LEZIONE 8

Esercizi sul calcolo del dominio di una funzione reale di variabile reale

N°51

$$y = \frac{\sqrt{-1+x}}{x^2+x+1}$$

IRRAZ. FRATTA

$$\begin{cases} -1+x \geq 0 \\ x^2+x+1 \neq 0 \quad \forall x \in \mathbb{R} \end{cases}$$

$-1+x \geq 0 \Rightarrow x \geq 1$

$x^2+x+1=0$

$\Delta = b^2-4ac = 1-4 = -3$

$\Delta < 0 \quad \nexists x \in \mathbb{R}$

$$\begin{cases} x \geq 1 \\ \forall x \in \mathbb{R} \end{cases}$$

$x \geq 1$

$\text{Dom } f = [1; +\infty[$

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$$y = \frac{x+3}{x^2-1}$$

RAZ. FRATTA

$x^2 - 1 \neq 0 \Rightarrow x^2 \neq 1 \Rightarrow x \neq \pm\sqrt{1} = \pm 1$

$x \neq \pm 1$

$Dom\ f = \mathbb{R} \setminus \{\pm 1\}$

$]-\infty; -1[\cup]-1; 1[\cup]1; +\infty[$

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$y = \frac{3x-1}{x^2-5x+6}$ RAZ. FRATTA

$x^2 - 5x + 6 \neq 0$

$\Delta = b^2 - 4ac = (-5)^2 - 4(1)(6) = 25 - 24 = 1$

$x_1, x_2 = -\frac{b \pm \sqrt{\Delta}}{2a} = -\frac{(-5) \pm \sqrt{1}}{2}$

$x_2 = \frac{5+1}{2}$ $x_1 = \frac{5-1}{2}$

$x_2 = \frac{6}{2} = 3$ $x_1 = \frac{4}{2}$

$x \neq 2 \wedge x \neq 3$

$\text{Dom } f = \mathbb{R} \setminus \{2, 3\}$

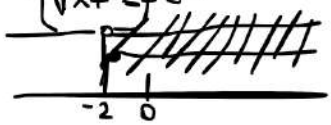
$\text{Dom } f =]-\infty; 2[\cup]2; 3[\cup]3; +\infty[$

54 $y = \frac{8x-1}{\sqrt{x+2}}$

IRRAZ. FRATTA

$\begin{cases} x+2 \geq 0 \\ \sqrt{x+2} \neq 0 \end{cases} \rightarrow \begin{cases} x \geq -2 \\ x+2 \neq 0 \Rightarrow x \neq -2 \end{cases}$

$\begin{cases} x \geq -2 \\ x \neq -2 \end{cases}$

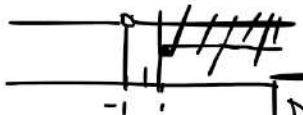


$x > -2$

55 $y = \frac{\sqrt{x-1}}{x+1}$

IRRAZ. F RATTA

$\begin{cases} x-1 \geq 0 \Rightarrow \boxed{x \geq 1} \\ x+1 \neq 0 \Rightarrow \boxed{x \neq -1} \end{cases}$



$x \geq 1$

$\boxed{Dom f = [1; +\infty[}$

60 $y = \underbrace{\sqrt[5]{x^3+4x^2-2}}_{\forall x \in \mathbb{R}} \cdot \sqrt{x}$ IRRAZ. INTERA $\sqrt[5]{-32} = -2$

$x \geq 0$ $\quad Dom f = \mathbb{R}_0^+$ REALI POSITIVI INCLUSO LO 0

$Dom f = [0, +\infty[$

$y = \sqrt[3]{\frac{5-x}{|x-1|}}$

IRRAZ. FRATTA

$|x-1| \neq 0$

$|x-1| \neq 0$

$x \neq 1$

$\text{Dom } f = \mathbb{R} \setminus \{1\}$

$\text{Dom } f =]-\infty ; 1[\cup]1 ; +\infty[$

$|x-1| = 0$

$\begin{cases} x-1 \geq 0 \\ x-1 = 0 \end{cases} \cup \begin{cases} x-1 < 0 \\ 1-x = 0 \end{cases}$

$x = 1$

$x = 1$

130 $y = \dfrac{\sqrt{16-x^2}}{x^2-6x+9} + \dfrac{x}{\sqrt{x^2-16}}$ IRRAZ. FRATTA

(A) $16-x^2 \geq 0$

(B) $x^2-6x+9 \neq 0$

(C) $\sqrt{x^2-16} \neq 0$

(A) $-4 \leq x \leq 4$

(A) $16-x^2 \geq 0$

$-x^2 \geq -16$

$x^2 \leq 16$ sol. intere

$x^2 = 16$ $\quad -4 \leq x \leq 4$

$x = \pm\sqrt{16} = \pm 4$

$x_1 = -4$; $x_2 = 4$

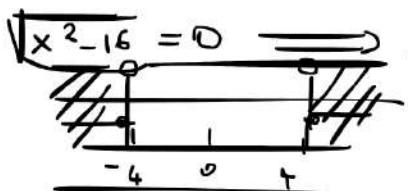
(B) $x^2 - 6x + 9 \neq 0$

$\Delta = b^2 - 4ac = (-6)^2 - 4(1)(9) = 36 - 36 = 0$

$x_1 = x_2 = -\frac{b}{2a} = -\frac{(-6)}{2 \cdot 1} = \frac{6}{2} = 3$

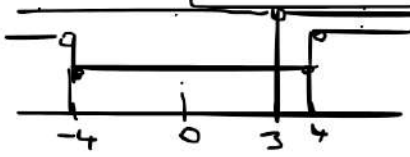
$\boxed{x \neq 3}$

c)



$\begin{cases} x^2-16 \geq 0 & x \leq -4 \vee x \geq 4 \\ 0 \geq 0 \Longrightarrow & \forall x \in \mathbb{R} \\ x^2-16 \neq 0 \Rightarrow & x \neq \pm 4 \end{cases}$

$x < -4 \vee x > 4$



$\begin{cases} -4 \leq x \leq 4 \\ x \neq 3 \\ x < -4 \vee x > 4 \end{cases}$

$\nexists x \in \mathbb{R}$ ~~Dom f = ∅~~

133 $y = \sqrt{2x - \sqrt{3-4x}}$ IRRAZ. INTERA

(A) $2x - \sqrt{3-4x} \geq 0$

(B) $3 - 4x \geq 0 \Rightarrow -4x \geq -3 \Rightarrow 4x \leq 3$

$\boxed{x \leq \frac{3}{4}}$

c $2x - \sqrt{3-4x} \geq 0$

$-\sqrt{3-4x} \geq -2x$

$\sqrt{3-4x} \leq 2x$

$\begin{cases} 3-4x \geq 0 \Rightarrow -4x \geq -3 \Rightarrow 4x \leq 3 \Rightarrow x \leq \frac{3}{4} \\ 2x \geq 0 \Rightarrow x \geq 0 \\ 3-4x \leq 4x^2 \Rightarrow -4x^2 - 4x + 3 \leq 0 \Rightarrow 4x^2 + 4x - 3 \geq 0 \end{cases}$

DISEQ. IRRAZIONALI

$\sqrt[n]{f(x)} \leq g(x)$ (n pari)

$\begin{cases} f(x) \geq 0 \\ g(x) \geq 0 \\ f(x) < [g(x)]^2 \end{cases}$

$\sqrt[n]{f(x)} \geq g(x)$ (n pari)

$\begin{cases} f(x) \geq 0 \\ g(x) < 0 \end{cases} \cup \begin{cases} g(x) \geq 0 \\ f(x) \geq [g(x)]^2 \end{cases}$

$4x^2 + 4x - 3 \geq 0$

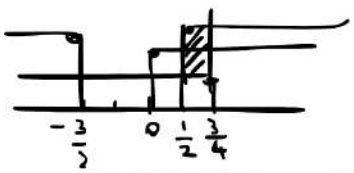
$\Delta = b^2 - 4ac = (4)^2 - 4(4)(-3) = 16 + 48 = 64$

$x_1, x_2 = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-4 \pm \sqrt{64}}{8}$ → $\frac{-4-8}{8} = -\frac{12}{8} = -\frac{3}{2}$; ↘ $\frac{-4+8}{8} = \frac{4}{8} = \frac{1}{2}$

$x_1 = -\frac{3}{2} \vee x_2 = \frac{1}{2}$

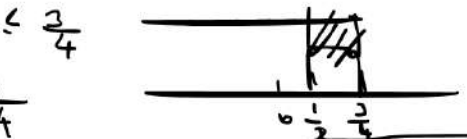
$x \leq -\frac{3}{2} \vee x \geq \frac{1}{2}$

$$\begin{cases} x \le \frac{3}{4} \\ x \ge 0 \\ x \le -\frac{3}{2} \vee x \ge \frac{1}{2} \end{cases}$$



$$\boxed{\frac{1}{2} \le x \le \frac{3}{4}}$$

$$\begin{cases} \frac{1}{2} \le x \le \frac{3}{4} \\ x \le \frac{3}{4} \end{cases}$$



$$\boxed{\frac{1}{2} \le x \le \frac{3}{4}}$$

$$\boxed{\mathcal{D}_f = \left[\frac{1}{2}; \frac{3}{4}\right]}$$

$$y = \sqrt{2x - \sqrt{3-4x}} \Rightarrow y\left(\frac{1}{2}\right) = \sqrt{2\cdot\frac{1}{2} - \sqrt{3-2\cdot\frac{1}{2}}} = \sqrt{1-1} = 0$$