LEZIONE 9

Calcolo domini funzioni algebriche

$|3| = 3$

$|-3| = 3$

91

$$y = \sqrt[3]{\frac{3x-6}{3-|x|}} + \frac{6x}{\sqrt{x-2}}$$

$$\begin{cases} 3-|x| \neq 0 & 1^{\text{a}} \text{ DEN} \\ x-2 \neq 0 & 2^{\text{a}} \text{ DEN} \end{cases}$$

$-|x| \neq -3 \Rightarrow |x| \neq 3 \Rightarrow \boxed{x \neq \pm 3}$

$x-2 \neq 0 \Rightarrow \boxed{x \neq 2}$

$$\begin{cases} x \neq \pm 3 \\ x \neq 2 \end{cases}$$

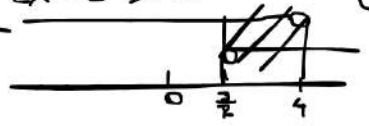
$$\boxed{\text{Dom } f = \mathbb{R} \setminus \{2; \pm 3\}}$$

92 $y = \frac{4x^2}{\sqrt{4-x}} - \frac{2x}{\sqrt{2x-3}}$

$\mathrm{Dom} f =]\frac{3}{2}; 4[$

$$\begin{cases} 4-x>0 \\ 2x-3>0 \end{cases} \Longrightarrow \begin{cases} -x>-4 \Rightarrow x<4 \\ 2x>3 \Rightarrow x>\frac{3}{2} \end{cases}$$

$\frac{3}{2} < x < 4$



93 $y = \dfrac{x+1}{\sqrt{x^2-6x+9}}$

$0 > 0$ No!!

$x^2 - 6x + 9 > 0$ $\quad$ $\text{Dom } f = \mathbb{R} \setminus \{3\}$

$(x-3)^2 > 0$

$x - 3 \neq 0 \Rightarrow \boxed{x \neq 3}$

94 $y = \frac{5x^4 - 3x + 3}{\sqrt[3]{2-x}} - \frac{x^2 - 2x + 1}{\sqrt{x^2 - 6x + 9}}$

$$\begin{cases} 2 - x \neq 0 \\ x^2 - 6x + 9 > 0 \end{cases} \Rightarrow \begin{cases} -x \neq -2 \Rightarrow \boxed{x \neq 2} \\ (x-3)^2 > 0 \Rightarrow \boxed{x \neq 3} \end{cases}$$

$\boxed{\text{Dom } f = \mathbb{R} \setminus \{2, 3\}}$ $\quad \boxed{x \neq 2 \wedge x \neq 3}$

95 $y = \dfrac{\sqrt[3]{2-x}}{\sqrt[4]{x^4-81}}$ Dom$\mathbb{R}$ =]−∞;−3[∪]3;+∞[

$x^4 - 81 > 0$

$(x^2-9)(x^2+9) > 0$

$(x-3)(x+3)(x^2+9) > 0$

$A^2 - B^2 = (A+B)(A-B)$

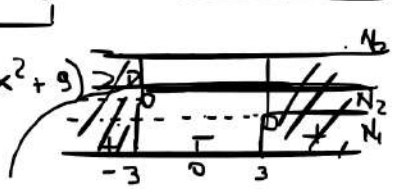
$x < -3 \vee x > 3$

N_1 $x-3>0$ $\boxed{x>3}$

N_2 $x+3>0$ $\boxed{x>-3}$

N_3 $x^2+3>0$

$x^2 > -9$ $\boxed{\forall x \in \mathbb{R}}$



96 $y = \sqrt{x - 3|x|}$ $\text{Dom} f = \{0\}$ $\left(\begin{cases} x \geq 0 \\ x \leq 0 \end{cases}\right.$ $\cup$ $\left.\begin{cases} x < 0 \\ x \geq 0 \end{cases}\right.$

$x - 3|x| \geq 0$

$\begin{cases} x \geq 0 \\ x - 3x \geq 0 \end{cases}$ $\cup$ $\begin{cases} x < 0 \\ x - 3(-x) \geq 0 \end{cases}$

$\begin{cases} x \geq 0 \\ -2x \geq 0 \Rightarrow x \leq 0 \end{cases}$ $\cup$ $\begin{cases} x < 0 \\ 4x \geq 0 \end{cases}$

$\boxed{x = 0}$ $\cup$ $\forall x \in \mathbb{R}$

0

$\boxed{x = 0}$ $\cup$ $\emptyset$

104 $y = \sqrt{\sqrt{x-1} - x + 3}$

$$\begin{cases} x-1 \geq 0 \Longrightarrow x \geq 1 \\ \sqrt{x-1} - x + 3 \geq 0 \Longrightarrow \boxed{\sqrt{x-1} \geq 3 - x} \end{cases}$$

!!!!!

$\sqrt{x-1} \geq 3-x$ $\quad n=2$ $\quad \boxed{\sqrt[n]{f(x)} \geq g(x)}$

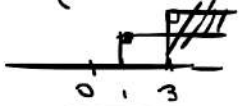
$\begin{cases} f(x) \geq 0 \\ g(x) < 0 \end{cases} \cup \begin{cases} g(x) \geq 0 \\ f(x) \geq [g(x)]^n \end{cases}$

$\begin{cases} x-1 \geq 0 \\ 3-x < 0 \end{cases} \cup \begin{cases} 3-x \geq 0 \\ x-1 \geq (3-x)^2 \end{cases}$

$\begin{cases} \boxed{x \geq 1} \\ -x < -3 \Rightarrow \boxed{x > 3} \end{cases} \cup \begin{cases} -x \geq -3 \Rightarrow \boxed{x \leq 3} \\ x-1 \geq 9+x^2-6x \end{cases}$

$\begin{cases} x \geq 1 \\ x > 3 \end{cases} \cup \begin{cases} x \leq 3 \\ -x^2+7x-10 \geq 0 \Rightarrow x^2-7x+10 \leq 0 \end{cases}$

$\begin{cases} x \geq 1 \\ x > 3 \end{cases}$



$x > 3$

$\cup$

$\begin{cases} x \leq 3 \\ x^2 - 7x + 10 \leq 0 \end{cases}$

$\Delta = 49 - 4(1)(10) = 49 - 40 = 9$

$x_1, x_2 = \dfrac{7 \pm 3}{2} \rightarrow x_1 = \dfrac{4}{2} = 2$; $x_2 = \dfrac{10}{2} = 5$

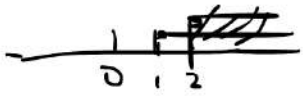
$2 \leq x \leq 5$

$\begin{cases} x \leq 3 \\ 2 \leq x \leq 5 \end{cases}$

$2 \leq x \leq 3$

$x \geq 3 \cup 2 \leq x \leq 3 \Longrightarrow \boxed{x \geq 2}$

$\begin{cases} x \geq 1 \\ x \geq 2 \end{cases}$



104 $y = \sqrt{\sqrt{x-1} - x + 3}$

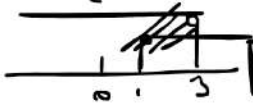
$$\begin{cases} x - 1 \geq 0 \Rightarrow x \geq 1 \\ \sqrt{x-1} - x + 3 \geq 0 \Rightarrow \boxed{\sqrt{x-1} \geq x - 3} \end{cases}$$

$\sqrt{x-1} \geq x-3$

$\begin{cases} x-1 \geq 0 \\ x-3 < 0 \end{cases} \cup \begin{cases} x-3 \geq 0 \\ x-1 \geq x^2+9-6x \end{cases}$

$\begin{cases} x \geq 1 \\ x < 3 \end{cases} \cup \begin{cases} x \geq 3 \\ x^2+7x-10 \geq 0 \Rightarrow x^2-7x+10 \leq 0 \end{cases}$

$\begin{cases} x \geq 3 \\ x^2-7x+10 \leq 0 \end{cases}$



$1 \leq x < 3$

$m=2$

$\begin{cases} f(x) \geq 0 \\ g(x) < 0 \end{cases} \cup \begin{cases} g(x) \geq 0 \\ f(x) \geq [g(x)]^2 \end{cases}$

$\sqrt{f(x)} \geq g(x)$

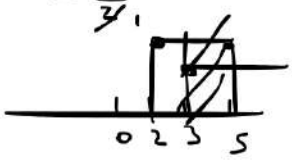
$x^2 - 7x + 10 \leq 0$

$\Delta = 49 - 4(1)(10) = 49 - 40 = 9$

$x_{1,2} = \frac{7 \pm 3}{2}$ → $\frac{4}{2} = 2$; $\frac{10}{2} = 5$

$2 \leq x \leq 5$

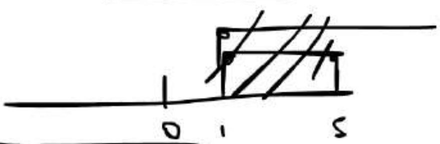
$\begin{cases} x \geq 3 \\ 2 \leq x \leq 5 \end{cases}$



$3 \leq x \leq 5$

$1 \leq x < 3 \cup 3 \leq x \leq 5 \Longrightarrow \boxed{1 \leq x \leq 5}$

$$\begin{cases} x \geq 1 \\ 1 \leq x \leq 5 \end{cases}$$



$\boxed{1 \leq x \leq 5}$

$Dom f = [1; 5]$

97

$$y = \frac{x^2-3}{e-|x^2-1|}$$

$$x \neq \pm\sqrt{1+e} \qquad \text{Dom } f = \mathbb{R} \setminus \{\pm\sqrt{1+e}\}$$

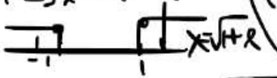
$$e-|x^2-1| \neq 0$$

$$-|x^2-1| \neq -e \Rightarrow |x^2-1| \neq e$$

$$|x^2-1| = e$$

$$\begin{cases} x^2-1 \geq 0 \\ x^2-1 = e \end{cases} \cup \begin{cases} x^2-1 < 0 \\ 1-x^2 = e \end{cases}$$

$$\begin{cases} x^2 \geq 1 \Rightarrow x \leq -1 \vee x \geq 1 \\ x^2 = e+1 \Rightarrow x = \pm\sqrt{1+e} \end{cases} \cup \begin{cases} -1 < x < 1 \\ -x^2 = e-1 \\ x^2 = 1-e \end{cases} \quad x = \pm\sqrt{1-e} \quad \nexists$$



$$\text{Dom } f = \left]-\infty; -\sqrt{1+e}\right[\cup \left]-\sqrt{1+e}; \sqrt{1+e}\right[\cup \left]\sqrt{1+e}; +\infty\right[$$