LEZIONE 7

Correzione espressioni goniometriche

ES 1

$$\cos(90^\circ-\alpha)+2\,\mathrm{sen}(360^\circ-\alpha)+\mathrm{sen}(180^\circ-\alpha)=$$

$$=\underline{\mathrm{sen}\,\alpha}+2(-\mathrm{sen}\,\alpha)\underline{+\mathrm{sen}\,\alpha}=$$

$$=2\,\mathrm{sen}\,\alpha-2\,\mathrm{sen}\,\alpha=0$$

ES 2

$$tg(180°+\alpha)\,sen(90°-\alpha)+2\,sen(180°-\alpha)=$$

$$= tg\,\alpha\cdot\cos\alpha+2\,sen\,\alpha=$$

$$=\frac{sen\,\alpha}{\cancel{\cos\alpha}}\cdot\cancel{\cos\alpha}+2\,sen\,\alpha=$$

$$sen\,\alpha+2\,sen\,\alpha=\underline{3\,sen\,\alpha}$$

ES. 3

$$\cos(270^\circ - \alpha) + tg(360^\circ - \alpha)\cos(180^\circ - \alpha)$$

$$sen\,\alpha = -\cos(270^\circ - \alpha)$$
$$\cos\alpha = -sen(270^\circ - \alpha)$$

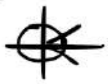
$$-sen\,\alpha - tg\,\alpha \cdot (-\cos\alpha) =$$
$$= -sen\,\alpha - \frac{sen\,\alpha}{\cancel{\cos\alpha}} \cdot (-\cancel{\cos\alpha}) = -sen\,\alpha + sen\,\alpha = \underline{0}$$

ES. 4

$$\cos\alpha \cdot \cos(360^\circ-\alpha) - \cos(90^\circ-\alpha)\,\text{sen}(360^\circ-\alpha) =$$

$$= \cos\alpha \cdot \cos\alpha - \text{sen}\,\alpha \cdot (-\text{sen}\,\alpha) =$$

$$= \cos^2\alpha + \text{sen}^2\alpha = 1$$



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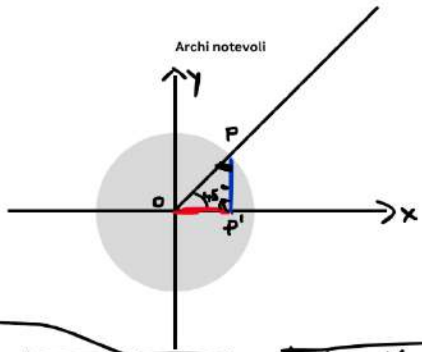


$\theta = 45°$

$\overline{OP} = 1$

$\overline{OP'} = \cos 45°$

$\overline{PP'} = \text{sen}\, 45°$

$P\hat{P'}O = 90°$

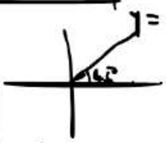
$P\hat{O}P' = 45°$

$P'\hat{P}O = 180° - 135° = 45°$

$\overline{OP'} = \overline{PP'} \Rightarrow \boxed{\cos 45° = \text{sen}\, 45°}$ $\boxed{P\hat{O}P = P'\hat{P}O = 45°} \Rightarrow$ TRIANGOLO ISOSCELE

$\boxed{\operatorname{sen} 45^\circ = \cos 45^\circ}$

$y = x$ $\boxed{\operatorname{tg} 45^\circ = \frac{\cancel{\operatorname{sen} 45^\circ}}{\cancel{\cos 45^\circ}} = 1}$ $\sqrt{2}$



$\boxed{m = \operatorname{tg} \theta}$

$\boxed{\operatorname{tg} 45^\circ = 1}$

$$\cos 45^\circ = \frac{1}{\sqrt{1 + \operatorname{tg}^2 45^\circ}} = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\operatorname{sen} \theta = \pm \frac{\operatorname{tg} \theta}{\sqrt{1 + \operatorname{tg}^2 \theta}}$$

$$\cos \theta = \pm \frac{1}{\sqrt{1 + \operatorname{tg}^2 \theta}}$$

$$\begin{cases} \cos 45^\circ = \frac{\sqrt{2}}{2} = \\ \operatorname{sen} 45^\circ = \frac{\sqrt{2}}{2} \end{cases}$$

$\underline{\operatorname{tg} 45^\circ = 1}$ $\underline{\operatorname{cotg} 45^\circ = 1}$

$$\overline{PP'} = \frac{\overline{PQ}}{2}$$

$$\Downarrow$$

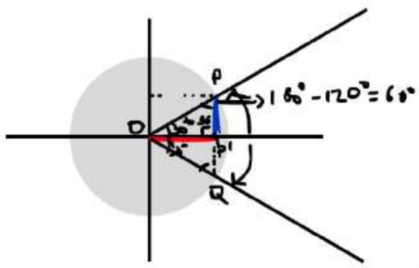
$$\overline{PP'} = \frac{1}{2}$$

$$\Downarrow$$

$$\boxed{\text{Sen}\,30° = \frac{1}{2}}$$

$$\overline{OP} = \overline{OQ} = 1$$

$$\Downarrow \overline{PQ} = 1$$



$\overline{OP'} = \text{Cos}\,30°$

$\overline{PP'} = \text{sen}\,30°$

$\boxed{\overline{OP} = 1}$

$Q\hat{O}P = 60°$

$O\hat{P}Q = 60°$

$O\hat{Q}P = 180° - 120° = 60°$

$\Downarrow$

TRIANGOLO EQUILATERO

(EQUIANGOLO)

$\Leftarrow$ $\overline{OP} = \overline{PQ} = \overline{OQ}$

$$\operatorname{sen} 30^\circ = \frac{1}{2} \qquad \cos^2\alpha = 1 - \operatorname{sen}^2\alpha$$

$$\Downarrow$$

$$\cos 30^\circ = \sqrt{1 - \operatorname{sen}^2 30^\circ} = \qquad \cos\alpha = \pm\sqrt{1-\operatorname{sen}^2\alpha}$$

$$= \sqrt{1 - \left(\frac{1}{2}\right)^2} = \sqrt{\frac{1}{1} - \frac{1}{4}} = \sqrt{\frac{4-1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2} \simeq 0,866$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\operatorname{sen} 30^\circ = \frac{1}{2}$$

$$\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}}$$

$$\operatorname{tg} 30^\circ = \frac{\operatorname{sen} 30^\circ}{\cos 30^\circ} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$$

RAZIONALIZZ.

$$\boxed{\operatorname{tg} 30^\circ = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}}$$

$$\begin{cases} \cos 60^\circ = \cos(90^\circ - 30^\circ) = \operatorname{sen} 30^\circ = \frac{1}{2} \\ \operatorname{sen} 60^\circ = \operatorname{sen}(90^\circ - 30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2} \end{cases}$$

$$\boxed{\operatorname{tg} 60^\circ = \frac{\operatorname{sen} 60^\circ}{\cos 60^\circ} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{1} = \sqrt{3} = \operatorname{cotg} 30^\circ}$$

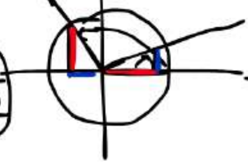
Espressioni goniometriche di riepilogo

$$\sqrt{3}\,\text{sen}(-30^\circ) - 2\cos 240^\circ\,\text{sen}\,120^\circ - \frac{1}{3}\,\text{tg}\,300^\circ =$$

$$= \sqrt{3}\left[-\text{sen}\,30^\circ\right] - 2\cos(180^\circ + 60^\circ) \cdot \underline{\text{sen}(90^\circ + 30^\circ)} - \frac{1}{3}\,\text{tg}(360^\circ - 60^\circ) =$$

$$= \sqrt{3}\left(-\frac{1}{2}\right) - 2\left[-\cos 60^\circ\right] \cdot \cos(30^\circ) - \frac{1}{3}\left[-\text{tg}\,60^\circ\right] =$$

$\text{sen}(180^\circ - 60^\circ)$



$$= -\frac{\sqrt{3}}{2} + \cancel{2} \cdot \frac{1}{\cancel{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{3} \cdot \sqrt{3} =$$

$$= \cancel{-\frac{\sqrt{3}}{2}} + \cancel{\frac{\sqrt{3}}{2}} + \frac{\sqrt{3}}{3}$$

$$\cos\frac{\pi}{6}\,tg\left(-\frac{\pi}{4}\right)+2\left(sen\,\frac{3}{4}\pi\right)^2+\cancel{\cos\frac{\pi}{2}\,sen\,\frac{9}{4}\pi}=$$

$$=\frac{\sqrt{3}}{2}\cdot\left[-tg\,\frac{\pi}{4}\right]+2\left[sen(30^\circ+45^\circ)\right]^2=$$

$$=\frac{\sqrt{3}}{2}(-1)+2\left[\cos 45^\circ\right]^2=$$

$$=-\frac{\sqrt{3}}{2}+2\left[\frac{\sqrt{2}}{2}\right]^2=$$

$$=-\frac{\sqrt{3}}{2}+\cancel{2}^{1}\cdot\frac{2}{\cancel{4}_2}=\boxed{-\frac{\sqrt{3}}{2}+1}$$

$\frac{45^\circ \; 9}{4\cdot 5}$

$\frac{\pi}{6}=30^\circ$

$\frac{\pi}{4}=45^\circ$

$\frac{3}{4}\pi=135^\circ$

$\frac{\pi}{2}=90^\circ$

$\frac{9\pi}{4}=405^\circ$

$$\sqrt{6}\,\operatorname{sen}\frac{3}{2}\pi + \cos\left(-\frac{\pi}{6}\right)\cos\frac{5}{4}\pi + \sqrt{2}\,\operatorname{tg}\frac{2}{3}\pi =$$

$$\frac{3}{2}\pi = 270^\circ$$

$$= \sqrt{6}\,(-1) + \cos\frac{\pi}{6}\cos(180^\circ + 45^\circ) + \sqrt{2}\,\operatorname{tg}(90^\circ + 30^\circ) =$$

$$\frac{\pi}{6} = 30^\circ$$

$$= -\sqrt{6} + \frac{\sqrt{3}}{2}\cdot\left[-\cos 45^\circ\right] + \sqrt{2}\left[-\operatorname{cotg} 30^\circ\right] =$$

$$\frac{5}{4}\pi = 225^\circ$$

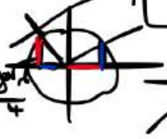
$$= -\sqrt{6} + \frac{\sqrt{3}}{2}\cdot\left(-\frac{\sqrt{2}}{2}\right) - \sqrt{2}\cdot\sqrt{3} =$$

$$\frac{2}{3}\pi = 120^\circ$$

$$\boxed{-\frac{9\sqrt{6}}{4}}$$

$$= \underline{-\sqrt{6}} - \frac{\sqrt{6}}{4} \underline{-\sqrt{6}} = -2\sqrt{6} - \frac{\sqrt{6}}{4}$$

$$= \frac{-8\sqrt{6} - \sqrt{6}}{4} = -\frac{9\sqrt{6}}{4}$$



$$\frac{\cos(90^\circ + 30^\circ)}{\operatorname{sen}(90^\circ + 30^\circ)}$$

$$\operatorname{sen}^2(180^\circ - \alpha) - 1 + \cos(360^\circ - \alpha) + \cos^2(-\alpha) =$$

$$= \underline{\operatorname{sen}^2 \alpha} - 1 + \cos\alpha + \underline{\cos^2 \alpha} =$$

$$= \cancel{1} - \cancel{1} + \cos\alpha$$

$$= \cos\alpha$$