Espressioni goniometriche

(1)

$$tg^2x - \frac{1}{\cos^2 x} =$$

$$= \frac{sen^2 x}{\cos^2 x} - \frac{1}{\cos^2 x} = \frac{sen^2 x - 1}{\cos^2 x} =$$

$$= \frac{sen^2 x - 1}{1 - sen^2 x} = \frac{\cancel{(sen^2 x - 1)}}{-1\cancel{(sen^2 x - 1)}} = \frac{1}{-1} = \underline{-1}$$

$$\boxed{\begin{array}{l} tg\,x = \frac{sen\,x}{\cos x} \\ sen^2 x + \cos^2 x = 1 \end{array}}$$

$$\boxed{\cos^2 x = 1 - sen^2 x}$$

2 $\cos^2 x - \operatorname{sen} x =$

$$\boxed{0 < x < \frac{\pi}{2}}$$

$$\operatorname{sen}^2 x + \cos^2 x = 1$$

$$\Downarrow$$

$$\boxed{\cos^2 x = 1 - \operatorname{sen}^2 x}$$

$= 1 - \operatorname{sen}^2 x - \operatorname{sen} x$

$$\operatorname{sen}^2 x = 1 - \cos^2 x$$

$$\boxed{\operatorname{sen} x = \pm\sqrt{1-\cos^2 x}}$$

3 $\sqrt{\underbrace{1 - \cos^4 x}_{A^2 - B^2}} = \sqrt{(1+\cos^2 x)\cdot(1-\cos^2 x)} =$

$(A+B)\,(A-B)$

$= \sqrt{1+\cos^2 x} \cdot \underbrace{\sqrt{1-\cos^2 x}}_{|\operatorname{sen} x|} = |\operatorname{sen} x|\sqrt{1+\cos^2 x} =$

$= |\operatorname{sen} x|\sqrt{1+1-\operatorname{sen}^2 x} = |\operatorname{sen} x|\sqrt{2-\operatorname{sen}^2 x}$

④ $\operatorname{tg} x + \dfrac{1}{\operatorname{tg} x} =$

$$\boxed{\begin{array}{l} \operatorname{sen}^2 x + \cos^2 x = 1 \\ \cos x = \pm\sqrt{1-\operatorname{sen}^2 x} \end{array}}$$

$$= \frac{\operatorname{tg}^2 x + 1}{\operatorname{tg} x} = \frac{\frac{\operatorname{sen}^2 x}{\cos^2 x} + \frac{1}{1}}{\frac{\operatorname{sen} x}{\cos x}} = \frac{\frac{\operatorname{sen}^2 x + \cos^2 x}{\cos^2 x}}{\frac{\operatorname{sen} x}{\cos x}} =$$

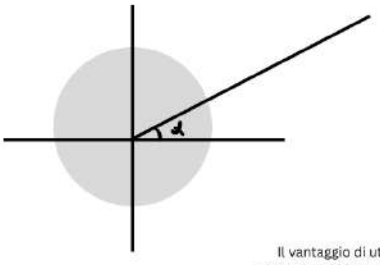
$$= \frac{\frac{1}{\cos^2 x}}{\frac{\operatorname{sen} x}{\cos x}} = \frac{1}{\cos^2 x} \cdot \frac{\cancel{\cos} x}{\operatorname{sen} x} = \frac{1}{\cos x \cdot \underline{\operatorname{sen} x}} = \frac{1}{\operatorname{sen} x \cdot \sqrt{1-\operatorname{sen}^2 x}}$$

5

$$\frac{tg\,x \cos^2 x}{sen^2 x \; cotg^2 x} =$$

$$= \frac{\frac{sen\,x}{\cancel{\cos x}} \cdot \cos^2 x}{\cancel{sen^2 x \cdot \frac{\cos^2 x}{\cancel{sen^2 x}}}} = \frac{sen\,x \,\cancel{\cos x}}{\cancel{\cos^2 x}} = \frac{sen\,x}{\cos x} = \frac{sen\,x}{\sqrt{1 - sen^2 x}}$$

ARCHI ASSOCIATI



Dall'angolo alfa io posso ricavare diverse tipologie di angolo

- angoli o archi che differiscono di 360°
- angoli o archi supplementari (la cui somma risulti 180°)
- angoli o archi che differiscono di 180°
- angoli o archi esplementari (la cui somma risulti 360°)
- angoli o archi opposti
- angoli o archi complementari (la cui somma risulti 90°)
- angoli o archi che differiscono di 90°
- angoli o archi che differiscono di 270° ***

Il vantaggio di utilizzare gli archi associati è il fatto che si possono conoscere rapidamente: seno, coseno, tangente, etc, etc di angoli non noti, sfruttando solo il fatto che sono archi associati di un angolo invece noto.

Archi che differiscono di 360°

$K \in \mathbb{Z} \quad K = \dots -2, -1, 0, 1, 2, \dots$

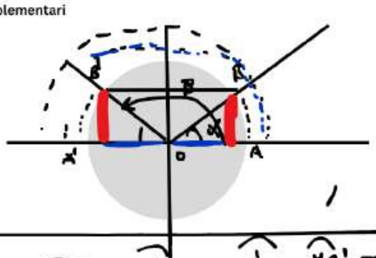
$$\operatorname{sen} \alpha = \operatorname{sen}(\alpha + K \cdot 360°) = \operatorname{sen}(\alpha + 2K\pi)$$

$$\cos \alpha = \cos(\alpha + K \cdot 360°) = \cos(\alpha + 2K\pi)$$

$$\operatorname{tg} \alpha = \operatorname{tg}(\alpha + K \cdot 360°) = \operatorname{tg}(\alpha + 2K\pi)$$

Due angoli che differiscono di 360° non modificano il valore del seno, del coseno e della tangente.

Angoli o archi supplementari

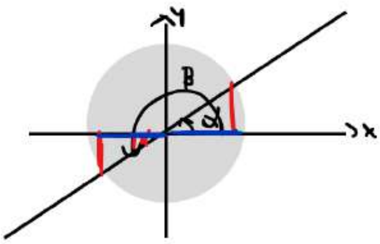


$\alpha + \beta = 180°$

$$\sin\alpha = \sin(180° - \alpha)$$
$$\cos\alpha = -\cos(180° - \alpha)$$
$$tg\,\alpha = -tg(180° - \alpha)$$

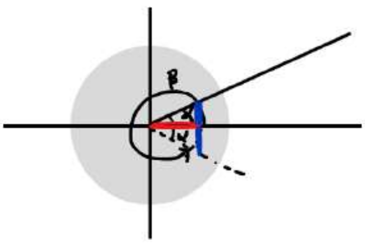
$$\widehat{AB} + \widehat{AB'} = \widehat{AA'} + \widehat{A'D} + \widehat{AA'} + \widehat{A'B'} =$$
$$= \underbrace{2\widehat{AA'}} + \widehat{A'B'} + \widehat{A'D} = \underbrace{360°} + 180°$$

Angoli o archi che differiscono di 180°



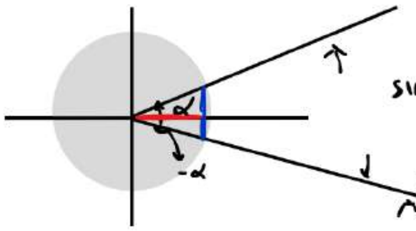
$$\operatorname{sen} \alpha = -\operatorname{sen}(\alpha + 180°)$$
$$\cos \alpha = -\cos(\alpha + 180°)$$
$$\operatorname{tg} \alpha = \operatorname{tg}(\alpha + 180°)$$

Angoli o archi esplementari



$$\operatorname{sen}\alpha = -\operatorname{sen}(360^\circ - \alpha)$$
$$\cos\alpha = \cos(360^\circ - \alpha)$$
$$\operatorname{tg}\alpha = -\operatorname{tg}(360^\circ - \alpha)$$

Angoli o archi opposti



$$\boxed{\cos\alpha = \cos(-\alpha)}$$

PARI $f(x)=f(-x)$
SIMMETRIA

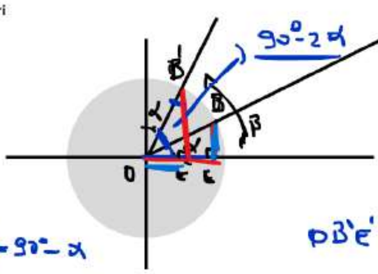
$$\boxed{\text{sen}\,\alpha = -\text{sen}(-\alpha)}$$

DISPARI
ANTISIMM. $f(x) = -f(-x)$

$$\boxed{\text{tg}\,\alpha = -\text{tg}(-\alpha)}$$

Archi o angoli complementari

$\boxed{\alpha + \beta = 90°}$



TRIANGOLO BED
RELATIVO AD α

TRIANGOLO B'DE'
RELATIVO A β

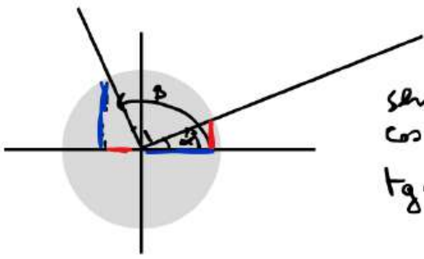
$\beta = 90° - 2\alpha + \alpha = 90° - \alpha$

$OB'E' = 180° - (90° + 90° - \alpha) =$
$= \alpha$

$\underbrace{O\hat{B}'E' = \beta\hat{O}E}$ $= \alpha \Rightarrow \underline{O\hat{B}E} = \underline{B'\hat{O}E} = \beta$

$\sin\alpha = \cos(90° - \alpha)$
$\cos\alpha = \sin(90° - \alpha)$
$\operatorname{tg}\alpha = \operatorname{cotg}(90° - \alpha)$

Archi o angoli che differiscono di 90°



$\operatorname{sen}\alpha = -\cos(90° + \alpha)$

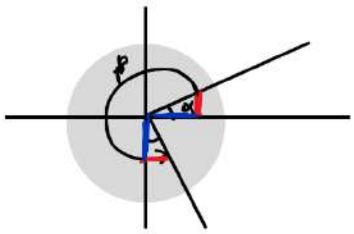
$\cos\alpha = \operatorname{sen}(90° + \alpha)$

$\operatorname{tg}\alpha = -\operatorname{cotg}(90° + \alpha)$

Archi che differiscono di 270°

$\alpha \neq \beta + 270^\circ$

$\beta = \alpha + 270^\circ$



$\operatorname{sen} \alpha = \cos(\alpha + 270^\circ)$

$\cos \alpha = -\operatorname{sen}(\alpha + 270^\circ)$

$\operatorname{tg} \alpha = -\operatorname{tg}(\alpha + 270^\circ)$

ESERCIZIO 1

$\boxed{\alpha = 42^\circ}$ $\boxed{\beta = 48^\circ}$ $\boxed{\beta = \underbrace{\frac{\pi}{2}}_{90} - \alpha}$

$\alpha + \beta = 90^\circ$ $\quad 42^\circ + 48^\circ = 90^\circ$

$\text{sen}\, 42^\circ = \cos 48^\circ$ $\quad \text{sen}\, \alpha = \cos(90^\circ - \alpha) = \cos\beta$

$\cos 42^\circ = \text{sen}\, 48^\circ$ $\quad \cos \alpha = \text{sen}\,(90^\circ - \alpha) = \text{sen}\,\beta$

$\text{tg}\, 42^\circ = \text{cotg}\, 48^\circ$ $\quad \text{tg}\, \alpha = \text{cotg}\,(90^\circ - \alpha) = \text{cotg}\,\beta$

$$\operatorname{cotg}(180^\circ-\alpha) + \frac{\cos(180^\circ-\alpha) - \operatorname{sen}(180^\circ+\alpha)}{\operatorname{tg}(360^\circ-\alpha)\cdot\cos(-\alpha)} + \operatorname{sen}(-\alpha)\operatorname{cotg}(180^\circ+\alpha) =$$

$$= -\operatorname{cotg}\alpha + \frac{-\cos\alpha - (-\operatorname{sen}\alpha)}{-\operatorname{tg}\alpha\,\cos\alpha} + \operatorname{sen}\alpha\cdot\operatorname{cotg}\alpha =$$

$$= -\operatorname{cotg}\alpha + \frac{\operatorname{sen}\alpha - \cos\alpha}{-\operatorname{tg}\alpha\cdot\cos\alpha} - \operatorname{sen}\alpha\cdot\operatorname{cotg}\alpha =$$

$$= -\frac{\cos\alpha}{\operatorname{sen}\alpha} + \frac{\operatorname{sen}\alpha - \cos\alpha}{-\frac{\operatorname{sen}\alpha}{\cancel{\cos\alpha}}\cdot\cancel{\cos\alpha}} - \cancel{\operatorname{sen}\alpha}\cdot\frac{\cos\alpha}{\cancel{\operatorname{sen}\alpha}}$$

$$-\frac{\cos\alpha}{\operatorname{sen}\alpha}+\frac{\operatorname{sen}\alpha-\cos\alpha}{-\operatorname{sen}\alpha}-\cos\alpha=$$

$$=\frac{-\cos\alpha-1(\operatorname{sen}\alpha-\cos\alpha)-\operatorname{sen}\alpha\cos\alpha}{\operatorname{sen}\alpha}=$$

$$=\frac{-\cancel{\cos\alpha}-\operatorname{sen}\alpha+\cancel{\cos\alpha}-\operatorname{sen}\alpha\cos\alpha}{\operatorname{sen}\alpha}=\frac{-\operatorname{sen}\alpha-\operatorname{sen}\alpha\cos\alpha}{\operatorname{sen}\alpha}$$

$$=\frac{-\cancel{\operatorname{sen}\alpha}(1+\cos\alpha)}{\cancel{\operatorname{sen}\alpha}}=\boxed{-(1+\cos\alpha)}$$