LEZIONE 5

Espressioni goniometriche

$$\boxed{\begin{aligned} &\text{sen}\,0° \cdot \cos 90° + 2\cdot\cos(180°) = \\ &= 0 + 2\cdot(-1) = 0 - 2 = \underline{-2} \end{aligned}}$$

$$\boxed{\begin{aligned} &5\,\text{tg}\,180° + \text{sen}\,90° + 2\cos(270°) = \\ &= \cancel{5\cdot 0} + 1 + \cancel{2\cdot 0} = \underline{1} \end{aligned}}$$

$$\text{tg}\,180° = \frac{\text{sen}\,180°}{\cos 180°} = \frac{0}{-1} = 0$$

$$1 - \cos 180^\circ + \cos 360^\circ + 2\,\text{sen}\,90^\circ =$$
$$= 1 - (-1) + 1 + 2 \cdot 1 = 1 + 1 + 1 + 2 = \underline{5}$$

$$15 \cdot \cos 180^\circ + 14 \cos 0^\circ + \cos 360^\circ =$$
$$= 15 \cdot (-1) + 14 \cdot 1 + 1 = -15 + 14 + 1 = \underline{0}$$

$$\text{sen}\,180^\circ - 4 \cos 90^\circ + 3 \cdot \text{sen}\,270^\circ =$$
$$= \cancel{0} \cancel{- 4 (0)} + 3 \cdot (-1) = \underline{-3}$$

LE FUNZIONI INVERSE GONIOMETRICHE

$y = \operatorname{sen} x$

$y = \cos x$

$y = \operatorname{tg} x$

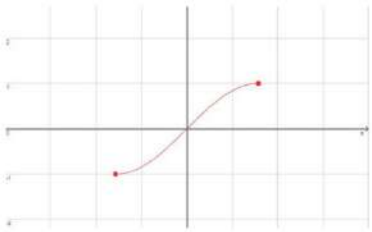
Queste funzioni associano a un valore angolare (un angolo) la lunghezza di un determinato segmento.

Le loro funzioni inverse associano la lunghezza di un determinato segmento ad un valore angolare (un angolo).

$$\Downarrow$$

Una funzione però, per essere invertita deve essere iniettiva e suriettiva, cioè biiettiva.

$y = \operatorname{sen} x$



$f : [-\frac{\pi}{2} ; \frac{\pi}{2}] \longrightarrow \mathbb{R}$

$\boxed{\text{INIETTIVA}}$

$\boxed{f : [-\frac{\pi}{2} ; \frac{\pi}{2}] \longrightarrow [-1 ; 1]}$

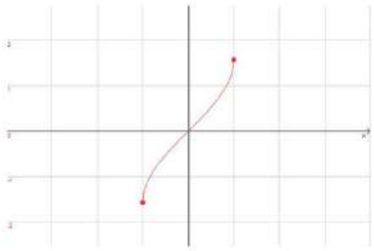
$\boxed{\text{INIETTIVA + SURIETTIVA = BIIETTIVA}}$

$\Downarrow$

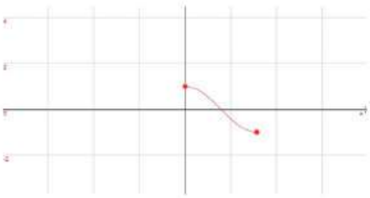
LA INVERTO

$$\boxed{y = \operatorname{sen}^{-1} x = \operatorname{arc\,sen}(x)}$$

$$\boxed{f^{-1}: [-1; 1] \longrightarrow \left[-\tfrac{\pi}{2}; \tfrac{\pi}{2}\right]}$$



$y = \cos x$



$f : [0, \pi] \longrightarrow \mathbb{R}$

$\boxed{\text{INIETTIVA}}$

$f : [0, \pi] \longrightarrow [-1, 1]$

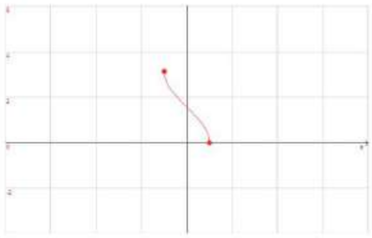
INIETTIVA SURIETTIVA = BIIETTIVA

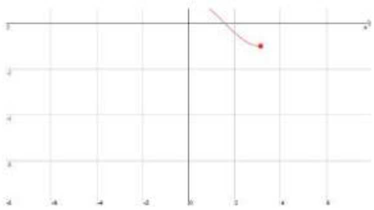
↓

LA INVERTO

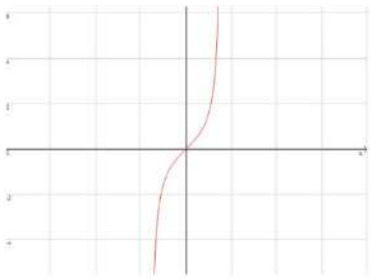
$y = \cos^{-1} x = \arccos x$

$\boxed{f^{-1} : [-1;1] \longrightarrow [0, \pi]}$





$y = \mathrm{tg}\, x$

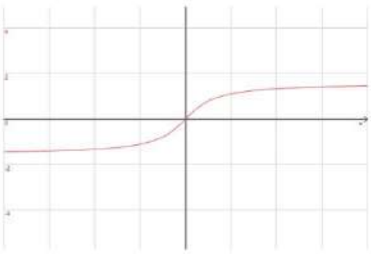


$f: \left]-\frac{\pi}{2}; \frac{\pi}{2}\right[\longrightarrow \mathbb{R}$

$\boxed{\text{INIETTIVA}}$

$\boxed{\text{E SURIETTIVA}}$

$$y = tg^{-1}x = arctg\, x$$

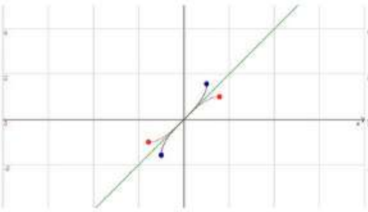


$$f^{-1}: \mathbb{R} \longrightarrow \left]-\frac{\pi}{2}; \frac{\pi}{2}\right[$$

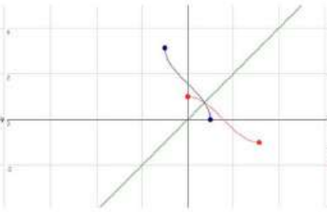
$$tg^{-1}x \neq \frac{1}{tg\, x}$$

↓

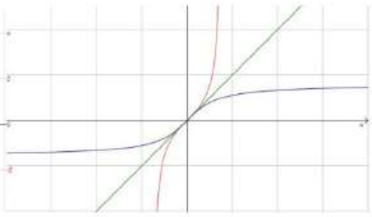
SIGNIFICA
FUNZIONE
INVERSA



Seno - Arco seno



Coseno - Arco coseno



Tangente - Arco tangente

Il grafico di una funzione quello della sua inversa risultano essere simmetrici rispetto alla bisettrice del 1° e del 3° quadrante.

$$\boxed{\begin{array}{l} sen^2\alpha + cos^2\alpha = 1 \\ \dfrac{sen\,\alpha}{cos\,\alpha} = tg\,\alpha \end{array}}$$

Relazioni fondamentali della goniometria

$$sen^2\alpha = 1 - cos^2\alpha \implies \boxed{sen\,\alpha = \pm\sqrt{1 - cos^2\alpha}}$$

$$cos^2\alpha = 1 - sen^2\alpha \implies \boxed{cos\,\alpha = \pm\sqrt{1 - sen^2\alpha}}$$

ESERCIZIO

$$\text{sen}\,\alpha = \frac{7}{25} \qquad 90° < \alpha < 180° \qquad \boxed{\begin{matrix}\cos\alpha = ?\\ \text{tg}\,\alpha = ?\end{matrix}}$$

$$\cos\alpha = \pm\sqrt{1-\text{sen}^2\alpha} = -\sqrt{1-\text{sen}^2\alpha}$$

$$\cos\alpha = -\sqrt{1-\left(\frac{7}{25}\right)^2} = -\sqrt{\frac{1}{1}-\frac{49}{625}} = -\sqrt{\frac{625-49}{625}}$$

$$\cos\alpha = -\sqrt{\frac{576}{625}} = -\frac{24}{25} \Longrightarrow \boxed{\cos\alpha = -\frac{24}{25}}$$

$$\text{tg}\,\alpha = \frac{\text{sen}\,\alpha}{\cos\alpha} = \frac{\frac{7}{25}}{-\frac{24}{25}} = \frac{7}{\cancel{25}}\cdot\left(-\frac{\cancel{25}}{24}\right) = -\frac{7}{24} \qquad \boxed{\text{tg}\,\alpha = -\frac{7}{24}}$$