Esercizi sulle formule di addizione e sottrazione

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$$\text{sen}(\alpha+\beta)\,\text{sen}(\alpha-\beta) =$$

$$= [\underbrace{\text{sen}\,\alpha\cos\beta}_{} + \underbrace{\cos\alpha\,\text{sen}\,\beta}_{}][\underbrace{\text{sen}\,\alpha\cos\beta}_{} - \underbrace{\cos\alpha\,\text{sen}\,\beta}_{}] =$$

$$= (A + B)(A - B)$$

$$= \text{sen}^2\alpha\cos^2\beta - \cos^2\alpha\,\text{sen}^2\beta =$$

$$= \text{sen}^2\alpha(1-\text{sen}^2\beta) - (1-\text{sen}^2\alpha)\,\text{sen}^2\beta =$$

$$= \text{sen}^2\alpha \underline{\cancel{- \text{sen}^2\alpha\,\text{sen}^2\beta}} - \text{sen}^2\beta \underline{\cancel{+ \text{sen}^2\alpha\,\text{sen}^2\beta}}$$

$$\underline{\text{sen}^2\alpha - \text{sen}^2\beta}$$

$$\text{sen}(\alpha+\beta) = \text{sen}\,\alpha\cos\beta + \cos\alpha\,\text{sen}\,\beta$$

$$\text{sen}(\alpha-\beta) = \text{sen}\,\alpha\cos\beta - \cos\alpha\,\text{sen}\,\beta$$

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$$\underbrace{\cos(\alpha+\beta)}\ \underbrace{\cos(\alpha-\beta)} =$$

$$= \underbrace{[\cos\alpha\cos\beta - \operatorname{sen}\alpha\operatorname{sen}\beta]}_{(A\ -\ B)}\ \underbrace{[\cos\alpha\cos\beta + \operatorname{sen}\alpha\operatorname{sen}\beta]}_{(A\ +\ B)} =$$

$$= \cos^2\alpha\cos^2\beta - \operatorname{sen}^2\alpha\operatorname{sen}^2\beta =$$

$$= \cos^2\alpha\,(1 - \operatorname{sen}^2\beta) - (1 - \cos^2\alpha)\operatorname{sen}^2\beta$$

$$= \cos^2\alpha - \cancel{\cos^2\alpha\operatorname{sen}^2\beta} - \operatorname{sen}^2\beta + \cancel{\cos^2\alpha\operatorname{sen}^2\beta} =$$

$$= \boxed{\cos^2\alpha - \operatorname{sen}^2\beta}$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \operatorname{sen}\alpha\operatorname{sen}\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \operatorname{sen}\alpha\operatorname{sen}\beta$$

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$$\cos(\alpha-\beta)\cos\alpha + \mathrm{sen}(\alpha-\beta)\,\mathrm{sen}\,\alpha =$$

$$= [\cos\alpha\cos\beta + \mathrm{sen}\,\alpha\,\mathrm{sen}\,\beta]\cos\alpha + [\mathrm{sen}\,\alpha\cos\beta - \cos\alpha\,\mathrm{sen}\,\beta]\,\mathrm{sen}\,\alpha =$$

$$= \cos^2\alpha\cos\beta + \cancel{\mathrm{sen}\,\alpha\cos\alpha\,\mathrm{sen}\,\beta} + \mathrm{sen}^2\alpha\cos\beta - \cancel{\mathrm{sen}\,\alpha\cos\alpha\,\mathrm{sen}\,\beta} =$$

$$= \cos^2\alpha\cos\beta + \mathrm{sen}^2\alpha\cos\beta = \cos\beta\,(\cos^2\alpha + \mathrm{sen}^2\alpha) = \boxed{\cos\beta}$$

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$$\frac{\cos(\alpha+\beta)+\cos(\alpha-\beta)}{\operatorname{sen}(\alpha+\beta)+\operatorname{sen}(\alpha-\beta)}$$

$$\frac{[\cos\alpha\cos\beta-\cancel{\operatorname{sen}\alpha\operatorname{sen}\beta}]+[\cos\alpha\cos\beta+\cancel{\operatorname{sen}\alpha\operatorname{sen}\beta}]}{[\operatorname{sen}\alpha\cos\beta+\cancel{\cos\alpha\operatorname{sen}\beta}]+[\operatorname{sen}\alpha\cos\beta-\cancel{\cos\alpha\operatorname{sen}\beta}]}$$

$$=\frac{\cancel{2}\cos\alpha\cancel{\cos\beta}}{\cancel{2}\operatorname{sen}\alpha\cancel{\cos\beta}}=\frac{\cos\alpha}{\operatorname{sen}\alpha}=\boxed{\operatorname{cotg}\alpha}$$

$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\operatorname{sen}\alpha\operatorname{sen}\beta$

$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\operatorname{sen}\alpha\operatorname{sen}\beta$

$\operatorname{sen}(\alpha+\beta)=\operatorname{sen}\alpha\cos\beta+\cos\alpha\operatorname{sen}\beta$

$\operatorname{sen}(\alpha-\beta)=\operatorname{sen}\alpha\cos\beta-\cos\alpha\operatorname{sen}\beta$

$$\frac{\cos(\alpha-\beta)}{\operatorname{sen}\beta\cos\alpha} =$$

$$= \frac{\cos\alpha\cos\beta + \operatorname{sen}\alpha\operatorname{sen}\beta}{\operatorname{sen}\beta\cos\alpha} =$$

$$= \frac{\cancel{\cos\alpha}\cos\beta}{\operatorname{sen}\beta\cancel{\cos\alpha}} + \frac{\operatorname{sen}\alpha\cancel{\operatorname{sen}\beta}}{\cancel{\operatorname{sen}\beta}\cos\alpha} =$$

$$= \frac{\cos\beta}{\operatorname{sen}\beta} + \frac{\operatorname{sen}\alpha}{\cos\alpha} = \underline{\operatorname{cotg}\beta + \operatorname{tg}\alpha}$$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \operatorname{sen}\alpha\operatorname{sen}\beta$

FORMULA DI ADDIZIONE E SOTTRAZIONE DELLA TANGENTE

$$tg(\alpha+\beta) = \frac{sen(\alpha+\beta)}{cos(\alpha+\beta)} = \frac{sen\alpha\cos\beta + cos\alpha\, sen\beta}{cos\alpha\cos\beta - sen\alpha\, sen\beta}$$

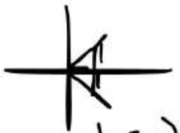
DIVIDO TUTTO PER $\boxed{cos\alpha\cos\beta}$

$$tg(\alpha+\beta) = \frac{\frac{sen\alpha\,\cancel{cos\beta}}{cos\alpha\,\cancel{cos\beta}} + \frac{\cancel{cos\alpha}\,sen\beta}{\cancel{cos\alpha}\,cos\beta}}{\frac{\cancel{cos\alpha\,cos\beta}}{\cancel{cos\alpha\,cos\beta}} - \frac{sen\alpha\,sen\beta}{cos\alpha\,cos\beta}} = \boxed{\frac{tg\alpha + tg\beta}{1 - tg\beta\cdot tg\alpha}}$$

$$\boxed{\operatorname{tg}(\alpha+\beta)=\frac{\operatorname{tg}\alpha+\operatorname{tg}\beta}{1-\operatorname{tg}\alpha\operatorname{tg}\beta}}$$

$$\operatorname{tg}(\alpha-\beta)=\frac{\operatorname{tg}\alpha+\operatorname{tg}(-\beta)}{1-\operatorname{tg}\alpha\operatorname{tg}(-\beta)}$$

$$\boxed{\operatorname{tg}(\alpha-\beta)=\frac{\operatorname{tg}\alpha-\operatorname{tg}\beta}{1+\operatorname{tg}\alpha\operatorname{tg}\beta}}$$



$\operatorname{tg}(-\beta)=-\operatorname{tg}\beta$