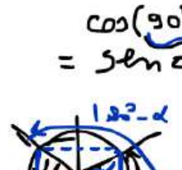
LEZIONE 7

Espressioni goniometriche con archi associati

$$\cos(90° - \alpha) + 2\,\text{sen}(360° - \alpha) + \text{sen}(180° - \alpha) =$$

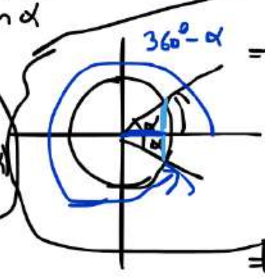
$$= \text{sen}\,\alpha - 2\,\text{sen}\,\alpha + \text{sen}\,\alpha$$



$\text{sen}(180° - \alpha) = \text{sen}\,\alpha$

$\cos(90° - \alpha)$

$\text{sen}\,\alpha$



$\text{sen}(360° - \alpha)$

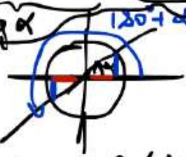
$= -\text{sen}\,\alpha$

$= \boxed{0}$ $-\text{sen}\,\alpha + \text{sen}\,\alpha$

$$\underbrace{tg(180^\circ+\alpha)}_{tg\,\alpha}\ \underbrace{sen(90^\circ-\alpha)}_{\cos\alpha} + 2\,\underbrace{sen(180^\circ-\alpha)}_{sen\,\alpha} = tg\,\alpha\cdot\cos\alpha + 2\,sen\,\alpha =$$

$$= \frac{sen\,\alpha}{\cancel{\cos\alpha}}\cdot\cancel{\cos\alpha} + 2\,sen\,\alpha =$$

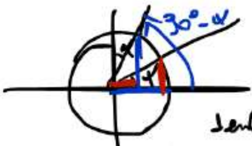
$$= sen\,\alpha + 2\,sen\,\alpha = \boxed{3\,sen\,\alpha}$$



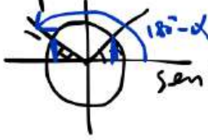
$$sen\,\alpha = -\,sen(180^\circ+\alpha)$$

$$\cos\alpha = -\,\cos(180^\circ+\alpha)$$

$$tg(180^\circ+\alpha) = \frac{sen(180^\circ+\alpha)}{\cos(180^\circ+\alpha)} = \frac{-\,sen\,\alpha}{-\,\cos\alpha} = tg\,\alpha$$

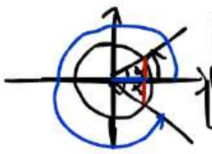


$$sen(90^\circ-\alpha) = \cos\alpha$$



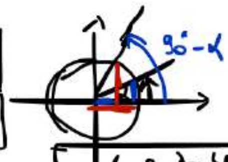
$$sen(180^\circ-\alpha) = sen\,\alpha$$

$\cos\alpha \cdot \cos(360° - \alpha) - \cos(90° - \alpha)\ \text{sen}(360° - \alpha) =$



$\boxed{\cos(360° - \alpha) = \cos\alpha}$

$\boxed{\text{sen}(360° - \alpha) = -\text{sen}\,\alpha}$

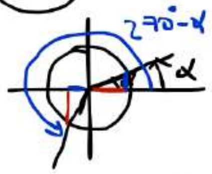


$\boxed{\cos(90° - \alpha) = \text{sen}\,\alpha}$

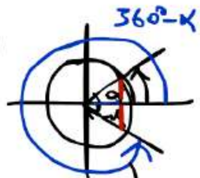
$= \cos\alpha \cdot \cos\alpha - \text{sen}\,\alpha \cdot (-\text{sen}\,\alpha) =$

$= \cos^2\alpha + \text{sen}^2\alpha = 1$

$$\underbrace{\cos(270^\circ-\alpha)}+\underline{tg(360^\circ-\alpha)}\cdot\cos(180^\circ-\alpha)=$$



$$\operatorname{sen}\alpha=-\cos(270^\circ-\alpha)$$
$$\cos\alpha=-\operatorname{sen}(270^\circ-\alpha)$$



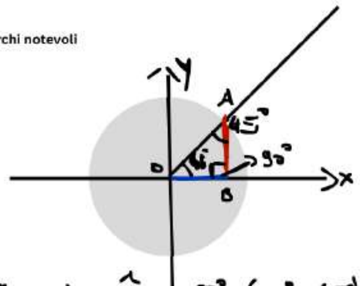
$$=-\operatorname{sen}\alpha+(-tg\,\alpha)\cdot(-\cos\alpha)=$$
$$=-\operatorname{sen}\alpha+tg\,\alpha\cos\alpha=-\operatorname{sen}\alpha+\frac{\operatorname{sen}\alpha\,\cancel{\cos\alpha}}{\cancel{\cos\alpha}}$$
$$-\operatorname{sen}\alpha+\operatorname{sen}\alpha=0$$

$$\cos(180^\circ-\alpha)=-\cos\alpha$$

$$\cos(360^\circ-\alpha)=+\cos\alpha$$
$$\operatorname{sen}(360^\circ-\alpha)=-\operatorname{sen}\alpha$$
$$\boxed{tg(360^\circ-\alpha)=-\frac{\operatorname{sen}\alpha}{\cos\alpha}=-tg\,\alpha}$$

$\boxed{\theta = 45^\circ = \frac{\pi}{4}}$

Archi notevoli



$\overline{OA} = 1$

$\overline{AB} = \text{sen}\,45^\circ$

$\overline{OB} = \cos 45^\circ$

Il triangolo OAB è rettangolo in B

$A\hat{B}O = 90^\circ \quad \underline{A\hat{O}B = 45^\circ} \implies \underline{B\hat{A}O = 180^\circ - (90^\circ + 45^\circ) = 180^\circ - 135^\circ = 45^\circ}$

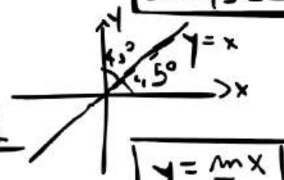
ABO È ISOSCELE

$A\hat{O}B = B\hat{A}O = 45^\circ \implies \overline{OB} = \overline{AB} \implies \boxed{\cos 45^\circ = \text{sen}\,45^\circ \;!!!}$

$$\mathrm{tg}\,45^\circ = \frac{\mathrm{sen}\,\cancel{45^\circ}}{\cos\cancel{45^\circ}} = 1$$

$$\mathrm{tg}\,45^\circ = 1$$

$$\boxed{\mathrm{sen}\,45^\circ = \cos 45^\circ}$$



$m = \mathrm{tg}\,\theta$

$$\boxed{m = \mathrm{tg}\,45^\circ = 1}$$

$$\boxed{y = \underline{m}x}$$

coeff. angol.

$$\boxed{\mathrm{sen}\,\theta = \pm\frac{\mathrm{tg}\,\theta}{\sqrt{1+\mathrm{tg}^2\theta}}}$$

$$\boxed{\cos\theta = \pm\frac{1}{\sqrt{1+\mathrm{tg}^2\theta}}}$$

$$\operatorname{sen}\theta = \frac{tg\theta}{\sqrt{1+tg^2\theta}} \Rightarrow \operatorname{sen}45^\circ = \frac{tg45^\circ}{\sqrt{1+tg^2 45^\circ}} = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}}$$

$$\operatorname{sen}45^\circ = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\boxed{\begin{array}{l} \operatorname{sen}45^\circ = \frac{\sqrt{2}}{2} \simeq 0{,}707 \\ \cos 45^\circ = \frac{\sqrt{2}}{2} \simeq 0{,}707 \\ tg\,45^\circ = 1 \\ \quad cotg\,45^\circ = 1 \end{array}}$$

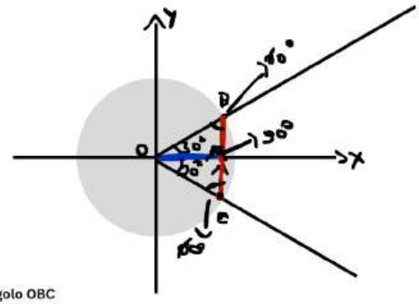
$\theta = 30°$

$\overline{BC} = 1$

$\Downarrow$

$\overline{AB} = \frac{\overline{BC}}{2} = \frac{1}{2}$

$$\boxed{\overline{AB} = sen 30° = \frac{1}{2}}$$



$\overline{AB} = sen 30°$

$\overline{OA} = cos 30°$

$\overline{OB} = 1$

TRIANGOLO OAB

$O\hat{B}A = O\hat{B}C =$

$180° - (30° + 30°) = 180° - 120°$

$= 60°$

Si è formato il triangolo OBC

$B\hat{O}C = 60°$ $O\hat{B}C = 60°$ $O\hat{C}B = 60° \Longrightarrow \boxed{\overline{OB} = \overline{BC} = \overline{OC} = 1}$

$$\text{sen}^2\theta + \cos^2\theta = 1 \implies \cos^2\theta = 1 - \text{sen}^2\theta \implies \boxed{\cos\theta = \pm\sqrt{1-\text{sen}^2\theta}}$$

$$\cos 30° = \sqrt{1 - \text{sen}^2 30°} = \sqrt{1 - \left(\frac{1}{2}\right)^2} = \sqrt{\frac{1}{1} - \frac{1}{4}} = \sqrt{\frac{4-1}{4}} = \sqrt{\frac{3}{4}}$$

$$\cos 30° = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2} \implies \cos 30° = \frac{\sqrt{3}}{2} \simeq 0{,}866$$

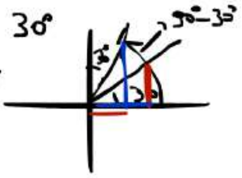
$$\text{sen}30° = \frac{1}{2} \qquad \text{tg}\,30° = \frac{\text{sen}\,30°}{\cos 30°} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\cos 30° = \frac{\sqrt{3}}{2}$$

$$\text{tg}\,30° = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3} \implies \text{tg}\,30° = \frac{\sqrt{3}}{3} \simeq 0{,}577$$

$$\boxed{\text{ctg}\,30° = \frac{\cos 30°}{\text{sen}\,30°} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{1} = \sqrt{3} \simeq 1{,}732}$$

$\theta = 60°$ COMPLEMENTARE DI 30°



$\text{sen}\, 60° = \text{sen}(90° - 30°) = \cos 30° = \frac{\sqrt{3}}{2}$

$\boxed{\text{sen}\, 60° = \frac{\sqrt{3}}{2}}$

$\cos 60° = \cos(90° - 30°) = \text{sen}\, 30° = \frac{1}{2}$

$\boxed{\cos 60° = \frac{1}{2}}$

$$\boxed{\begin{array}{l} \text{tg}\, 60° = \text{cotg}\, 30° = \sqrt{3} \\ \text{cotg}\, 60° = \text{tg}\, 30° = \frac{\sqrt{3}}{3} \end{array}}$$

$$\sqrt{6}\,\text{sen}\,\frac{3}{2}\pi + \cos\left(-\frac{\pi}{6}\right)\,\cos\frac{5}{4}\pi + \sqrt{2}\,\text{tg}\,\frac{2}{3}\pi =$$

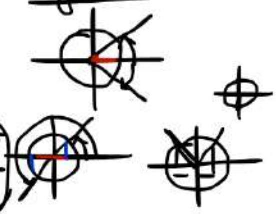
$$= \sqrt{6}\,\text{sen}\,270° + \cos(-30°)\cdot\cos 225° + \sqrt{2}\,\text{tg}(120°) =$$

$$= \sqrt{6}(-1) + \cos 30°\cdot\cos(180°+45°) + \sqrt{2}\,\text{tg}(180°-60°) =$$

$$= -\sqrt{6} + \left(\frac{\sqrt{3}}{2}\right)\cdot(-\cos 45°) - \sqrt{2}\,\text{tg}\,60° =$$

$$= -\sqrt{6} + \left(\frac{\sqrt{3}}{2}\right)\cdot\left(-\frac{\sqrt{2}}{2}\right) - \sqrt{2}\cdot\sqrt{3} =$$

$$= -\frac{\sqrt{6}}{1} - \frac{\sqrt{6}}{4} - \frac{\sqrt{6}}{1} = \frac{-4\sqrt{6} - \sqrt{6} - 4\sqrt{6}}{4} = -\frac{9\sqrt{6}}{4}$$



$\frac{3}{\cancel{2}_1}\cdot \cancel{180°}^{90°}$

$\frac{180°}{6} = 30°$

$\frac{5}{\cancel{4}_1}\cdot \cancel{180°}^{45°} = 225°$

$\frac{2}{\cancel{3}_1}\cdot \cancel{180°}^{60°}$