LEZIONE 5

Grafici delle funzioni inverse goniometriche

$y = \operatorname{sen} x \qquad f: \left[-\frac{\pi}{2}; \frac{\pi}{2}\right] \longrightarrow [-1; 1]$

$$\boxed{y = \operatorname{arcsen} x \qquad f^{-1}: [-1; 1] \longrightarrow \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]}$$

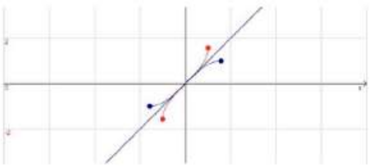

Grafico che mostra la simmetria delle due funzioni: seno e arco seno rispetto la bisettrice del 1° e del 3° quadrante

$y = \cos x$ $\qquad f: [0, \pi] \longrightarrow [-1; 1]$

$y = \arccos x$ $\qquad f^{-1}: [-1; 1] \longrightarrow [0, \pi]$

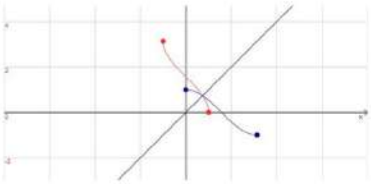

Grafico che mostra la simmetria delle due funzioni: coseno e arco coseno rispetto la bisettrice del 1° e del 3° quadrante

$y = \operatorname{tg} x \qquad f: \left]-\frac{\pi}{2}; \frac{\pi}{2}\right[\longrightarrow \mathbb{R}$

$y = \operatorname{arctg} x \qquad f^{-1}: \mathbb{R} \longrightarrow \left]-\frac{\pi}{2}; \frac{\pi}{2}\right[$

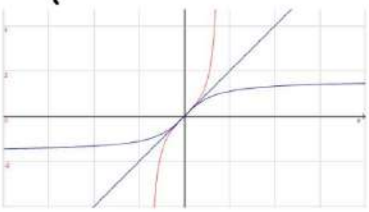

Grafico che mostra la simmetria delle funzioni: tangente e arcotangente rispetto la bisettrice del 1° e del 3° quadrante.

$$\boxed{sen^2\alpha + cos^2\alpha = 1}$$

$$cos^2\alpha = 1 - sen^2\alpha \Rightarrow \boxed{cos\,\alpha = \pm\sqrt{1 - sen^2\alpha}}$$

$$\boxed{sen\,\alpha = \pm\sqrt{1 - cos^2\alpha}}$$

Ricavo seno e coseno conoscendo l'uno o l'altro.

Basta conoscere o il seno o il coseno e sono a posto!!!

ESERCIZIO 1

$\boxed{\operatorname{sen}\alpha = \frac{7}{25}}$ $\quad$ $\boxed{90^\circ < \alpha < 180^\circ} \Rightarrow \cos\alpha < 0$!!!

$\cos\alpha = ?$

$$\cos\alpha = -\sqrt{1 - \operatorname{sen}^2\alpha} = -\sqrt{1 - \left(\frac{7}{25}\right)^2} =$$

$$= -\sqrt{1 - \frac{49}{625}} = -\sqrt{\frac{625-49}{625}} = -\sqrt{\frac{576}{625}} = -\frac{\sqrt{576}}{\sqrt{625}} = -\frac{24}{25}$$

$\boxed{\cos\alpha = -\frac{24}{25}}$ ✓

ESERCIZIO 2

$\operatorname{sen}\alpha = -\frac{5}{13}$ $\cos\alpha = ?$

$\operatorname{tg}\alpha = ?$

$$\cos\alpha = \pm\sqrt{1-\operatorname{sen}^2\alpha} = \pm\sqrt{1-\left(\frac{5}{13}\right)^2} = \pm\sqrt{\frac{1}{1}-\frac{25}{169}} = \sqrt{\frac{169-25}{169}} = \pm\sqrt{\frac{144}{169}}$$

$\cos\alpha = \pm\frac{12}{13}$ $\boxed{k \in \mathbb{Z}}$ $k = \ldots, -2, -1, 0, 1, 2, \ldots$

3° QUADRANTE $\boxed{\pi + 2k\pi < \alpha < \frac{3}{2}\pi + 2k\pi}$ $\operatorname{sen}\alpha = -\frac{5}{13}$

$\cos\alpha = -\frac{12}{13}$

$\boxed{\operatorname{tg}\alpha = \frac{5}{12}}$ $\operatorname{tg}\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha} = \frac{\frac{5}{13}}{\frac{12}{13}} = \frac{5}{\cancel{13}} \cdot \frac{\cancel{13}}{12} = \frac{5}{12}$

4^{a} QUADRANTE $\boxed{\frac{3}{2}\pi + 2k\pi < \alpha < 2\pi + 2k\pi}$

$\cos\alpha = \frac{12}{13}$ $\quad \sin\alpha = -\frac{5}{13}$ $\quad \boxed{\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = -\frac{5}{12}}$

Nota la tangente dell'angolo posso ricavare seno e coseno dell'angolo stesso

$$\cos^2\alpha + \operatorname{sen}^2\alpha = 1$$

Divido tutta la relazione per cos^2

$$\Downarrow$$

$$1 + tg^2\alpha = \frac{1}{\cos^2\alpha}$$

$$\cos^2\alpha\,(1 + tg^2\alpha) = \frac{1}{\cancel{\cos^2\alpha}} \cdot \cancel{\cos^2\alpha}$$

2° principio di equivalenza

$$\cos^2\alpha\,(1 + tg^2\alpha) = 1$$

DIdivo tutto per (1+tg^2)

$$\cos^2\alpha = \frac{1}{1 + tg^2\alpha} \Rightarrow \boxed{\cos\alpha = \pm\sqrt{\frac{1}{1 + tg^2\alpha}}}$$

$$\operatorname{sen}^2\alpha = 1 - \cos^2\alpha = \frac{1}{1} - \frac{1}{1+\operatorname{tg}^2\alpha}$$

$$\operatorname{sen}^2\alpha = \frac{\cancel{1} + \operatorname{tg}^2\alpha - \cancel{1}}{1+\operatorname{tg}^2\alpha} \Rightarrow \operatorname{sen}^2\alpha = \frac{\operatorname{tg}^2\alpha}{1+\operatorname{tg}^2\alpha}$$

$$\boxed{\operatorname{sen}\alpha = \pm\sqrt{\frac{\operatorname{tg}^2\alpha}{1+\operatorname{tg}^2\alpha}} = \pm\frac{\operatorname{tg}\alpha}{\sqrt{1+\operatorname{tg}^2\alpha}}}$$

$$\boxed{\begin{cases} \cos\alpha = \pm \dfrac{1}{\sqrt{1+tg^2\alpha}} \\ sen\,\alpha = \pm \dfrac{tg\,\alpha}{\sqrt{1+tg^2\alpha}} \end{cases}}$$

ESERCIZIO 3

$tg\alpha = 2\sqrt{2}$ $\quad 0° < \alpha < 90°$ $\quad$ $sen\alpha = ?$ $\quad cos\alpha = ?$

$$sen\,\alpha = \frac{tg\,\alpha}{\sqrt{1+tg^2\alpha}} = \frac{2\sqrt{2}}{\sqrt{1+(2\sqrt{2})^2}} = \frac{2\sqrt{2}}{\sqrt{1+8}} = \frac{2\sqrt{2}}{3}$$

$\boxed{\begin{array}{l} sen\alpha > 0 \\ cos\alpha > 0 \end{array}}$!!!

$$cos\,\alpha = \frac{1}{\sqrt{1+tg^2\alpha}} = \frac{1}{3} \Rightarrow \boxed{cos\alpha = \frac{1}{3}} \quad \boxed{sen\,\alpha = \frac{2\sqrt{2}}{3}}$$

ESERCIZIO 2

$$\operatorname{tg}\alpha = -2\sqrt{6}$$

$$\operatorname{sen}\alpha = \frac{\operatorname{tg}\alpha}{\sqrt{1+\operatorname{tg}^2\alpha}} = \frac{-2\sqrt{6}}{\sqrt{1+24}} = -\frac{2\sqrt{6}}{5}$$

$$\cos\alpha = -\frac{1}{\sqrt{1+\operatorname{tg}^2\alpha}} = -\frac{1}{\sqrt{1+24}} = -\frac{1}{5}$$

2° QUADRANTE se K PARI

$$\boxed{\frac{\pi}{2} + K\pi < \alpha < \pi + K\pi}$$

$\operatorname{sen}\alpha > 0$

$\cos\alpha < 0$

$\operatorname{sen}\alpha = ?$

$\cos\alpha =$ [illegible]

4° QUADRANTE

SE K DISPARI

ESERCIZIO 5

$$\cos\Big[\underbrace{\operatorname{arcsen}\left(-\frac{\sqrt{2}}{4}\right)}_{\alpha}\Big]$$

PONGO

$$\boxed{\alpha = \operatorname{arcsen}\left(-\frac{\sqrt{2}}{4}\right)}$$

$$\operatorname{sen}\alpha = -\frac{\sqrt{2}}{4}$$

$$\operatorname{sen}\alpha < 0$$

4° QUADRANTE $\boxed{\cos\alpha > 0}$

$$\cos\alpha = \sqrt{1-\operatorname{sen}^2\alpha} = \sqrt{1-\left(-\frac{\sqrt{2}}{4}\right)^2} =$$

$$= \sqrt{1-\frac{\cancel{2}^{1}}{\cancel{16}_{8}}} = \sqrt{1-\frac{1}{8}} = \sqrt{\frac{8-1}{8}} = \frac{\sqrt{7}}{\sqrt{8}} = \frac{\sqrt{7}}{2\sqrt{2}}$$

$$\frac{\sqrt{7}}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{14}}{4} \qquad \boxed{\cos\alpha = \frac{\sqrt{14}}{4}} \quad !!!$$

ESERCIZIO 6 $\dfrac{\operatorname{sen}\alpha + \operatorname{tg}^2\alpha}{1 + \sec^2\alpha}$

DEVE COMPARIRE SOLO SENO...!!!

$$\frac{\frac{\operatorname{sen}\alpha}{1} + \frac{\operatorname{sen}^2\alpha}{\cos^2\alpha}}{1 + \frac{1}{\cos^2\alpha}} = \frac{\frac{\operatorname{sen}\alpha\cos^2\alpha + \operatorname{sen}^2\alpha}{\cancel{\cos^2\alpha}}}{\frac{\cos^2\alpha + 1}{\cancel{\cos^2\alpha}}} =$$

$$= \frac{\operatorname{sen}\alpha\cos^2\alpha + \operatorname{sen}^2\alpha}{\cos^2\alpha + 1} = \frac{\operatorname{sen}\alpha\,(1 - \operatorname{sen}^2\alpha) + \operatorname{sen}^2\alpha}{1 - \operatorname{sen}^2\alpha + 1}$$

$$\frac{\operatorname{sen}\alpha - \operatorname{sen}^3\alpha + \operatorname{sen}^2\alpha}{2 - \operatorname{sen}^2\alpha} = \boxed{\frac{\operatorname{sen}\alpha\,(1 - \operatorname{sen}^2\alpha + \operatorname{sen}\alpha)}{2 - \operatorname{sen}^2\alpha}} \quad \checkmark$$