LEZIONE 10

Formule di duplicazione

$sen(2\alpha) = ?$

$cos(2\alpha) = ?$

$tg(2\alpha) = ?$

$sen(\alpha \pm \beta) = sen\alpha\cos\beta \pm sen\beta\cos\alpha$

$cos(\alpha \pm \beta) = cos\alpha\cos\beta \mp sen\alpha\, sen\beta$

$\boxed{tg(\alpha \pm \beta) = \dfrac{tg\alpha \pm tg\beta}{1 \mp tg\alpha\, tg\beta}}$

$\underline{\beta = \alpha}$

$sen(\alpha + \alpha) = sen(2\alpha) = \underline{sen\alpha\cos\alpha} + \underline{sen\alpha\cos\alpha} = 2\, sen\alpha\cos\alpha$

$\boxed{sen(2\alpha) = 2\, sen\alpha\cos\alpha}$

$\cos(2\alpha) = \underline{\cos\alpha \cdot \cos\alpha} - \operatorname{sen}\alpha \cdot \operatorname{sen}\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha$

$$\boxed{\cos 2\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha}$$

$tg(2\alpha)$

$$\boxed{tg(\alpha+\alpha) = \frac{tg\alpha + tg\alpha}{1 - tg^2\alpha} = \frac{2\,tg\alpha}{1 - tg^2\alpha}}$$

$$\boxed{\begin{cases} sen(2\alpha) = 2\,sen\alpha\cos\alpha \\ cos(2\alpha) = \cos^2\alpha - sen^2\alpha \\ tg(2\alpha) = \dfrac{2\,tg\alpha}{1 - tg^2\alpha} \end{cases}}$$

FORMULE
DI
DUPLICAZIONE

$\text{sen}\,\underset{2\alpha}{36^\circ} = \text{sen}\,2\cdot\underbrace{18^\circ}_{\alpha} = 2\,\text{sen}\,18^\circ\cos 18^\circ$

$$\boxed{\text{sen}\,18^\circ = \frac{\sqrt{5}-1}{4} \qquad \cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}}$$

$$= \cancel{2}\cdot\left(\frac{\sqrt{5}-1}{\cancel{4}_{2}}\right)\cdot\frac{\sqrt{10+2\sqrt{5}}}{4} =$$

$$= \frac{(\sqrt{5}-1)\cdot\sqrt{10+2\sqrt{5}}}{8} = \frac{\sqrt{(\sqrt{5}-1)^2\cdot(10+2\sqrt{5})}}{8} =$$

$$= \frac{\sqrt{(5+1-2\sqrt{5})\cdot 2(5+\sqrt{5})}}{8} = \frac{\sqrt{(6-2\sqrt{5})\cdot 2(5+\sqrt{5})}}{8} =$$

$$= \frac{\sqrt{2(3-\sqrt{5})\cdot 2\cdot(5+\sqrt{5})}}{8} =$$

$$= \frac{\sqrt{4(3-\sqrt{5})(5+\sqrt{5})}}{8} = \frac{\cancel{2}^{1}}{\cancel{8}_{4}}\sqrt{15+3\sqrt{5}-5\sqrt{5}-5}$$

$$= \boxed{\frac{1}{4}\sqrt{10-2\sqrt{5}}} \qquad \boxed{\text{sen}\,36^\circ = \frac{1}{4}\sqrt{10-2\sqrt{5}}}$$

$$\underbrace{\operatorname{sen}(\alpha+\beta)}\,\underbrace{\operatorname{sen}(\alpha-\beta)} =$$

$$\underbrace{\operatorname{sen}(\alpha \pm \beta) = \operatorname{sen}\alpha\cos\beta \pm \operatorname{sen}\beta\cos\alpha}$$

$$=$$

$$(\underbrace{\operatorname{sen}\alpha\cos\beta}_{A} + \underbrace{\operatorname{sen}\beta\cos\alpha}_{B})\cdot(\underbrace{\operatorname{sen}\alpha\cos\beta}_{A} - \underbrace{\operatorname{sen}\beta\cos\alpha}_{B}) =$$

$$(A + B)\,(A - B) = A^2 - B^2$$

$$= \operatorname{sen}^2\alpha\cos^2\beta - \operatorname{sen}^2\beta\cos^2\alpha =$$

$$\cos^2\beta = 1 - \operatorname{sen}^2\beta$$

$$\cos^2\alpha = 1 - \operatorname{sen}^2\alpha$$

$$= \operatorname{sen}^2\alpha\cdot(1 - \operatorname{sen}^2\beta) - \operatorname{sen}^2\beta\,(1 - \operatorname{sen}^2\alpha) =$$

$$= \operatorname{sen}^2\alpha \cancel{- \operatorname{sen}^2\alpha\operatorname{sen}^2\beta} - \operatorname{sen}^2\beta \cancel{+ \operatorname{sen}^2\alpha\operatorname{sen}^2\beta} = \operatorname{sen}^2\alpha - \operatorname{sen}^2\beta$$

$$\cos(\alpha+\beta)\cos(\alpha-\beta) \qquad\qquad \cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\operatorname{sen}\alpha\operatorname{sen}\beta$$

$$[\underbrace{\cos\alpha\cos\beta}_{A}-\underbrace{\operatorname{sen}\alpha\operatorname{sen}\beta}_{B}][\cos\alpha\cos\beta+\operatorname{sen}\alpha\operatorname{sen}\beta]$$

$$=[A-B]\,[A+B]=A^2-B^2$$

$$=\cos^2\alpha\,\underbrace{\cos^2\beta}-\underbrace{\operatorname{sen}^2\alpha}\operatorname{sen}^2\beta= \qquad \cos^2\beta=1-\operatorname{sen}^2\beta \quad \operatorname{sen}^2\alpha=1-\cos^2\alpha$$

$$=\cos^2\alpha(1-\operatorname{sen}^2\beta)-(1-\cos^2\alpha)\operatorname{sen}^2\beta=$$

$$=\cos^2\alpha-\cancel{\cos^2\alpha\operatorname{sen}^2\beta}-\operatorname{sen}^2\beta+\cancel{\cos^2\alpha\operatorname{sen}^2\beta}=\underline{\cos^2\alpha-\operatorname{sen}^2\beta}$$

FORMULE PARAMETRICHE

Lo scopo è quello di poter scrivere il seno e il coseno di un angolo in funzione della tangente dell'angolo dimezzato.

A

$$\operatorname{sen} 2\alpha = 2 \operatorname{sen}\alpha \cos\alpha$$

B

$$\cos 2\alpha = \cos^2\alpha - \operatorname{sen}^2\alpha$$

Dividiamo il secondo membro sia di A che di B per la quantità:

$$\boxed{\cos^2\frac{\alpha}{2} + \operatorname{sen}^2\frac{\alpha}{2}}$$

$$\operatorname{sen}\alpha = 2 \operatorname{sen}\frac{\alpha}{2}\cos\frac{\alpha}{2}$$

$$\cos\alpha = \cos^2\frac{\alpha}{2} - \operatorname{sen}^2\frac{\alpha}{2}$$

$$\operatorname{sen}\alpha = \frac{2\operatorname{sen}\frac{\alpha}{2}\cos\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}+\operatorname{sen}^2\frac{\alpha}{2}} \quad ; \quad \cos\alpha = \frac{\cos^2\frac{\alpha}{2}-\operatorname{sen}^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}+\operatorname{sen}^2\frac{\alpha}{2}}$$

Dividamo tutto per $\cos^2\frac{\alpha}{2} \neq 0$ $\boxed{\frac{\alpha}{2} \neq \frac{\pi}{2}+K\pi \quad (90^\circ) \ K\cdot 180^\circ}$

$$\operatorname{sen}\alpha = \frac{\frac{2\operatorname{sen}\frac{\alpha}{2}\cancel{\cos\frac{\alpha}{2}}}{\cos^{\cancel{2}}\frac{\alpha}{2}}}{\frac{\cancel{\cos^2\frac{\alpha}{2}}}{\cancel{\cos^2\frac{\alpha}{2}}}+\frac{\operatorname{sen}^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}} \qquad \cos\alpha = \frac{\frac{\cancel{\cos^2\frac{\alpha}{2}}}{\cancel{\cos^2\frac{\alpha}{2}}}-\frac{\operatorname{sen}^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}}{\frac{\cancel{\cos^2\frac{\alpha}{2}}}{\cancel{\cos^2\frac{\alpha}{2}}}+\frac{\operatorname{sen}^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}}$$

$$\boxed{\operatorname{sen}\alpha = \frac{2\operatorname{tg}\frac{\alpha}{2}}{1+\operatorname{tg}^2\frac{\alpha}{2}} \qquad \cos\alpha = \frac{1-\operatorname{tg}^2\frac{\alpha}{2}}{1+\operatorname{tg}^2\frac{\alpha}{2}}}$$

$$\begin{cases} \operatorname{sen}\alpha = \dfrac{2\operatorname{tg}\frac{\alpha}{2}}{1+\operatorname{tg}^2\frac{\alpha}{2}} \\ \cos\alpha = \dfrac{1-\operatorname{tg}^2\frac{\alpha}{2}}{1+\operatorname{tg}^2\frac{\alpha}{2}} \end{cases} \Rightarrow \begin{cases} \operatorname{sen}\alpha = \dfrac{2t}{1+t^2} \\ \cos\alpha = \dfrac{1-t^2}{1+t^2} \end{cases} \qquad \operatorname{tg}\frac{\alpha}{2} = t$$

$\boxed{\text{FORMULE PARAMETRICHE}}$

Es. $\boxed{tg\,\frac{\alpha}{2} = -\frac{2}{3}}$ $\boxed{\begin{array}{l} sen\,\alpha = ? \\ cos\,\alpha = ? \end{array}}$

$$sen\,\alpha = \frac{2\,tg\,\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}} = \frac{2\left(-\frac{2}{3}\right)}{1+\left(-\frac{2}{3}\right)^2} = \frac{-\frac{4}{3}}{1+\frac{4}{9}} = \frac{-\frac{4}{3}}{\frac{9+4}{9}} = -\frac{4}{\cancel{3}}\cdot\frac{\cancel{9}^{\,3}}{13} = -\frac{12}{13}$$

$$cos\,\alpha = \frac{1-tg^2\frac{\alpha}{2}}{1+tg^2\frac{\alpha}{2}} = \frac{1-\left(-\frac{2}{3}\right)^2}{1+\left(-\frac{2}{3}\right)^2} = \frac{1-\frac{4}{9}}{1+\frac{4}{9}} = \frac{\frac{9-4}{9}}{\frac{9+4}{9}}$$

$$= \frac{5}{\cancel{9}}\cdot\frac{\cancel{9}}{13} = \frac{5}{13}$$

$$\boxed{\begin{array}{l} sen\,\alpha = -\frac{12}{13} \\ cos\,\alpha = \frac{5}{13} \end{array}}$$

② $\operatorname{sen}\alpha + \cos\alpha + \operatorname{tg}\alpha =$

$$= \frac{4\operatorname{tg}\frac{\alpha}{2}}{1+\operatorname{tg}^2\frac{\alpha}{2}} + \frac{1-\operatorname{tg}^2\frac{\alpha}{2}}{1+\operatorname{tg}^2\frac{\alpha}{2}} + \frac{2\operatorname{tg}\frac{\alpha}{2}}{1-\operatorname{tg}^2\frac{\alpha}{2}} =$$

$$\operatorname{tg}\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha}$$

$$= \frac{\dfrac{2\operatorname{tg}\frac{\alpha}{2}}{\cancel{1+\operatorname{tg}^2\frac{\alpha}{2}}}}{\dfrac{1-\operatorname{tg}^2\frac{\alpha}{2}}{\cancel{1+\operatorname{tg}^2\frac{\alpha}{2}}}}$$

$$\operatorname{tg}\frac{\alpha}{2} = t$$

$$= \frac{4t}{1+t^2} + \frac{1-t^2}{1+t^2} + \frac{2t}{1-t^2} =$$

$$= \frac{4t(1-t^2) + (1-t^2)^2 + 2t(1+t^2)}{(1-t^2)(1+t^2)} =$$

$$= \frac{4t - 4t^3 + 1 + t^4 - 2t^2 + 2t + 2t^3}{(1-t^2)(1+t^2)} =$$

$$= \frac{t^4 - 2t^3 - 2t^2 + 6t + 1}{(1-t^2)(1+t^2)}$$

FORMULE DI BISEZIONE

Riscrivere un'espressione goniometrica per seno e coseno dell'angolo dimezzato

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$$

$$\cos 2\alpha = 1 - \sin^2\alpha - \sin^2\alpha$$

$$\cos 2\alpha = 1 - 2\sin^2\alpha$$

$$2\sin^2\alpha = 1 - \cos 2\alpha$$

$$\sin^2\alpha = \frac{1-\cos 2\alpha}{2}$$

$$\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2}$$

$$\boxed{\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}}$$

$$\cos 2\alpha = \cos^2\alpha - (1-\cos^2\alpha)$$

$$\cos 2\alpha = 2\cos^2\alpha - 1$$

$$+2\cos^2\alpha = +1 + \cos 2\alpha$$

$$\cos^2\alpha = \frac{1+\cos 2\alpha}{2}$$

$$\cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2}$$

$$\boxed{\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}}$$

BISEZIONE SENO E COSENO

$$\tan\frac{\alpha}{2} = \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = \frac{\pm\sqrt{\frac{1-\cos\alpha}{2}}}{\pm\sqrt{\frac{1+\cos\alpha}{2}}} = \pm\frac{\sqrt{1-\cos\alpha}}{\cancel{\sqrt{2}}} \cdot \frac{\cancel{\sqrt{2}}}{\sqrt{1+\cos\alpha}}$$

$$\boxed{\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}}$$

BISEZIONE TANGENTE