LEZIONE 8

Revisione espressioni goniometriche contenenti archi associati e archi notevoli

$\boxed{134}$

$$\operatorname{sen}\frac{11}{6}\pi + \cos\frac{\pi}{3} + 5\,\operatorname{tg}\frac{7}{4}\pi =$$

$$= \operatorname{sen}330° + \cos 60° + 5\,\operatorname{tg}315° =$$

$$= \operatorname{sen}(360° - 30°) + \frac{1}{2} + 5\,\operatorname{tg}(45° + 270°)$$

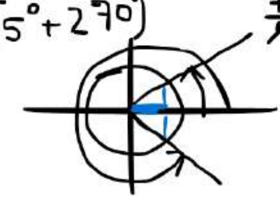
$$= -\operatorname{sen}30° + \frac{1}{2} - 5\,\operatorname{tg}(45°) =$$

$$= -\cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - 5 \cdot 1 = -5$$

$$\frac{11}{\cancel{6}_1} \cdot \cancel{180°}^{\,3}$$

$$\frac{180°}{3} = 60°$$

$$\frac{7}{\cancel{4}_1} \cdot \cancel{180}^{\,45} = 315°$$



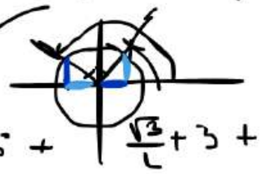
$\boxed{135}$

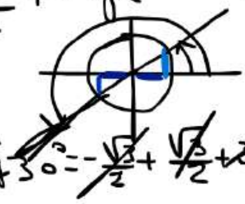
$$\operatorname{sen}\frac{\pi}{2}\cdot\cos\frac{5}{6}\pi + \frac{\sqrt{3}}{2} + 3\,tg\,\frac{7}{6}\pi =$$

$$= \operatorname{sen}90^\circ \cos150^\circ + \frac{\sqrt{3}}{2} + 3\,tg\,210^\circ =$$

$$= 1\cdot\cos(90^\circ+60^\circ) + \frac{\sqrt{3}}{2} + 3\,tg(180^\circ+30^\circ) =$$

$$=$$





$$-\operatorname{sen}60^\circ + \frac{\sqrt{3}}{2} + 3 + 3\cdot tg\,30^\circ = -\cancel{\frac{\sqrt{3}}{2}} + \cancel{\frac{\sqrt{3}}{2}} + \cancel{3}\cdot\frac{\sqrt{3}}{\cancel{3}} = \sqrt{3}$$

$$\frac{1}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}$$

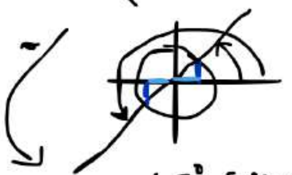
$$\frac{\pi}{2} = \frac{180^\circ}{2}$$

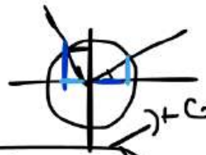
$$\frac{5}{6}\pi = \frac{180^\circ}{6} = 150^\circ$$

$$\frac{7}{6}\pi = \frac{7}{\cancel{6}}\cdot\cancel{180}^{30} = 210^\circ$$

136 $\operatorname{sen}\frac{5}{4}\pi \cdot \operatorname{sen}\frac{\pi}{4} + \operatorname{sen}\frac{2}{3}\pi \cos\frac{\pi}{6} =$

$= \operatorname{sen}(180^\circ + 45^\circ) \cdot \operatorname{sen} 45^\circ + \operatorname{sen} 120^\circ \cos 30^\circ =$





$\frac{5}{4}\pi = \frac{5}{\cancel{4}_1} \cdot \overset{45^\circ}{\cancel{180}} = 225$

$\frac{2}{3}\pi = \frac{2}{\cancel{3}_1} \cdot \overset{60^\circ}{\cancel{180}}$

$= -\operatorname{sen} 45^\circ \cdot \operatorname{sen} 45^\circ + \operatorname{sen}(90^\circ + 30^\circ) \cos 30^\circ =$

$) + \cos 30^\circ$

$= -\operatorname{sen}^2 45^\circ + \cos^2 30^\circ = -\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = -\frac{2}{4} + \frac{3}{4} = \frac{1}{4}$

137

$$4\cos\frac{3}{4}\pi \cdot \operatorname{sen}\frac{5}{3}\pi - 6\operatorname{sen}\frac{7}{4}\pi \cdot \operatorname{tg}\left(-\frac{\pi}{6}\right) =$$

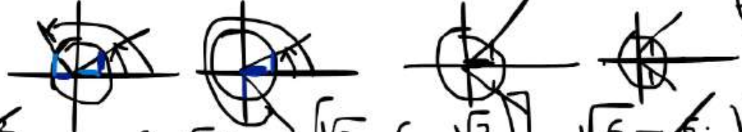
$$= 4\cos 135° \cdot \operatorname{sen} 300° - 6\operatorname{sen} 315° \cdot \operatorname{tg}(-30°) =$$

$$= 4\cos(90°+45°) \cdot \operatorname{sen}(270°+30°) - 6\operatorname{sen}(270°+45°) \cdot \operatorname{tg}(-30°) =$$

$$= -4\operatorname{sen}45° \cdot (-\cos 30°) + 6\operatorname{sen}45° \left[-\operatorname{tg}30°\right]$$

$$\frac{3}{4}\pi = \frac{3}{4} \cdot \frac{180°}{1} = 135°$$

$$\frac{5}{3}\pi = 300°$$

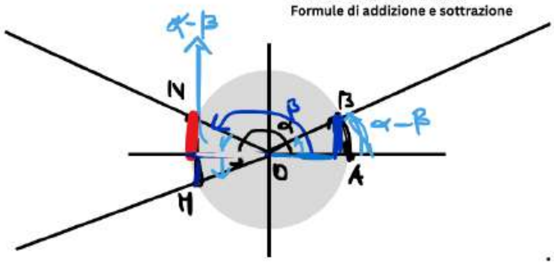


$$= -\cancel{4}^{2} \cdot \frac{\sqrt{2}}{\cancel{2}} \cdot \left(-\frac{\sqrt{3}}{\cancel{2}}\right) + 6\left[\frac{\sqrt{2}}{2} \cdot \left(-\frac{\sqrt{3}}{3}\right)\right] = \sqrt{6} - \cancel{6} \cdot \frac{\sqrt{6}}{\cancel{6}}$$

$$= \sqrt{6} - \sqrt{6} = \boxed{0}$$

FORMULE GONIOMETRICHE

Formule di addizione e sottrazione



α, β

$sen(\alpha \pm \beta) = ?$

$cos(\alpha \pm \beta) = ?$

$\boxed{OA = OB = ON = OM = 1}$

I triangoli OAB e OMN sono uguali per il 1º criterio di uguaglianza
e questo implica che MN = AB

Your paragraph text

$A(1,0) \quad B(\cos(\alpha-\beta), \operatorname{sen}(\alpha-\beta))$

$N(\cos\beta, \operatorname{sen}\beta) \quad M(\cos\alpha, \operatorname{sen}\alpha)$

$\overline{AB} = \overline{MN}$

$$\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} = \sqrt{(x_M - x_N)^2 + (y_M - y_N)^2}$$

$$\sqrt{(\cos(\alpha-\beta) - 1)^2 + (\operatorname{sen}(\alpha-\beta) - 0)^2} = \sqrt{(\cos\alpha - \cos\beta)^2 + (\operatorname{sen}\alpha - \operatorname{sen}\beta)^2}$$

$$[\cos(\alpha-\beta) - 1]^2 + \operatorname{sen}^2(\alpha-\beta) = (\cos\alpha - \cos\beta)^2 + (\operatorname{sen}\alpha - \operatorname{sen}\beta)^2$$

$$\cos^2(\alpha-\beta) + 1 - 2\cos(\alpha-\beta) + \operatorname{sen}^2(\alpha-\beta) = \cos^2\alpha + \cos^2\beta - 2\cos\alpha\cos\beta + \operatorname{sen}^2\alpha + \operatorname{sen}^2\beta - 2\operatorname{sen}\alpha\operatorname{sen}\beta$$

$$\underbrace{[\cos^2(\alpha-\beta) + \operatorname{sen}^2(\alpha-\beta)]}_{1} + 1 - 2\cos(\alpha-\beta) = \underbrace{(\cos^2\alpha + \operatorname{sen}^2\alpha)}_{1} + \underbrace{(\cos^2\beta + \operatorname{sen}^2\beta)}_{1} - 2\cos\alpha\cos\beta - 2\operatorname{sen}\alpha\operatorname{sen}\beta$$

$$\cancel{1+1} - 2\cos(\alpha-\beta) = \cancel{2} - 2\cos\alpha\cos\beta - 2\operatorname{sen}\alpha\operatorname{sen}\beta$$

$$-2\cos(\alpha-\beta) = -2\cos\alpha\cos\beta - 2\operatorname{sen}\alpha\operatorname{sen}\beta$$

$$\boxed{\cos(\alpha-\beta) = \cos\alpha\cos\beta + \operatorname{sen}\alpha\operatorname{sen}\beta}$$

DIVIDO PER 2
E MOLTIPLICO
PER −1

Formula di sottrazione del coseno

$$\cos(\alpha - (-\beta)) = \cos(\alpha + \beta) = \cos\alpha \cos(-\beta) + \text{sen}\,\alpha\, \text{sen}(-\beta)$$
$$= \cos\alpha \cos\beta - \text{sen}\, \text{sen}\,\beta$$

Formula di addizione del coseno

α

$$\cos((90^\circ - \alpha) - \beta) = \cos(90^\circ - \alpha)\cos\beta + \text{sen}(90^\circ - \alpha)\,\text{sen}\,\beta =$$

$\cos(-\beta) = \cos\beta$
$\text{sen}(-\beta) = -\text{sen}\,\beta$

$$= \text{sen}\,\alpha \cos\beta + \cos\alpha\, \text{sen}\,\beta$$

$$\cos(90^\circ - (\alpha + \beta)) = \text{sen}(\alpha + \beta)$$

$$\boxed{\text{sen}(\alpha + \beta) = \text{sen}\,\alpha \cos\beta + \cos\alpha\, \text{sen}\,\beta}$$

$$\boxed{\text{sen}(\alpha + (-\beta)) = \text{sen}\,\alpha \cos(-\beta) + \cos\alpha\, \text{sen}(-\beta) = \text{sen}\,\alpha \cos\beta - \cos\alpha\, \text{sen}\,\beta}$$

Formule di addizione e sottrazione del seno

$$\boxed{\begin{aligned} \cos(\alpha \mp \beta) &= \cos\alpha \cos\beta \pm \sin\alpha \sin\beta \\ \sin(\alpha \pm \beta) &= \sin\alpha \cos\beta \pm \cos\alpha \sin\beta \end{aligned}}$$