESERCIZIO 1

$$\operatorname{cosec}\alpha \cdot \cos\alpha \cdot \operatorname{tg}\alpha =$$

$$= \frac{1}{\cancel{\operatorname{sen}\alpha}} \cdot \cancel{\cos\alpha} \cdot \frac{\cancel{\operatorname{sen}\alpha}}{\cancel{\cos\alpha}} = 1 \quad \checkmark$$

ESERCIZIO 2

$$\boxed{\operatorname{sen}^2\alpha+\cos^2\alpha=1}$$

$$\frac{(\sec^2\alpha+\operatorname{cosec}^2\alpha)\operatorname{tg}^2\alpha}{\sec^3\alpha}=$$

$$=\frac{\left(\frac{1}{\cos^2\alpha}+\frac{1}{\operatorname{sen}^2\alpha}\right)\cdot\frac{\operatorname{sen}^2\alpha}{\cos^2\alpha}}{\frac{1}{\cos^3\alpha}}=\frac{\left(\frac{\operatorname{sen}^2\alpha+\cos^2\alpha}{\operatorname{sen}^2\alpha\cdot\cos^2\alpha}\right)\cdot\frac{\operatorname{sen}^2\alpha}{\cos^2\alpha}}{\frac{1}{\cos^3\alpha}}=$$

$$=\frac{\left(\frac{1}{\cancel{\operatorname{sen}^2\alpha}\cdot\cos^2\alpha}\right)\cdot\frac{\cancel{\operatorname{sen}^2\alpha}}{\cos^2\alpha}}{\frac{1}{\cos^3\alpha}}=\frac{\frac{1}{\cos^4\alpha}}{\frac{1}{\cos^3\alpha}}=\frac{1}{\cancel{\cos^4}\alpha}\cdot\cancel{\cos^3}\alpha=\frac{1}{\cos\alpha}=\boxed{\sec\alpha}$$

√

ESERCIZIO 3

$$\boxed{\operatorname{sen}^2\alpha + \cos^2\alpha = 1}$$

$$\cos^2\alpha\,(\operatorname{tg}\alpha + \operatorname{cotg}\alpha)^2 + (\operatorname{cosec}^2\alpha + \sec^2\alpha)\operatorname{sen}^2\alpha =$$

$$= \cos^2\alpha\left(\frac{\operatorname{sen}\alpha}{\cos\alpha} + \frac{\cos\alpha}{\operatorname{sen}\alpha}\right)^2 + \left(\frac{1}{\operatorname{sen}^2\alpha} + \frac{1}{\cos^2\alpha}\right)\operatorname{sen}^2\alpha =$$

$$= \cos^2\alpha\left(\frac{\operatorname{sen}^2\alpha + \cos^2\alpha}{\operatorname{sen}\alpha\cos\alpha}\right)^2 + \left(\frac{\cos^2\alpha + \operatorname{sen}^2\alpha}{\operatorname{sen}^2\alpha\cos^2\alpha}\right)\cdot\operatorname{sen}^2\alpha =$$

$$\boxed{= \frac{1}{\operatorname{sen}^2\alpha\cos^2\alpha}}$$

$$= \cos^2\alpha\cdot\left(\frac{1}{\operatorname{sen}\alpha\cos\alpha}\right)^2 + \frac{1}{\operatorname{sen}^2\alpha\cos^2\alpha}\cdot\operatorname{sen}^2\alpha =$$

$$= \cancel{\cos^2\alpha}\cdot\frac{1}{\operatorname{sen}^2\alpha\,\cancel{\cos^2\alpha}} + \frac{1}{\cancel{\operatorname{sen}^2\alpha}\cos^2\alpha}\cdot\cancel{\operatorname{sen}^2\alpha} = \frac{1}{\operatorname{sen}^2\alpha} + \frac{1}{\cos^2\alpha} = \frac{\overbrace{\cos^2\alpha + \operatorname{sen}^2\alpha}^{1}}{\operatorname{sen}^2\alpha\cos^2\alpha}$$

ESERCIZIO 4

$\boxed{\text{sen } 90° = \cos 0° = 1}$

$$\frac{\text{sen } 90° \, tg x}{\cos x \cdot \cos 0°} + (\cos 90° - \cos 180°)(\sec x + \cos x) + \frac{\text{sen } 270° - \cos^2 x}{\cos x} =$$

$$= \frac{tg x}{\cos x} + (0 - (-1))\left(\frac{1}{\cos x} + \cos x\right) + \frac{-1 - \cos^2 x}{\cos x} =$$

$$= \frac{tg x}{\cos x} + \left(\frac{1}{\cos x} + \cos x\right) - \frac{(1 + \cos^2 x)}{\cos x} =$$

$$= \frac{tg x}{\cos x} + \frac{1}{\cos x} + \cos x - \frac{1 + \cos^2 x}{\cos x} =$$

$$= \frac{\frac{\operatorname{sen} x}{\cos x}}{\cos x} + \frac{1}{\cos x} + \cos x - \frac{1+\cos^2 x}{\cos x} =$$

$$= \frac{\operatorname{sen} x}{\cos^2 x} + \frac{1}{\cos x} + \frac{\cos x}{1} - \frac{1+\cos^2 x}{\cos x} =$$

$$= \frac{\operatorname{sen} x + \cos x + \cos^3 x - (1+\cos^2 x)\cdot\cos x}{\cos^2 x} =$$

$$= \frac{\operatorname{sen} x + \cancel{\cos x} + \cancel{\cos^3 x} - \cancel{\cos x} - \cancel{\cos^3 x}}{\cos^2 x} = \boxed{\frac{\operatorname{sen} x}{\cos^2 x}}$$

<u>ESERCIZIO 5</u>

$\boxed{\operatorname{sen}\pi = 0}$ $\boxed{\cos\frac{3}{2}\pi = 0}$ $\boxed{\cos\frac{\pi}{2} = 0}$

$$\sec^2\alpha \cdot \operatorname{sen}\alpha \left(\cos\alpha - \cancel{\cos\frac{3}{2}\pi}\right) + \operatorname{cotg}\alpha + 2\sec\alpha \operatorname{cosec}\alpha \left(\cancel{\cos\frac{\pi}{2}} + \operatorname{sen}\alpha\right) \cdot \left(\cancel{\operatorname{sen}}(\pi - \operatorname{sen}\alpha)\right) =$$

$$= \frac{1}{\cos^{\cancel{2}}\alpha} \operatorname{sen}\alpha \cdot \cancel{\cos}\,\alpha + \frac{\cos\alpha}{\operatorname{sen}\alpha} + 2\,\frac{1}{\cos\alpha} \cdot \frac{1}{\cancel{\operatorname{sen}\alpha}}\, \cancel{\operatorname{sen}\alpha} \cdot (-\operatorname{sen}\alpha) =$$

$$= \cancel{\frac{\operatorname{sen}\alpha}{\cos\alpha}} + \frac{\cos\alpha}{\operatorname{sen}\alpha} - \cancel{\frac{\operatorname{sen}\alpha}{\cos\alpha}} = \frac{\cos\alpha}{\operatorname{sen}\alpha} = \boxed{\operatorname{cotg}\alpha}$$

ESERCIZIO 6

$\operatorname{sen}\frac{3}{2}\pi = -1 \qquad \operatorname{sen}\left(\frac{3}{2}\pi\right) = \operatorname{sen}\left(\underbrace{\frac{3}{2}\pi + 2K\pi}\right)$

$$\frac{\operatorname{sen}\alpha \cdot \operatorname{sen}\frac{27}{2}\pi + \operatorname{tg}\alpha}{\cos 8\pi \cos\alpha} \cdot \frac{\operatorname{cotg}^2\alpha \operatorname{sen}\frac{15}{2}\pi}{\operatorname{cosec}\alpha \cos 13\pi} + 1 =$$

$$= \frac{\operatorname{sen}\alpha \cdot \operatorname{sen}\left(\frac{3}{2}\pi + 12\pi\right) + \frac{\operatorname{sen}\alpha}{\cos\alpha}}{\cos(2\pi + 6\pi) \cos\alpha} \cdot \frac{\frac{\cos^2\alpha}{\operatorname{sen}^2\alpha} \operatorname{sen}\left(\frac{3}{2}\pi + 6\pi\right)}{\frac{1}{\operatorname{sen}\alpha} \cdot \cos(\pi + 12\pi)} + 1 =$$

$$= \frac{\operatorname{sen}\alpha \cdot (-1) + \frac{\operatorname{sen}\alpha}{\cos\alpha}}{\cos\alpha} \cdot \frac{\frac{\cos^2\alpha}{\operatorname{sen}^2\alpha} (\cancel{-1})}{\frac{1}{\operatorname{sen}\alpha} (\cancel{-1})} + 1 =$$

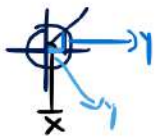
$$\frac{\frac{-\operatorname{sen}\alpha}{1} + \frac{\operatorname{sen}\alpha}{\cos\alpha}}{\cos\alpha} \cdot \frac{\cos^2\alpha}{\operatorname{sen}^2\alpha} \cdot \cancel{\operatorname{sen}\alpha} + 1 =$$

$$= \frac{\frac{-\operatorname{sen}\alpha\cos\alpha + \operatorname{sen}\alpha}{\cos\alpha}}{\cos\alpha} \cdot \frac{\cos^2\alpha}{\operatorname{sen}\alpha} + 1 =$$

$$= \frac{\cancel{\operatorname{sen}\alpha}(1-\cos\alpha)}{\cancel{\cos^2\alpha}} \cdot \frac{\overset{(-1)}{\cancel{\cos^2\alpha}}}{\cancel{-\operatorname{sen}\alpha}} + 1 = (1-\cos\alpha)(-1) + 1 =$$

$$= \cos\alpha - \cancel{1} + \cancel{1} = \boxed{\cos\alpha}$$

FUNZIONI GONIOMETRICHE INVERSE



$$\boxed{f : \mathbb{R} \to \mathbb{R}}$$

$$y = f(x) = \operatorname{sen} x$$

La funzione seno all'angolo x associa la lunghezza del segmento in blu

$$y = f(x) = \cos x$$

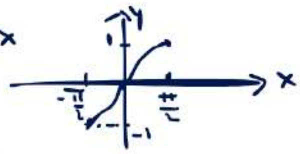
La funzione coseno all'angolo x associa la lunghezza del segmento in azzurro chiaro

Se voglio invertire le funzione seno e coseno devo poterle rendere iniettive, cioè in modo tale che ad un unico elemento del dominio corrisponda uno e un solo elemento del codominio

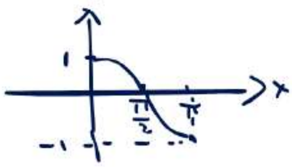
Per ottenere senx e cosx in forma di funzioni iniettive l'unico modo è operare una restrizione sul dominio di entrambe le funzioni

Your paragraph text

$f: \underbrace{\left[-\frac{\pi}{2} ; \frac{\pi}{2}\right]} \longrightarrow \mathbb{R} \quad y = f(x) = \operatorname{sen} x$



$f: \underbrace{[0, \pi]} \longrightarrow \mathbb{R} \quad y = f(x) = \cos x$



INIETTIVE,
MA NON SURIETTIVE!!!

Per renderle suriettive restringo anche i codomini

$$\boxed{\begin{array}{ll} y = \operatorname{sen} x & f : \left[-\frac{\pi}{2} ; \frac{\pi}{2}\right] \to [-1 ; 1] \\ y = \cos x & f : [0 ; \pi] \to [-1 ; 1] \end{array}}$$

$\boxed{g = f^{-1}}$

ORA RISULTANO ESSERE INVERTIBILI PERCHE' BIIETTIVE, CIOE'
SIA INIETTIVE CHE SURIETTIVE!!!
Queste si chiamano arco seno e arco coseno.

$$y = \operatorname{sen}^{-1} x = \operatorname{arcsen} x \qquad g : [-1 ; 1] \longrightarrow \left[-\frac{\pi}{2} ; \frac{\pi}{2}\right]$$

$$y = \cos^{-1} x = \arccos x \qquad g : [-1 , 1] \longrightarrow [0 , \pi]$$

$y = tg\,x$ $\quad f: \left]-\frac{\pi}{2}; \frac{\pi}{2}\right[\longrightarrow \mathbb{R}$

SIA INIETTIVA CHE SURIETTIVA

$\Downarrow$

INVERTIBILE

$$\boxed{y = \tan^{-1} x = arctg\, x}$$

ARCO TANGENTE, OVVERO FUNZIONE INVERSA DELLA TANGENTE CON
CON LE OPPORTUNE RESTRIZIONI AL DOMINIO DELLA FUNZIONE TANGENTE