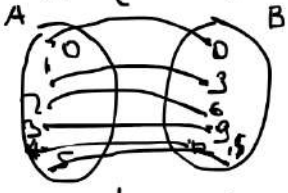
LEZIONE 16

Revisione esercizi verifica 4-5-6

$A = \{x \in \mathbb{N} \mid x < 6\}$



$\text{Im} f = B = \{0, 3, 6, 9, 12, 15\}$

$f: A \to B$

$f(0) = 0$

$f(1) = 3$

$f(2) = 6$

$\vdots$

$f(3) = 9$

$f(4) = 12$

$f(5) = 15$

Legge generale

$y = f(x) = 3x$

E' una funzione che associa ad ogni numero naturale il triplo di tale numero.

$\forall x \in A \; \exists! \; y \in B : y = 3x$

f INIETTIVA E SURRETTIVA

$\Downarrow$

f BIIETTIVA

f BIIETTIVA $\Longrightarrow \exists f^{-1}$ FUNZ. INVERSA

$y = 3x \Longrightarrow \boxed{x = \frac{y}{3}} \Longrightarrow \boxed{x = f^{-1}(y) = \frac{y}{3}}$

INVERSA

$f^{-1}: B \longrightarrow A$

$\Downarrow$

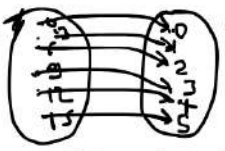
ASSOCIA AI NUMERI DI B $\frac{1}{3}$ DI QUESTI NUMERI

$f^{-1}(0) = 0$ $\quad f^{-1}(12) = 4$

$f^{-1}(3) = 1$ $\quad f^{-1}(15) = 5$

$f^{-1}(6) = 2$

$f^{-1}(9) = 3$



CONTROIMMAGINI (IMMAG. FUNZ. INVERSA)

$f(x) = 2x + 1$

$f : A \longrightarrow \mathbb{Z}$

$A = \{x \in \mathbb{Z} \mid -1 \leq x < 7\}$

$C = \operatorname{Im} f = ?$

$C \cup D = ?$

$A = \{-1, 0, 1, 2, 3, 4, 5, 6\}$

$g(x) = 2(5 - x) - 1$

$g : B \longrightarrow \mathbb{Z}$

$B = \{x \in \mathbb{N} \mid 12 : x = m, m \in \mathbb{N}\}$

$D = \operatorname{Im} g = ?$

$C \cap D = ?$

$B = \{1, 2, 3, 4, 6, 12\}$

$A = \{-1, 0, 1, 2, 3, 4, 5, 6\}$ $\quad f(x) = 2x + 1$

$f(-1) = 2 \cdot (-1) + 1 = -2 + 1 = \underline{-1}$

$f(0) = 2 \cdot (0) + 1 = 0 + 1 = \underline{1}$

$f(1) = 2 \cdot (1) + 1 = 2 + 1 = \underline{3}$

$f(2) = 2 \cdot (2) + 1 = 4 + 1 = \underline{5}$

$f(3) = 2 \cdot (3) + 1 = 6 + 1 = \underline{7}$

$f(4) = 2 \cdot (4) + 1 = 8 + 1 = \underline{9}$

$f(5) = 2 \cdot (5) + 1 = 10 + 1 = \underline{11}$

$f(6) = 2 \cdot (6) + 1 = 12 + 1 = \underline{13}$

$C = Imf = \{-1, 1, 3, 5, 7, 9, 11, 13\}$

$B = \{1, 2, 3, 4, 6, 12\}$ $\quad g(x) = 2(5-x) - 1$

$g(1) = 2 \cdot (5-1) - 1 = 2 \cdot 4 - 1 = 8 - 1 = 7$

$g(2) = 2 \cdot (5-2) - 1 = 2 \cdot 3 - 1 = 6 - 1 = 5$

$g(3) = 2 \cdot (5-3) - 1 = 2 \cdot 2 - 1 = 4 - 1 = 3$

$g(4) = 2 \cdot (5-4) - 1 = 2 \cdot 1 - 1 = 2 - 1 = 1$

$g(6) = 2 \cdot (5-6) - 1 = 2(-1) - 1 = -2 - 1 = -3$

$g(12) = 2 \cdot (5-12) - 1 = 2 \cdot (-7) - 1 = -14 - 1 = -15$

$\text{Im } g = D = \{-15, -3, 1, 3, 5, 7\}$

$C = \{-1, 1, 3, 5, 7, 9, 11, 13\}$ $D = \{-15, -3, 1, 3, 5, 7\}$

$C \cup D = \{-15, -3, -1, 1, 3, 5, 7, 9, 11, 13\}$

$C \cap D = \{1, 3, 5, 7\}$

$f: Q \longrightarrow Q$ $\qquad \boxed{y = f(x) = \frac{3}{4}x + \frac{1}{4}}$ LEGGE DI f

$f(-1) = \frac{3}{4}(-1) + \frac{1}{4} = -\frac{3}{4} + \frac{1}{4} = -\frac{2^1}{4_2} = -\frac{1}{2}$ $\boxed{f(-1) = -\frac{1}{2}}$

$y = \frac{3}{4}x + \frac{1}{4} \Rightarrow -\frac{3}{4}x - \frac{1}{4} = -y \Rightarrow \frac{3}{4}x + \frac{1}{4} = y$

$\Rightarrow 3x + 1 = 4y \Rightarrow 3x = 4y - 1 \Rightarrow \boxed{x = \frac{4y-1}{3}}$ $\boxed{x = f^{-1}(y) = \frac{4y-1}{3}}$

$$f^{-1}(y) = \frac{4y-1}{3}$$

LEGGE O FUNZIONE INVERSA

$$f^{-1}(1) = \frac{4(-1)-1}{3} = -\frac{4-1}{3} = -\frac{5}{3}$$

$f^{-1}(-1) = -\frac{5}{3}$

CONTROIMM. DI -1
IMM. f^{-1} IN -1

$f(x) = \frac{3}{4}x + \frac{1}{4}$ $\qquad$ $f^{-1}(x) = \frac{4x-1}{3}$

$f(x) = f^{-1}(x)$

$\frac{3}{4}x + \frac{1}{4} = \frac{4x-1}{3} \Rightarrow \frac{9x+3}{12} = \frac{4(4x-1)}{12}$

$9x + 3 = 16x - 4$

$9x - 16x = -4 - 3$

$-7x = -7 \Rightarrow 7x = 7 \Rightarrow \boxed{x = 1}$

plug $x = 1$

$\boxed{f(x) = f^{-1}(x)}$