256

$$\boxed{\frac{1}{2}\,tg\,2\alpha \cdot (1+tg\,\alpha)} =$$

$$\boxed{tg\,2\alpha = \frac{2\,tg\,\alpha}{1-tg^2\alpha}}$$

$$= \frac{1}{\cancel{2}}\left(\frac{\cancel{2}\,tg\,\alpha}{\underset{(1-tg\,\alpha)}{\cancel{(1-tg^2\alpha)}}}\right)\cdot \cancel{(1+tg\,\alpha)}^{\,1} = \boxed{\frac{tg\,\alpha}{1-tg\,\alpha}}$$

$$(1+tg\,\alpha)(1-tg\,\alpha) = 1-tg^2\alpha$$

257

$$2\,tg^2\alpha\, sen^2\frac{\alpha}{2} + tg\alpha\, sen\alpha =$$

$$= 2\,tg^2\alpha\cdot\left(\pm\sqrt{\frac{1-\cos\alpha}{2}}\right)^2 + tg\alpha\, sen\alpha =$$

$$= \cancel{2}\,tg^2\alpha\cdot\left(\frac{1-\cos\alpha}{\cancel{2}}\right) + tg\alpha\, sen\alpha =$$

$$= tg^2\alpha\,(1-\cos\alpha) + tg\alpha\cdot sen\alpha =$$

$$= tg\alpha\,[tg\alpha(1-\cos\alpha) + sen\alpha] =$$

$$= tg\alpha\,[tg\alpha - tg\alpha\cos\alpha + sen\alpha] =$$

$$= tg\alpha\left[tg\alpha - \frac{sen\alpha}{\cancel{\cos\alpha}}\cdot\cancel{\cos\alpha} + sen\alpha\right] =$$

$$= tg\alpha\,[tg\alpha - \cancel{sen\alpha} + \cancel{sen\alpha}] = tg\alpha\cdot tg\alpha = \underline{tg^2\alpha}$$

$$\boxed{sen\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}}$$

$$(\sqrt{2})^2 = 2$$

$$tg\alpha = \frac{sen\alpha}{\cos\alpha}$$

258 $\frac{1-\cos\alpha}{2}\left(\operatorname{ctg}^2\frac{\alpha}{2}-1\right)=$ $\qquad$ $\operatorname{tg}\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$

$=\frac{1-\cos\alpha}{2}\left[\left(\pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}\right)^2-1\right]=$ $\qquad$ $\operatorname{ctg}\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}$

$=\frac{1-\cos\alpha}{2}\cdot\left[\frac{1+\cos\alpha}{1-\cos\alpha}-\frac{1}{1}\right]=$

$=\frac{1-\cos\alpha}{2}\cdot\left[\frac{\cancel{1}+\cos\alpha-(\cancel{1}-\cos\alpha)}{1-\cos\alpha}\right]=\frac{\cancel{1-\cos\alpha}}{\cancel{2}}\cdot\frac{\cancel{2}\cos\alpha}{\cancel{1-\cos\alpha}}=\boxed{\cos\alpha}$

259*

$$\frac{1}{1-tg^2\frac{\alpha}{2}} \cdot \frac{1}{1-ctg^2\frac{\alpha}{2}} =$$

$$tg\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

$$ctg\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}$$

$$= \frac{1}{1-\left(\frac{1-\cos\alpha}{1+\cos\alpha}\right)} \cdot \frac{1}{1-\left(\frac{1+\cos\alpha}{1-\cos\alpha}\right)} =$$

$$= \frac{1}{\frac{\cancel{1}+\cos\alpha-\cancel{1}+\cos\alpha}{1+\cos\alpha}} \cdot \frac{1}{\frac{\cancel{1}-\cos\alpha-\cancel{1}-\cos\alpha}{1-\cos\alpha}} = \frac{1}{\frac{2\cos\alpha}{1+\cos\alpha}} \cdot \frac{1}{\frac{-2\cos\alpha}{1-\cos\alpha}}$$

$$= \frac{(1+\cos\alpha)}{2\cos\alpha} \cdot \frac{(1-\cos\alpha)}{-2\cos\alpha} = \frac{1-\cos^2\alpha}{-4\cos^2\alpha} = \frac{\cancel{-}1(\cos^2\alpha-1)}{\cancel{-}4\cos^2\alpha} =$$

$$= \frac{1}{4} \cdot \frac{\cos^2\alpha-1}{\cos^2\alpha} = \frac{1}{4} \cdot \left[\frac{\cos^2\alpha}{\cos^2\alpha} - \frac{1}{\cos^2\alpha}\right] = \frac{1}{4}\left[1-\frac{1}{\cos^2\alpha}\right]$$

259 $$\frac{1}{1-\operatorname{tg}^2\frac{\alpha}{2}} - \frac{1}{1-\operatorname{cotg}^2\frac{\alpha}{2}} =$$

$$\operatorname{tg}\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

$$\operatorname{cotg}\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}$$

$$= \frac{1}{1-\left(\frac{1-\cos\alpha}{1+\cos\alpha}\right)} - \frac{1}{1-\left(\frac{1+\cos\alpha}{1-\cos\alpha}\right)} =$$

$$= \frac{1}{\frac{\cancel{1}+\cos\alpha-\cancel{1}+\cos\alpha}{1+\cos\alpha}} - \frac{1}{\frac{\cancel{1}-\cos\alpha-\cancel{1}-\cos\alpha}{1-\cos\alpha}} =$$

$$= \frac{1}{\frac{2\cos\alpha}{1+\cos\alpha}} - \frac{1}{\frac{-2\cos\alpha}{1-\cos\alpha}}$$

$$= \frac{1+\cos\alpha}{2\cos\alpha} - \frac{(1-\cos\alpha)}{-2\cos\alpha} =$$

$$= \frac{1+\cos\alpha}{2\cos\alpha} + \frac{1-\cos\alpha}{2\cos\alpha} = \frac{1+\cancel{\cos\alpha}+1-\cancel{\cos\alpha}}{2\cos\alpha} =$$

$$= \frac{\cancel{2}}{\cancel{2}\cos\alpha} = \boxed{\frac{1}{\cos\alpha}}$$

Formule di Prostaferesi

Attraverso queste formule riesco ad esprimere la somma e la differenza di seni e coseni di angoli differenti in funzione di prodotti di seni e coseni.
Queste formule sono dirette discendenti delle formule di addizione e sottrazione del seno e del coseno.

$$\begin{cases} \operatorname{sen}(\alpha+\beta) = \operatorname{sen}\alpha\cos\beta + \operatorname{sen}\beta\cos\alpha \\ \operatorname{sen}(\alpha-\beta) = \operatorname{sen}\alpha\cos\beta - \operatorname{sen}\beta\cos\alpha \end{cases}$$

Formule di addizione e sottrazione del seno

$$\boxed{\operatorname{sen}(\alpha+\beta) + \operatorname{sen}(\alpha-\beta) = 2\operatorname{sen}\alpha\cos\beta}$$

LE SOMMO MEMBRO A MEMBRO

$$\boxed{\operatorname{sen}(\alpha+\beta) - \operatorname{sen}(\alpha-\beta) = 2\operatorname{sen}\beta\cos\alpha}$$

LE SOTTRAGGO MEMBRO A MEMBRO

$$\boxed{\begin{aligned} &\operatorname{sen}(\alpha+\beta)+\operatorname{sen}(\alpha-\beta)=2\operatorname{sen}\alpha\cos\beta \\ &\operatorname{sen}(\alpha-\beta)-\operatorname{sen}(\alpha-\beta)=2\operatorname{sen}\beta\cos\alpha \end{aligned}}$$

$$\Downarrow$$

$$\boxed{\begin{cases} \operatorname{sen}p+\operatorname{sen}q=2\operatorname{sen}\left(\dfrac{p+q}{2}\right)\cos\left(\dfrac{p-q}{2}\right) \\ \operatorname{sen}p-\operatorname{sen}q=2\operatorname{sen}\left(\dfrac{p-q}{2}\right)\cos\left(\dfrac{p+q}{2}\right) \end{cases}}$$

$$\begin{cases} \alpha+\beta=p \\ \alpha-\beta=q \end{cases}$$

$$\begin{cases} \alpha=p-\beta \\ p-\beta-\beta=q \end{cases}$$

$$-2\beta=q-p$$

$$\boxed{\alpha=\frac{p+q}{2}} \quad \boxed{\beta=\frac{p-q}{2}}$$

$$\alpha=p-\left(\frac{p-q}{2}\right)=\frac{2p-p+q}{2}$$

$$\begin{cases} \cos(\alpha+\beta) = \cos\alpha\cos\beta - \operatorname{sen}\alpha\operatorname{sen}\beta \\ \cos(\alpha-\beta) = \cos\alpha\cos\beta + \operatorname{sen}\alpha\operatorname{sen}\beta \end{cases}$$

$$\alpha = \frac{p+q}{2}$$

$$\beta = \frac{p-q}{2}$$

$$\cos(\alpha+\beta) + \cos(\alpha-\beta) = 2\cos\alpha\cos\beta$$

$$\cos(\alpha+\beta) - \cos(\alpha-\beta) = -2\operatorname{sen}\alpha\operatorname{sen}\beta$$

$$\boxed{\begin{aligned} \cos p + \cos q &= 2\cos\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right) \\ \cos p - \cos q &= -2\operatorname{sen}\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right) \end{aligned}}$$

$$\boxed{tg\alpha \pm tg\beta = \frac{sen\alpha}{cos\alpha} \pm \frac{sen\beta}{cos\beta} = \frac{sen\alpha cos\beta \pm sen\beta cos\alpha}{cos\alpha cos\beta} = \frac{sen(\alpha \pm \beta)}{cos\alpha cos\beta} = \frac{sen(p \pm q)}{cos\left(\frac{p+q}{2}\right) cos\left(\frac{p-q}{2}\right)}}$$

$$\alpha = \frac{p+q}{2}$$

$$\beta = \frac{p-q}{2}$$

$$sen\alpha + cos\beta = \underbrace{sen\alpha + sen(90° - \beta)}_{\text{APPLICO PROSTAFERESI QUI}}$$

$$\boxed{cos\beta = sen(90° - \beta)}$$

$$1 + \operatorname{sen} x = \operatorname{sen} 90^\circ + \operatorname{sen} x$$

$$\operatorname{sen} p + \operatorname{sen} q = 2 \operatorname{sen}\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right)$$

$$p = 90^\circ$$

$$q = x$$

$$\operatorname{sen} 90^\circ + \operatorname{sen} x = 2 \operatorname{sen}\left(\frac{90^\circ + x}{2}\right) \cdot \cos\left(\frac{90^\circ - x}{2}\right) =$$

$$= 2 \operatorname{sen}\left(45^\circ + \frac{x}{2}\right)\cos\left(45^\circ - \frac{x}{2}\right) =$$

$$\cos\left(45^\circ - \frac{x}{2}\right) = \operatorname{sen}\left[90^\circ - \left(45^\circ - \frac{x}{2}\right)\right]$$

$$= 2 \operatorname{sen}\left(45^\circ + \frac{x}{2}\right)\operatorname{sen}\left[90^\circ - \left(45^\circ - \frac{x}{2}\right)\right]$$

$$= 2 \operatorname{sen}\left(45^\circ + \frac{x}{2}\right)\operatorname{sen}\left(45^\circ + \frac{x}{2}\right) = \boxed{2 \operatorname{sen}^2\left(45^\circ + \frac{x}{2}\right)}$$

$$\operatorname{sen}^2 5\alpha - \operatorname{sen}^2 3\alpha =$$

$$= (\operatorname{sen} 5\alpha + \operatorname{sen} 3\alpha)(\operatorname{sen} 5\alpha - \operatorname{sen} 3\alpha)$$

$$= \left[2 \operatorname{sen}\left(\frac{5\alpha+3\alpha}{2}\right) \cos\left(\frac{5\alpha-3\alpha}{2}\right)\right]\left[2 \cdot \operatorname{sen}\left(\frac{5\alpha-3\alpha}{2}\right) \cdot \cos\left(\frac{5\alpha+3\alpha}{2}\right)\right]$$

$$= 4 \operatorname{sen}\frac{\cancel{8}^{4}\alpha}{\cancel{2}_1} \cos\frac{\cancel{2}^{1}\alpha}{\cancel{2}_1} \operatorname{sen}\frac{\cancel{2}^{1}\alpha}{\cancel{2}_1} \cdot \cos\frac{\cancel{8}^{4}\alpha}{\cancel{2}_1} =$$

$$= 4 \operatorname{sen} 4\alpha \cos 4\alpha \operatorname{sen}\alpha \cos\alpha =$$

$$= (2 \operatorname{sen} 4\alpha \cos 4\alpha)(2 \operatorname{sen}\alpha \cos\alpha) = \underline{\operatorname{sen} 8\alpha \cdot \operatorname{sen} 2\alpha}$$

$p = 5\alpha$

$q = 3\alpha$

$$\operatorname{sen} p + \operatorname{sen} q = 2 \operatorname{sen}\frac{p+q}{2} \cos\frac{p-q}{2}$$

$$\operatorname{sen} p - \operatorname{sen} q = 2 \operatorname{sen}\frac{p-q}{2} \cos\frac{p+q}{2}$$

$$\cos 30^\circ + \cos 50^\circ + 2\cos 40^\circ =$$

$$\boxed{\begin{aligned} &\cos p + \cos q = \\ &= 2\cos\frac{p+q}{2}\cos\frac{p-q}{2} \end{aligned}}$$

$$= 2\cos\frac{50^\circ+30^\circ}{2}\cos\frac{50^\circ-30^\circ}{2} + 2\cos 40^\circ =$$

$$= 2\cos 40^\circ \cos 10^\circ + 2\cos 40^\circ =$$

$$= \cos 40^\circ\left(2\cos 10^\circ + 2\right) =$$

$$= 2\cos 40^\circ\left(\cos 10^\circ + 1\right) = 2\cos 40^\circ \cdot \left(\cos 10^\circ + \cos 0^\circ\right) =$$

$$= 2\cos 40^\circ \cdot \left(2\cos\frac{10^\circ}{2} \cdot \cos\frac{10^\circ}{2}\right) = 4\cos 40^\circ \cos^2 5^\circ$$

Formule di Werner

Le formule di Werner non sono altro che l'inverso delle formule di Prostaferesi, ovvero contrariamente alle formule di Prostaferesi trasformano un prodotto di due seni, di due coseni o di un seno e un coseno in una somma algebrica fra seni o coseni.

$$\operatorname{sen}(\alpha+\beta) + \operatorname{sen}(\alpha-\beta) = 2\,\underbrace{\operatorname{sen}\alpha\cos\beta}$$

$$\boxed{\operatorname{sen}\alpha\cos\beta = \frac{1}{2}\left[\operatorname{sen}(\alpha+\beta) + \operatorname{sen}(\alpha-\beta)\right]}$$

$$\cos(\alpha+\beta) + \cos(\alpha-\beta) = 2\cos\alpha\cos\beta$$

$$\boxed{\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]}$$

$$\text{sen}(\alpha+\beta)-\text{sen}(\alpha-\beta)=2\,\text{sen}\,\beta\cos\alpha$$

$$\boxed{\text{sen}\,\beta\cos\alpha=\frac{1}{2}\left[\text{sen}(\alpha+\beta)-\text{sen}(\alpha-\beta)\right]}$$

$$\cos(\alpha+\beta)-\cos(\alpha-\beta)=-2\,\text{sen}\,\alpha\,\text{sen}\,\beta$$

$$\boxed{\text{sen}\,\alpha\,\text{sen}\,\beta=-\frac{1}{2}\left[\cos(\alpha+\beta)-\cos(\alpha-\beta)\right]=\frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]}$$