## LEZIONE 6

### Correzione delle espressioni goniometriche

$\operatorname{sen}^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \operatorname{sen}^2 x$

$$\operatorname{tg}^2 x - \frac{1}{\cos^2 x} =$$

$$= \left(\frac{\operatorname{sen} x}{\cos x}\right)^2 - \frac{1}{\cos^2 x} = \frac{\operatorname{sen}^2 x}{\cos^2 x} - \frac{1}{\cos^2 x} =$$

$$= \frac{\operatorname{sen}^2 x - 1}{\cos^2 x} = \frac{\operatorname{sen}^2 x - 1}{\cos^2 x} = \frac{\operatorname{sen}^2 x - 1}{1 - \operatorname{sen}^2 x} = \frac{\cancel{(\operatorname{sen}^2 x - 1)}}{-1\cancel{(\operatorname{sen}^2 x - 1)}} = \boxed{-1}$$

$\cos^2 x - \text{sen}\, x =$

$= 1 - \text{sen}^2 x - \text{sen}\, x$

$\text{sen}^2 x + \cos^2 x = 1$

$\cos^2 x = 1 - \text{sen}^2 x$

$$\sqrt{1-\cos^4 x} =$$

$$(A^2-B^2)=(A+B)(A-B)$$

$$= \sqrt{(1-\cos^2 x)(1+\cos^2 x)} =$$

$$\boxed{0 < x < \frac{\pi}{2}}$$

$$= \sqrt{\text{sen}^2 x\cdot(1+\cos^2 x)} =$$

$$= \sqrt{\text{sen}^2 x\,(1+1-\text{sen}^2 x)} = \sqrt{\underline{\text{sen}^2 x}\cdot\underline{(2-\text{sen}^2 x)}} =$$

$$= \sqrt{\text{sen}^2 x}\cdot\sqrt{2-\text{sen}^2 x} = \boxed{\text{sen}\,x\cdot\sqrt{2-\text{sen}^2 x}}$$

$$\frac{tg\,x}{1} + \frac{1}{tg\,x} = \frac{tg^2x + 1}{tg\,x} = \frac{\frac{sen^2x}{cos^2x} + \frac{1}{1}}{\frac{sen\,x}{cos\,x}} =$$

$$\boxed{0 < x < \frac{\pi}{2}}$$

$$= \frac{\frac{sen^2x + cos^2x}{cos^2x}}{\frac{sen\,x}{cos\,x}} = \frac{\frac{1}{cos^2x}}{\boxed{\frac{sen\,x}{cos\,x}}} = \frac{1}{\underset{cos\,x}{\cancel{cos^2x}}} \cdot \frac{\overset{1}{\cancel{cos\,x}}}{sen\,x} =$$

$$\boxed{tg\,x = \frac{sen\,x}{cos\,x}}$$

$$= \frac{1}{sen\,x\,cos\,x} = \boxed{\frac{1}{sen\,x\sqrt{1 - sen^2x}}}$$

$$sen^2x + cos^2x = 1$$

$$cos^2x = 1 - sen^2x$$

$$cos\,x = \pm\sqrt{1 - sen^2x}$$

$$\frac{tg\,x\ \cos^2 x}{sen^3 x\ cotg^2 x} = \frac{\frac{sen\,x}{\cancel{\cos x}} \cdot \cancel{\cos^2 x}\,\cos x}{\cancel{sen^3 x}\,sen\,x \cdot \frac{\cancel{\cos^2 x}}{\cancel{sen^2 x}}\,1} =$$

$$= \frac{s\cancel{en}x \cdot \cancel{\cos x}^{1}}{\cancel{sen}\,x \cdot \cancel{\cos^2 x}_{\cos x}} = \frac{1}{\cos x} = \boxed{\frac{1}{\sqrt{1 - sen^2 x}}}$$

$tg\,x = \frac{sen\,x}{\cos x}$

$cotg\,x = \frac{\cos x}{sen\,x}$

$\cos^2 x = 1 - sen^2 x$

$\cos x = \pm\sqrt{1 - sen^2 x}$

$$\boxed{\begin{array}{l} \text{sen}\,\alpha = \pm\sqrt{1-\cos^2\alpha} \\ \cos\alpha = \pm\sqrt{1-\text{sen}^2\alpha} \end{array}}$$

Ricaviamo adesso le due relazioni che mi permettono di ricavare il seno e il coseno in funzione della tangente.

$$\cos^2\alpha + \operatorname{sen}^2\alpha = 1$$

$$\frac{\cos^2\alpha}{\cos^2\alpha} + \frac{\operatorname{sen}^2\alpha}{\cos^2\alpha} = \frac{1}{\cos^2\alpha}$$

$$\cos^2\alpha\left(1 + \operatorname{tg}^2\alpha\right) = \frac{1}{\cancel{\cos^2\alpha}} \qquad \cancel{\cos^2\alpha}$$

$$\frac{\cos^2\alpha\left(1+\cancel{\operatorname{tg}^2\alpha}\right)}{\cancel{\left(1+\operatorname{tg}^2\alpha\right)}} = \frac{1}{1+\operatorname{tg}^2\alpha}$$

DIVIDIAMO PER $\cos^2\alpha$

$$\boxed{\cos^2\alpha \neq 0 \Rightarrow \cos\alpha \neq 0}$$

$$\boxed{\begin{gathered}\alpha \neq \frac{\pi}{2} + K\pi \\ 90^\circ + K \cdot 180^\circ\end{gathered}}$$

DIVIDO
TUTTO PER $1 + \operatorname{tg}^2\alpha$

$$\cos^2\alpha = \frac{1}{1+\operatorname{tg}^2\alpha} \Rightarrow \boxed{\cos\alpha = \pm\frac{1}{\sqrt{1+\operatorname{tg}^2\alpha}}}$$

$$sen^2\alpha = 1 - \cos^2\alpha \qquad \cos^2\alpha = \frac{1}{1+tg^2\alpha}$$

$$sen^2\alpha = \frac{1}{1} - \frac{1}{1+tg^2\alpha} = \frac{\cancel{1}+tg^2\alpha-\cancel{1}}{1+tg^2\alpha}$$

$$sen^2\alpha = \frac{tg^2\alpha}{1+tg^2\alpha} \Longrightarrow sen\,\alpha = \pm\sqrt{\frac{tg^2\alpha}{1+tg^2\alpha}} = \pm\frac{tg\,\alpha}{\sqrt{1+tg^2\alpha}}$$

$$\boxed{sen\,\alpha = \pm\frac{tg\,\alpha}{\sqrt{1+tg^2\alpha}}}$$

$$\boxed{\begin{aligned} \cos\alpha &= \pm\frac{1}{\sqrt{1+tg^2\alpha}} \\ sen\,\alpha &= \pm\frac{tg\,\alpha}{\sqrt{1+tg^2\alpha}} \end{aligned}}$$

Queste formule mi permettono di ricavare il seno ed il coseno di un arco o angolo nota la tangente di tale arco o angolo.

$tg\,\alpha = 2\sqrt{2}$ $\boxed{0^\circ < \alpha < 90^\circ}$ $\boxed{\begin{array}{l} sen\,\alpha = ? \\ \cos\alpha = ? \end{array}}$

$(\sqrt{2})^2 = 2$

$(2^{\frac{1}{2}})^2 = 2$

$$sen\,\alpha = \frac{tg\,\alpha}{\sqrt{1+tg^2\alpha}} = \frac{2\sqrt{2}}{\sqrt{1+(2\sqrt{2})^2}} = \frac{2\sqrt{2}}{\sqrt{1+8}}$$

$$sen\,\alpha = \frac{2\sqrt{2}}{\sqrt{9}} = \frac{2\sqrt{2}}{3} \Longrightarrow \boxed{sen\,\alpha = \frac{2\sqrt{2}}{3}} \qquad \boxed{\cos\alpha = \frac{1}{3}}$$

$$\cos\alpha = \frac{1}{\sqrt{1+tg^2\alpha}} = \frac{1}{\sqrt{1+(2\sqrt{2})^2}} = \frac{1}{\sqrt{1+8}} = \frac{1}{\sqrt{9}} = \frac{1}{3}$$

$$sen^2\alpha + \cos^2\alpha = \left(\frac{1}{3}\right)^2 + \left(\frac{2\sqrt{2}}{3}\right)^2 = \frac{1}{9} + \frac{8}{9} = \frac{\cancel{9}}{\cancel{9}} = 1$$

$\operatorname{tg}\alpha = -2\sqrt{6}$

$$\boxed{\begin{matrix} \operatorname{sen}\alpha = ? \\ \cos\alpha = ? \end{matrix}}$$

$$\operatorname{sen}\alpha = \pm \frac{\operatorname{tg}\alpha}{\sqrt{1+\operatorname{tg}^2\alpha}} = \pm \frac{(-2\sqrt{6})}{\sqrt{1+(-2\sqrt{6})^2}} = \pm \frac{(-2\sqrt{6})}{\sqrt{1+24}} = \pm \frac{(-2\sqrt{6})}{\sqrt{25}}$$

$$\operatorname{sen}\alpha = \pm \frac{(-2\sqrt{6})}{5}$$

$$0 < \alpha < \frac{\pi}{2} \vee \frac{\pi}{2} < \alpha < \pi \quad \oplus$$

$$\operatorname{sen}\alpha = -\frac{2\sqrt{6}}{5}$$

$$\cos\alpha = \pm\sqrt{1-\operatorname{sen}^2\alpha} = \pm\sqrt{1-\left(\frac{2\sqrt{6}}{5}\right)} =$$

$$\pi < \alpha \frac{3}{2}\pi \vee \frac{3}{2}\pi < \alpha < 2\pi \quad \ominus$$

$$= \pm\sqrt{1-\frac{24}{25}} = \pm\sqrt{\frac{25-24}{25}} = \pm\sqrt{\frac{1}{25}}$$

$$\operatorname{sen}\alpha = \frac{2\sqrt{6}}{5}$$

$$\cos\alpha = \pm\frac{1}{5}$$

$$\alpha\alpha < \frac{\pi}{2} \vee \frac{3}{2}\pi < \alpha < 2\pi$$

$$\cos\alpha = \frac{1}{5}$$

$$\frac{\pi}{2} < \alpha < \pi \vee \pi < \alpha < \frac{3}{2}\pi$$

$$\cos\alpha = -\frac{1}{5}$$

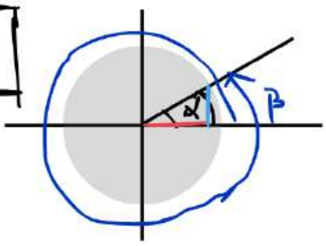
## ARCHI O ANGOLI ASSOCIATI

Quando parlo di archi o angoli associati intendo angoli che rispetto ad un altro angolo di riferimento possono avere una certa proprietà fissa e fra questi classifichiamo:

- archi o angoli che differiscono di un angolo giro (360°)
- archi o angoli supplementari
- archi o angoli che differiscono di un angolo piatto (180°)
- archi o angoli esplementari
- archi o angoli opposti
- archi o angoli complementari
- archi e angoli che differiscono di un angolo retto (90°)
- archi o angoli che differiscono di un angolo pari a 270°

## Archi che differiscono di un angolo di 360°

$$\boxed{\begin{aligned}\beta &= \alpha + 2\pi \\ &\alpha + 360^\circ\end{aligned}}$$



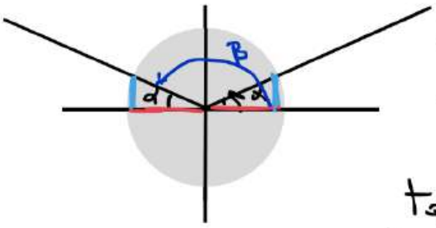
$$\text{sen}\,\alpha = \text{sen}(\alpha + 360^\circ)$$

$$\cos\alpha = \cos(\alpha + 360^\circ)$$

$$\text{tg}\,\alpha = \text{tg}(\alpha + 360^\circ)$$

## Archi supplementari

$$\boxed{\begin{array}{l}\alpha + \beta = \pi \\ \alpha + \beta = 180°\end{array}}$$



$$\text{sen}\,\alpha = \text{sen}\,\beta$$

$$\boxed{\text{sen}\,\alpha = \text{sen}\,(180° - \alpha)}$$

$$\cos\alpha = -\cos\beta$$

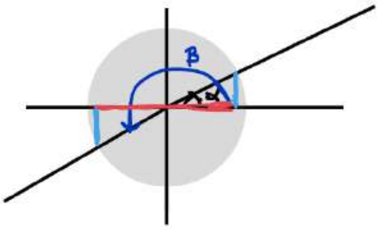
$$\boxed{\cos\alpha = -\cos(180° - \alpha)}$$

$$\text{tg}\,\alpha = -\text{tg}\,\beta$$

$$\boxed{\text{tg}\,\alpha = -\text{tg}(180° - \alpha)}$$

$$\boxed{\begin{gathered}\beta = \alpha + 180^\circ \\ \beta = \alpha + \pi\end{gathered}}$$

## Archi che differiscono di 180°



$$\operatorname{sen}\alpha = -\operatorname{sen}\beta$$

$$\boxed{\operatorname{Sen}\alpha = -\operatorname{sen}(\alpha+180^\circ)}$$

$$\cos\alpha = -\cos\beta$$

$$\boxed{\cos\alpha = -\cos(\alpha+180^\circ)}$$

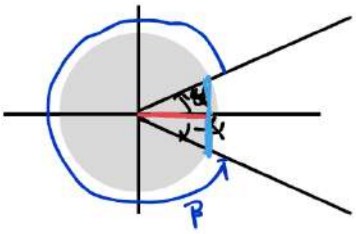
$$\operatorname{tg}\alpha = \operatorname{tg}\beta$$

$$\boxed{\operatorname{tg}\alpha = \operatorname{Tg}(\alpha+180^\circ)}$$

## Archi esplementari

$$\boxed{\begin{array}{l}\alpha + \beta = 2\pi \\ \alpha + \beta = 360^\circ\end{array}}$$

$$\beta = 360^\circ - \alpha$$



$$\text{sen}\,\alpha = -\,\text{sen}\,\beta$$

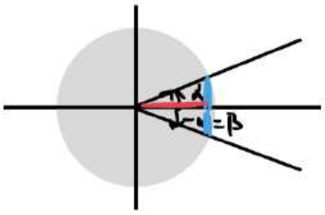
$$\boxed{\text{sen}\,\alpha = -\text{sen}(360^\circ - \alpha)}$$

$$\text{Cos}\,\alpha = \cos\beta$$

$$\boxed{\cos\alpha = \cos(360^\circ - \alpha)}$$

$$\boxed{\text{tg}\,\alpha = -\,\text{tg}(360^\circ - \alpha)}$$

### Archi opposti



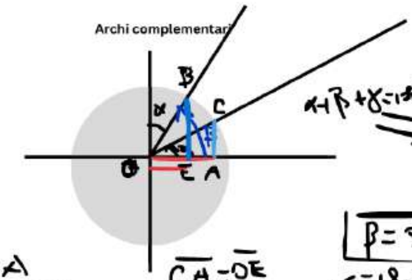
$$\boxed{\operatorname{sen}\alpha = -\operatorname{sen}(-\alpha)}$$

$$\boxed{\cos\alpha = \cos(-\alpha)}$$

$$\boxed{\operatorname{tg}\alpha = -\operatorname{tg}(-\alpha)}$$

$$\boxed{\begin{aligned}\alpha+\beta&=90^\circ\\ \alpha+\beta&=\frac{\pi}{2}\end{aligned}}$$

Archi complementari



$\overline{CA}=\overline{OE}$

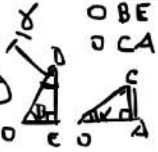
$\overline{BE}=\overline{OA}$

TRIANGOLI

OBE

OCA

$\alpha+\beta+\gamma=180^\circ$



$$\boxed{\beta=90^\circ-\alpha}$$

$\gamma=180^\circ-(90^\circ-\alpha+90^\circ)=\alpha$

OBE E OCA

$$\boxed{\begin{aligned}\operatorname{sen}\alpha&=\cos(90^\circ-\alpha)\\ \cos\alpha&=\operatorname{sen}(90^\circ-\alpha)\end{aligned}}$$

$$\boxed{\operatorname{tg}\alpha=\operatorname{cotg}(90^\circ-\alpha)}$$