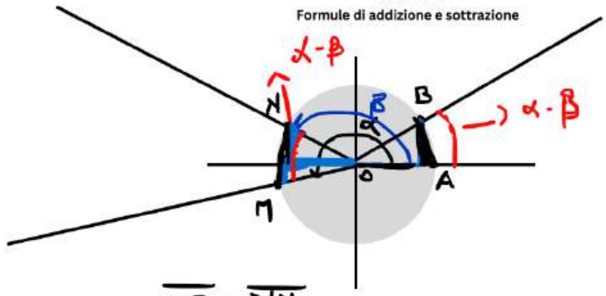
LEZIONE 8

Formule di addizione e sottrazione



$\overline{AB} = \overline{NM}$

$\alpha - \beta$

$A\,(1;0)$

$B\,(\cos(\alpha-\beta), \operatorname{sen}(\alpha-\beta))$

$N\,(\cos\beta, \operatorname{sen}\beta)$

$M\,(\cos\alpha, \operatorname{sen}\alpha)$

$$\overline{AB} = \underline{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}} \qquad \overline{MN} = \underline{\sqrt{(x_N - x_M)^2 + (y_N - y_M)^2}}$$

$$\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} = \sqrt{(x_N - x_M)^2 + (y_N - y_M)^2}$$

$$\sqrt{[\cos(\alpha - \beta) - 1]^2 + [\operatorname{sen}(\alpha - \beta) - 0]^2} = \sqrt{(\cos\beta - \cos\alpha)^2 + (\operatorname{sen}\beta - \operatorname{sen}\alpha)^2}$$

$$[\cos(\alpha - \beta) - 1]^2 + \operatorname{sen}^2(\alpha - \beta) = (\cos\beta - \cos\alpha)^2 + (\operatorname{sen}\beta - \operatorname{sen}\alpha)^2$$

$$\cos^2(\alpha - \beta) + 1 - 2\cos(\alpha - \beta) + \operatorname{sen}^2(\alpha - \beta) = \cos^2\beta + \cos^2\alpha - 2\cos\beta\cos\alpha + \operatorname{sen}^2\beta + \operatorname{sen}^2\alpha - 2\operatorname{sen}\beta\operatorname{sen}\alpha$$

$$\underline{\cos^2(\alpha-\beta)} + 1 - 2\cos(\alpha-\beta) \underline{+ \operatorname{sen}^2(\alpha-\beta)} = \underline{\cos^2\beta} + \underline{\cos^2\alpha} - 2\cos\beta\cos\alpha + \underline{\operatorname{sen}^2\beta} + \underline{\operatorname{sen}^2\alpha} - 2\operatorname{sen}\beta\operatorname{sen}\alpha$$

$$\cancel{1+}1 - 2\cos(\alpha-\beta) = \cancel{1+}1 - 2\cos\beta\cos\alpha - 2\operatorname{sen}\beta\operatorname{sen}\alpha$$

$$\frac{\cancel{-2}\cos(\alpha-\beta)}{\cancel{-2}} = \frac{\cancel{-2}\cos\beta\cos\alpha \cancel{-2}\operatorname{sen}\beta\operatorname{sen}\alpha}{\cancel{-2}}$$

$$\boxed{\cos(\alpha-\beta) = \cos\beta\cos\alpha + \operatorname{sen}\beta\operatorname{sen}\alpha}$$

$$\cos^2\alpha + \operatorname{sen}^2\alpha = 1$$

$$\cos^2\beta + \operatorname{sen}^2\beta = 1$$

$$\cos^2(\alpha-\beta) + \operatorname{sen}^2(\alpha-\beta) = 1$$

$$\boxed{\cos(\alpha-\beta)=\cos\beta\cos\alpha+\operatorname{sen}\beta\operatorname{sen}\alpha}$$

FORMULA DI SOTTRAZIONE DEL COSENO

Prendendo al posto dell'angolo beta il suo opposto, cioè -beta

$$\cos(\alpha-(-\beta))=\cos(-\beta)\cos\alpha+\operatorname{sen}(-\beta)\operatorname{sen}\alpha$$

$$\boxed{\cos(\alpha+\beta)=\cos\beta\cos\alpha-\operatorname{sen}\beta\operatorname{sen}\alpha}$$

$\cos(-\beta)=\cos\beta$

$\operatorname{sen}(-\beta)=-\operatorname{sen}\beta$

FORMULA DI ADDIZIONE DEL COSENO

$$\cos(90^\circ - \alpha - \beta) = \cos\beta \cos(90^\circ - \alpha) + \operatorname{sen}\beta \operatorname{sen}(90^\circ - \alpha)$$

$$\cos(90^\circ - (\alpha + \beta)) = \operatorname{sen}\alpha \cos\beta + \cos\alpha \operatorname{sen}\beta$$

$$\boxed{\operatorname{sen}(\alpha + \beta) = \operatorname{sen}\alpha \cos\beta + \cos\alpha \operatorname{sen}\beta}$$

FORMULA DI ADDIZIONE DEL SENO

$$\operatorname{sen}(\alpha - \beta) = \operatorname{sen}\alpha \cos(-\beta) + \cos\alpha \operatorname{sen}(-\beta)$$

$$\boxed{\operatorname{sen}(\alpha - \beta) = \operatorname{sen}\alpha \cos\beta - \cos\alpha \operatorname{sen}\beta}$$

FORMULA DI SOTTRAZIONE DEL SENO

$$\operatorname{sen}\alpha = \cos(\underbrace{90^\circ - \alpha}_{\beta})$$

AL POSTO DI $\alpha \longrightarrow 90^\circ - \alpha$ NELLA FORMULA DI SOTTRAZIONE

$$\boxed{\begin{aligned}\cos(\alpha-\beta)&=\cos\alpha\cos\beta+\operatorname{sen}\alpha\operatorname{sen}\beta\\ \cos(\alpha+\beta)&=\cos\alpha\cos\beta-\operatorname{sen}\alpha\operatorname{sen}\beta\\ \operatorname{sen}(\alpha+\beta)&=\operatorname{sen}\alpha\cos\beta+\cos\alpha\operatorname{sen}\beta\\ \operatorname{sen}(\alpha-\beta)&=\operatorname{sen}\alpha\cos\beta-\cos\alpha\operatorname{sen}\beta\end{aligned}}$$

FORMULE DI ADDIZIONE E SOTTRAZIONE
DI SENO E COSENO

$\boxed{\cos 15^\circ} = ?$ $\cos(15^\circ) = \cos(\underline{45}^\circ - \underline{30}^\circ)$

$\alpha = 45^\circ$
$\beta = 30^\circ$

$\boxed{\cos(\alpha - \beta) = \cos\alpha \cos\beta + \operatorname{sen}\alpha \operatorname{sen}\beta}$

$\cos(45^\circ - 30^\circ) = \cos 45^\circ \cdot \cos 30^\circ + \operatorname{sen} 45^\circ \cdot \operatorname{sen} 30^\circ =$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6} + \sqrt{2}}{4}}$

$$\boxed{\operatorname{sen}\alpha = \frac{3}{5}} \qquad 0^\circ < \alpha < 90^\circ \qquad \boxed{\frac{4+3\sqrt{3}}{10}}$$

$$\cos(\alpha + 60^\circ) = ? \qquad \cos(\alpha - 60^\circ) = ? \qquad \beta = 60^\circ$$

$$\boxed{\cos(\alpha \mp \beta) = \cos\alpha \cos\beta \pm \operatorname{sen}\alpha \operatorname{sen}\beta}$$

$$\cos^2\alpha = 1 - \operatorname{sen}^2\alpha \implies \cos\alpha = \sqrt{1-\operatorname{sen}^2\alpha} = \sqrt{1-\left(\frac{3}{5}\right)^2} =$$

$$= \sqrt{1-\frac{9}{25}} = \sqrt{\frac{25-9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5} \implies \boxed{\cos\alpha = \frac{4}{5}}$$

$$\cos(\alpha - 60^\circ) = \frac{4}{5}\cdot\cos 60^\circ + \frac{3}{5}\cdot\operatorname{sen} 60^\circ = \frac{\cancel{4}}{5}\cdot\frac{1}{\cancel{2}} + \frac{3}{5}\cdot\frac{\sqrt{3}}{2} = \frac{2}{5} + \frac{3\sqrt{3}}{10}$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \operatorname{sen}\alpha\operatorname{sen}\beta =$$

$$= \frac{2}{5} - \frac{3\sqrt{3}}{10} = \boxed{\frac{4-3\sqrt{3}}{10}}$$

$$\boxed{\begin{aligned} \cos(\alpha-60^\circ) &= \frac{4+3\sqrt{3}}{10} \\ \cos(\alpha+60^\circ) &= \frac{4-3\sqrt{3}}{10} \end{aligned}}$$

$$tg(\alpha+\beta) = \frac{sen(\alpha+\beta)}{cos(\alpha+\beta)} = \frac{sen\alpha\cos\beta + \cos\alpha\, sen\beta}{\cos\alpha\cos\beta - sen\alpha\, sen\beta}$$

DIVIDIAMO TUTTO PER $\cos\alpha\cos\beta$

$\cos\alpha \neq 0 \qquad k \in \mathbb{Z}$

$\alpha \neq \frac{\pi}{2} + k\pi$

$\cos\beta \neq 0$

$\beta \neq \frac{\pi}{2} + k\pi$

$tg\alpha = -tg\alpha$

$$tg(\alpha+\beta) = \frac{\frac{sen\alpha\cancel{\cos\beta}}{\cos\alpha\cancel{\cos\beta}} + \frac{\cancel{\cos\alpha}\, sen\beta}{\cancel{\cos\alpha}\cos\beta}}{\frac{\cancel{\cos\alpha\cos\beta}}{\cancel{\cos\alpha\cos\beta}} - \frac{sen\alpha\, sen\beta}{\cos\alpha\cos\beta}}$$

$$\boxed{tg(\alpha+\beta) = \frac{tg\alpha + tg\beta}{1 - tg\alpha\, tg\beta}}$$

FORMULA DI ADDIZIONE DELLA TANGENTE

AL POSTO DI $\beta \to -\beta$

$$\boxed{tg(\alpha-\beta) = \frac{tg\alpha + tg(-\beta)}{1 - tg(\alpha)\, tg(-\beta)} = \frac{tg\alpha - tg\beta}{1 + tg\alpha\, tg\beta}}$$

FORMULA DI SOTTRAZIONE DELLA TANGENTE

$tg(75^\circ) = ?$ $\boxed{tg\, 75^\circ = 2+\sqrt{3}}$ $75^\circ = 45^\circ + 30^\circ$ $\alpha = 45^\circ$, $\beta = 30^\circ$

$$\boxed{tg(\alpha+\beta) = \frac{tg\,\alpha + tg\,\beta}{1 - tg\alpha\, tg\beta}}$$

$tg(45^\circ + 30^\circ) =$

$$= \frac{tg\,45^\circ + tg\,30^\circ}{1 - tg\,45^\circ \cdot tg\,30^\circ} = \frac{1 + \frac{\sqrt{3}}{3}}{1 - \frac{\sqrt{3}}{3}} = \frac{\frac{3+\sqrt{3}}{3}}{\frac{3-\sqrt{3}}{3}} = \frac{3+\sqrt{3}}{\cancel{3}} \cdot \frac{\cancel{3}}{3-\sqrt{3}} =$$

$$= \frac{3+\sqrt{3}}{3-\sqrt{3}} \cdot \frac{3+\sqrt{3}}{3+\sqrt{3}} = \frac{(3+\sqrt{3})^2}{9-3} = \frac{9+3+6\sqrt{3}}{6} = \frac{12+6\sqrt{3}}{6} = \frac{\cancel{6}(2+\sqrt{3})}{\cancel{6}}$$

$(a-b)\quad(a+b)\quad(a^2-b^2)$

$\cos\alpha = \frac{5}{13}$ $\qquad\qquad$ $\cos(45^\circ - \alpha) = ?$

$\operatorname{sen}\alpha = \pm\sqrt{1-\cos^2\alpha} = \pm\sqrt{1-\left(\frac{5}{13}\right)^2} = \sqrt{1-\frac{25}{169}} =$

$= \sqrt{\frac{169-25}{169}} = \sqrt{\frac{144}{169}} = \frac{12}{13} \Longrightarrow \boxed{\operatorname{sen}\alpha = \mp\frac{12}{13}}$ $\qquad$ Sen[illegible] PER

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \operatorname{sen}\alpha\operatorname{sen}\beta$

$\cos(45^\circ-\alpha) = \cos 45^\circ\cos\alpha + \operatorname{sen}45^\circ\operatorname{sen}\alpha =$

$= \frac{\sqrt{2}}{2}\cdot\frac{5}{13} + \frac{\sqrt{2}}{2}\cdot\frac{12}{13} = \frac{5\sqrt{2}}{26} + \frac{12\sqrt{2}}{26} = \boxed{\frac{17\sqrt{2}}{26}}$

$$\boxed{\cos(45^\circ-\alpha) = -\frac{7\sqrt{2}}{26}}$$

FORMULE DI DUPLICAZIONE

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \operatorname{sen}\alpha\operatorname{sen}\beta$$

$$\boxed{\alpha = \beta}$$

$$\Downarrow$$

$$\cos(\alpha+\alpha) = \cos\alpha\cos\alpha - \operatorname{sen}\alpha\operatorname{sen}\alpha$$

$$\Downarrow$$

$$\boxed{\cos(2\alpha) = \cos^2\alpha - \operatorname{sen}^2\alpha}$$

FORMULA DI DUPLICAZIONE DEL COSENO

$$\operatorname{sen}(\alpha+\beta) = \operatorname{sen}\alpha\cos\beta + \operatorname{sen}\beta\cos\alpha \qquad \alpha=\beta$$

$$\operatorname{sen}(\alpha+\alpha) = \operatorname{sen}\alpha\cos\alpha + \operatorname{sen}\alpha\cos\alpha$$

$$\boxed{\operatorname{sen}(2\alpha) = 2\operatorname{sen}\alpha\cos\alpha}$$

FORMULA DI DUPLICAZIONE DEL SENO

$$\operatorname{tg}(\alpha+\beta) = \frac{\operatorname{tg}\alpha + \operatorname{tg}\beta}{1-\operatorname{tg}\alpha\operatorname{tg}\beta} =$$

$$\operatorname{tg}(\alpha+\alpha) = \frac{\operatorname{tg}\alpha+\operatorname{tg}\alpha}{1-\operatorname{tg}\alpha\operatorname{tg}\alpha} \Longrightarrow \boxed{\operatorname{tg}(2\alpha) = \frac{2\operatorname{tg}\alpha}{1-\operatorname{tg}^2\alpha}}$$

FORMULA DI DUPLICAZIONE DELLA TANGENTE

$$\mathrm{sen}\,2\alpha = 2\,\mathrm{sen}\,\alpha\cos\alpha$$

$$\cos 2\alpha = \underline{\cos^2\alpha - \mathrm{sen}^2\alpha} = 1 - \mathrm{sen}^2\alpha - \mathrm{sen}^2\alpha = \underline{1 - 2\,\mathrm{sen}^2\alpha} =$$

$$= \cos^2\alpha - (1 - \cos^2\alpha)$$

$$= \underline{2\cos^2\alpha - 1}$$

$$\mathrm{tg}\,2\alpha = \frac{2\,\mathrm{tg}\,\alpha}{1 - \mathrm{tg}^2\alpha}$$

FORMULE DI DUPLICAZIONE

$$\text{sen}\,60^\circ = \text{sen}(\underbrace{2\cdot 30^\circ}_{2\alpha}) = 2\,\text{sen}\,\underset{\alpha}{30^\circ}\cdot\cos 30^\circ = \cancel{2}\cdot\frac{1}{2}\cdot\frac{\sqrt{3}}{\cancel{2}} = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \cos(2\cdot 30^\circ) = \cos^2 30^\circ - \text{sen}^2 30^\circ = \left(\frac{\sqrt{3}}{2}\right)$$