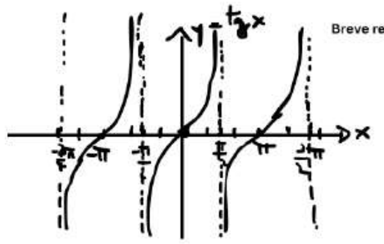


LEZIONE 4

Breve recap su tangente e cotangente

$$\boxed{tg\,x = \frac{sen\,x}{cos\,x}}$$

$$D = \{x \in \mathbb{R} \mid \cos x \neq 0\}$$

DOMINIO DI $y = tg\,x$

$$\boxed{x \neq \underbrace{\frac{\pi}{2}}_{90^\circ} + \underbrace{K \cdot \pi}_{180^\circ}}$$

INTERVALLI DI VARIAZIONE E PERIODICITA' DELLA COTANGENTE

$$\boxed{\cot g\, x = \frac{\cos x}{\operatorname{sen} x}}$$

$$\cot g\, 0° = \frac{1}{tg\, 0°} = \frac{\cos 0°}{\operatorname{sen} 0°} = \frac{1}{0} \quad \nexists \text{ alcun valore}$$

$$90° \rightarrow \frac{\pi}{2}$$

$$180° \rightarrow \pi$$

$$270° \rightarrow \frac{3\pi}{2}$$

$$360° \rightarrow 2\pi$$

$$\cot g\, 90° = \frac{1}{tg\, 90°} = \frac{\cos 90°}{\operatorname{sen} 90°} = \frac{0}{1} = 0$$

$$\cot g\, 180° = \frac{1}{tg\, 180°} = \frac{\cos 180°}{\operatorname{sen} 180°} = -\frac{1}{0} \quad \nexists \text{ alcun valore}$$

$$\cot g\, 270° = \frac{1}{tg\, 270°} = \frac{\cos 270°}{\operatorname{sen} 270°} = \frac{0}{-1} = 0$$

$$\cot g\, 360° = \nexists \text{ alcun valore}$$

$$\boxed{\cot\alpha = \cot(\alpha + k\pi)}$$

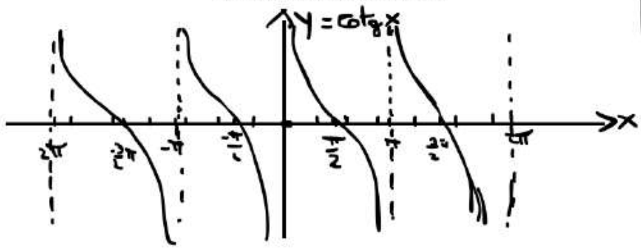
Periodicità della cotangente uguale a quella della tangente, cioè ogni 180° la funzione cotangente ripete il suo valore

$k \in \mathbb{Z} \quad k = \ldots -2, -1, 0, 1, 2, \ldots$

$$y = \cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

$D = \{x \in \mathbb{R} \mid \sin x \neq 0\}$ $\boxed{x \neq k\pi \; ; k \in \mathbb{Z}}$

GRAFICO FUNZIONE COTANGENTE



$0 < x < 90^\circ$

cotg x ↓ cotg > 0

$90^\circ < x < 180^\circ$

cotg x ↓ cotg < 0

$180^\circ < x < 270^\circ$

cotg x ↓ cotg > 0

$270^\circ < x < 360^\circ$

cotg x ↓ cotg < 0

Seno sen x → $\boxed{\operatorname{cossec} x = \frac{1}{\operatorname{sen} x}}$ → COSSECANTE

Cosseno cos x → $\boxed{\sec x = \frac{1}{\cos x}}$ → SECANTE

Tangente tg x → $\boxed{\operatorname{cotg} x = \frac{1}{\operatorname{tg} x}}$ → COTANGENTE

Esempi di espressioni goniometriche

1) $$\operatorname{cosec}\alpha \cdot \cos\alpha \cdot \operatorname{tg}\alpha =$$

Il trucchetto per semplificare questo genere di espressioni è ricondurre tutto a seno e coseno.

$$= \frac{1}{{}_{1}\cancel{\operatorname{sen}\alpha}} \cdot {}^{1}\cancel{\cos\alpha} \cdot \frac{\cancel{\operatorname{sen}\alpha}^{1}}{{}_{1}\cancel{\cos\alpha}} = 1$$

$$2)\quad \frac{(\sec^2\alpha + \operatorname{cosec}^2\alpha)\,\operatorname{tg}^2\alpha}{\sec^3\alpha} = \qquad \operatorname{sen}^2\alpha + \cos^2\alpha = 1$$

$$= \frac{\left(\frac{1}{\cos^2\alpha} + \frac{1}{\operatorname{sen}^2\alpha}\right)\cdot\frac{\operatorname{sen}^2\alpha}{\cos^2\alpha}}{\frac{1}{\cos^3\alpha}} = \frac{\left(\frac{\operatorname{sen}^2\alpha + \cos^2\alpha}{\operatorname{sen}^2\alpha\cdot\cos^2\alpha}\right)\cdot\frac{\operatorname{sen}^2\alpha}{\cos^2\alpha}}{\frac{1}{\cos^3\alpha}}$$

$$= \frac{\frac{1}{\cancel{\operatorname{sen}^2\alpha}\cos^2\alpha}\cdot\frac{\cancel{\operatorname{sen}^2\alpha}}{\cos^2\alpha}}{\frac{1}{\cos^3\alpha}} = \frac{\frac{1}{\cos^4\alpha}}{\frac{1}{\cos^3\alpha}} = \frac{1}{\cancel{\cos^4\alpha}}\cdot\frac{\cancel{\cos^3\alpha}}{1} = \frac{1}{\cos\alpha} = \boxed{\sec\alpha}$$

$$\cos^2\alpha \cdot (\operatorname{tg}\alpha + \operatorname{cotg}\alpha)^2 + (\operatorname{cosec}^2\alpha + \sec^2\alpha)\operatorname{sen}^2\alpha =$$

$$= \cos^2\alpha \cdot \left(\frac{\operatorname{sen}\alpha}{\cos\alpha} + \frac{\cos\alpha}{\operatorname{sen}\alpha}\right)^2 + \left(\frac{1}{\operatorname{sen}^2\alpha} + \frac{1}{\cos^2\alpha}\right)\operatorname{sen}^2\alpha =$$

$$= \cos^2\alpha \cdot \left(\frac{\operatorname{sen}^2\alpha + \cos^2\alpha}{\operatorname{sen}\alpha\cos\alpha}\right)^2 + \left(\frac{\cos^2\alpha + \operatorname{sen}^2\alpha}{\operatorname{sen}^2\alpha \cdot \cos^2\alpha}\right) \cdot \operatorname{sen}^2\alpha =$$

$$= \cos^2\alpha \cdot \left(\frac{1}{\operatorname{sen}\alpha\cos\alpha}\right)^2 + \left(\frac{1}{\cancel{\operatorname{sen}^2}\alpha \cdot \cos^2\alpha}\right) \cdot \cancel{\operatorname{sen}^2\alpha} =$$

$$= \cancel{\cos^2\alpha} \cdot \frac{1}{\operatorname{sen}^2\alpha\,\cancel{\cos^2\alpha}} + \frac{1}{\cos^2\alpha} = \frac{1}{\operatorname{sen}^2\alpha} + \frac{1}{\cos^2\alpha} = \frac{\cos^2\alpha + \operatorname{sen}^2\alpha}{\operatorname{sen}^2\alpha\cos^2\alpha}$$

$$= \boxed{\frac{1}{\operatorname{sen}^2\alpha\cos^2\alpha}}$$

4) $\dfrac{\operatorname{sen} 90° \operatorname{tg} x}{\cos x \cos 0°} + (\cos 90° - \cos 180°)(\sec x + \cos x) + \dfrac{\operatorname{sen}^2 270° - \cos^2 x}{\cos x} =$

$= \dfrac{\operatorname{tg} x}{\cos x} + (0 - (-1))\left[\dfrac{1}{\cos x} + \dfrac{\cos x}{1}\right] + \left[\dfrac{1 - \cos^2 x}{\cos x}\right] -$

$= \dfrac{\dfrac{\operatorname{sen} x}{\cos x}}{\cos x} + \left[\dfrac{1 + \cos^2 x}{\cos x}\right] + \left[-\dfrac{1 - \cos^2 x}{\cos x}\right] = \boxed{\dfrac{\operatorname{sen} x}{\cos^2 x}}$

$\cancel{\cos(1+\cos^2 x)}$

$= \dfrac{\operatorname{sen} x}{\cos^2 x} + \dfrac{1 + \cos^2 x}{\cos x} - \dfrac{1 + \cos^2 x}{\cos x} = \cancel{\dfrac{\operatorname{sen} x + \cos x(1 + \cos^2 x) -}{\cos^2 x}}$