

## Operazioni con i polinomi

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Gli esercizi sono proposti in ordine di difficoltà crescente.

**nota:** in un file così lungo e complesso può accadere che sia presente un errore di diversa natura nonostante gli esercizi siano stati controllati più volte. Saremo grati di ricevere segnalazioni di eventuali refusi o suggerimenti di qualsiasi natura.

## 1. somma algebrica di polinomi



1	$a + (1 - a)$	1
2	$(1 - a) + 2a$	$1 + a$
3	$x - (x - 1)$	1
4	$a - (a + 1 - a^2)$	$a^2 - 1$
5	$(a + 1) + (a - 1)$	$2a$
6	$(a + 1) - (a - 1)$	2
7	$(x - 1) - (x + 1)$	-2
8	$-(a^2 + b^2) - (-a^2 - b^2)$	0
9	$(a^2 - 1) + (1 - a - a^2)$	-a
10	$(x^2 + y^2 + 1) - (1 - x^2)$	$2x^2 + y^2$
11	$(3 - a + a^2) + (3 + 2a - 2a^2)$	$-a^2 + a + 6$
12	$(3 - a + a^2) - (3 + 2a - 2a^2)$	$-3a + 3a^2$
13	$-(3 - a + a^2) + (3 + 2a - 2a^2)$	$3a - 3a^2$
14	$-(3 - a + a^2) - (3 + 2a - 2a^2)$	$-6 - a + a^2$

15	$\left(2x^3 - \frac{8}{3}x^2 + \frac{1}{4}x + 1\right) - \left(\frac{1}{5}x^3 - \frac{5}{2}x^2 + x + 1\right)$	$\frac{9}{5}x^3 - \frac{1}{6}x^2 - \frac{3}{4}x$
16	$(3x^2b^3 + 7x^2 - b^2) - (+8x^2 - 4x^2b^3 + b^2)$	$7x^2b^3 - x^2 - 2b^2$
17	$\left(\frac{3}{4}mn - n^3 + \frac{1}{4}an^3\right) - \left(\frac{1}{4}mn - \frac{1}{2}an^3\right)$	$\frac{1}{2}mn - n^3 + \frac{3}{4}an^3$
18	$\left(\frac{1}{3}a^2 + a - 5\right) + \left(\frac{2}{3}a^2 - a\right) - (a^2 - 3a + 1)$	$3a - 6$
19	$[2x - (a - b)] + (3a + b) - (2a + 3b)$	$2x - b$
20	$-\frac{2}{3}a - \left\{-\left[\frac{1}{2}b - \left(2a - \frac{1}{6}\right)\right] + b - \frac{1}{3}a\right\}$	$\frac{1}{6} - \frac{7}{3}a - \frac{1}{2}b$
21	$(0, \bar{6}m^2 - m) - (m^2 - 3m + 1) - \left(\frac{1}{3}m^2 + m - 5\right)$	$4 + m - \frac{2}{3}m^2$
22	$(x^2 - x + 1) - (3x^2 + 2) - (5 - 2x^2)$	$-x - 6$
23	$[-(a^2b^2c + 3ab^2c) + (7ab^2c + 8a^2b^3c) + a^2b^2c] - 3a^2b^3c + -(-ab^2c + 5a^2bc)$	$5ab^2c - 5a^2bc + 5a^2b^3c$

## 2. prodotto di un polinomio per un monomio



24	$2(a + b)$	$2a + 2b$
25	$a(x + 1)$	$ax + a$
26	$b(b - 1)$	$b^2 - b$
27	$-x(x - x^2)$	$x^3 - x^2$
28	$(a - x)3x$	$3ax - 3x^2$

29	$b(a + b - c)$	$ab + b^2 - bc$
30	$(2x^2 + 3ax - b^2)(-4ab)$	$-8abx^2 - 12a^2bx + 4ab^3$
31	$(a^2 - 3x + 2)3x$	$3a^2x - 9x^2 + 6x$
32	$-3a^2x(ax - 4x + 2)$	$-3a^3x^2 + 12a^2x^2 - 6a^2x$
33	$2a(2a^2 - 4a - 3)$	$4a^3 - 8a^2 - 6a$
34	$\frac{1}{3}a(x - 3y + 6z)$	$\frac{1}{3}ax - ay + 2az$
35	$\frac{1}{2}b^2x^3(-2x + 4b - 6x^2 + 8b^2)$	$-b^2x^4 + 2b^3x^3 - 3b^2x^5 + 4b^4x^3$
36	$-0,5xy(xy + 4x^2y + 4xy^2 - 0,6x^2y^2)$	$-\frac{1}{2}x^2y^2 - 2x^3y^2 - 2x^2y^3 + \frac{1}{3}x^3y^3$
37	$3mn(0,6m^2n + 0,2mn - m^2n^2)$	$2m^3n^2 + \frac{3}{5}m^2n^2 - 3m^3n^3$
38	$\left(-\frac{2}{3}m^2n + 0,3m^2n^2 - \frac{2}{3}mn^2t^2\right)(3m^2n^2t^3)$	$-2m^4n^3t^3 + m^4n^4t^3 - 2m^3n^4t^5$
39	$\left(-3xy + 4xy^2 - \frac{2}{3}x^2y\right)\left(-\frac{1}{2}xy\right)$	$\frac{3}{2}x^2y^2 - 2x^2y^3 + \frac{1}{3}x^3y^2$
40	$2b^2x^3(0,5b + 0,25x - 0,16x^2 - 0,125b^2)$	$b^3x^3 + \frac{1}{2}b^2x^4 - \frac{1}{3}b^2x^5 - \frac{1}{4}b^4x^3$
41	$3a(a - b) + b(2b + 3a) - 3(a^2 - b^2)$	$5b^2$
42	$p(p + q + 1) - q(p - 1) - p(p + q) + q(p + 1)$	$p + 2q$

43	$7a^2b^2\left(-\frac{4}{7}b + \frac{5}{7}a\right) + (-a^2b)(5ab - 4b^2 + 8a^2)$	$-8a^4b$
44	$\left(mn + 5n^2 - \frac{1}{3}mn^3\right)(3m^2n) - \left(\frac{1}{5}m - n - mn\right)15m^2n^2$	$15m^3n^3 - m^3n^4 + 30m^2n^3$
45	$a^2(2a - 3b)5b - \frac{1}{2}(ab - 4b^2)a^2 - \left(-\frac{1}{2}a^3b\right)$	$10a^3b - 13a^2b^2$
46	$\frac{x^2}{2}(x - y + 1) - \frac{2}{3}x(-x^2 + x) - x^2(y - 3) - \frac{23}{6}x^2$	$\frac{7}{6}x^3 - x^2 - \frac{3}{2}x^2y$
47	$xy(x^2 - y^3) - (2xy^2 + x)x^2y + (x^2 - 2y)\left(-\frac{xy^3}{2}\right)$	$-\frac{5}{2}x^3y^3$
48	$\left[2(x^2 - 3xy) + \left(\frac{1}{2}x - 3y\right)2x - (3x - 6y)\frac{1}{3}x\right]\frac{1}{2}x$	$x^3 - 5x^2y$
49	$\left[\frac{1}{3}a^2\left(\frac{4}{3}a - b\right) - \frac{1}{4}a(a^2 - 2ab)\right]\left(-\frac{3}{4}ab\right) - b^2\left(\frac{1}{24}a^3 - a^2b\right) + \frac{1}{3}ab\left(a^3 + \frac{1}{2}a^2b\right)$	$\frac{3}{16}a^4b + a^2b^3$
50	$x^2 - xy(y - z) - \frac{x}{2}(yz + x - 6) - 3x - \frac{1}{2}xyz$	$\frac{1}{2}x^2 - xy^2$
51	$(2 + x)\left\{-\left[2xy\left(\frac{1}{2}x - \frac{3}{4}y\right)\right] + x^2y\right\}^2 - 9x^2y^4\left(\frac{1}{4}x + \frac{1}{2}\right)$	0
52	$a\left\{a^3 - \left[(-4a^2 + 5b^2 + 2ab)(-a) + \left(\frac{5}{2}b + a\right)2ab\right]\right\}$	$-3a^4$
53	$\left\{x^2 - \left[3y\left(x - \frac{y}{9}\right) - \frac{2}{3}x\left(y - \frac{3}{4}x\right)\right]\right\}\left(-\frac{2}{3}x\right) - \frac{1}{9}x(-2xy + 2y^2 - 3x^2)$	$\frac{16}{9}x^2y - \frac{4}{9}xy^2$
54	$a^2\left(a + \frac{1}{3}\right) + \left(-\frac{3}{5}a - \frac{3}{2}a^2 + \frac{1}{5}a^3\right)\left(\frac{5}{9}a\right) + \frac{1}{3}\left(a + \frac{1}{9}a^4\right) - \frac{4}{27}a^4 - 2a\left(a^2 + \frac{1}{6}\right)$	$-\frac{11}{6}a^3$
55	$\left[(-5a^2 + 2b^2)ab + \left(\frac{5}{2}a^2b - b^3\right)(2a)\right]^0$	<i>perde di significato</i>
56	$(y - x - 2)(-y) + 2(-2y) + (x - 2y + 3)\left(\frac{1}{2}x\right) + 2y(1 + y) - \frac{1}{4}x(2 + 2x)$	$x + y^2$

57	$(x^2 + 1)2x^2 - 3x^2(x^2 - x + 1) - (-x^3 + x^2 - x)x$	$2x^3$
58	$x^2y^2 + \left[xy\left(\frac{x}{2} + \frac{3}{2}y + 1\right) - x\left(xy + \frac{3}{2}y^2 + y\right)\right]^2 xy - \frac{1}{4}x^5y^3$	$x^2y^2$

## 3. prodotto di polinomi



59	$(2 - a)(a + 3)$	$-a^2 - a + 6$
60	$(x + 3)(x - 4)$	$x^2 - x - 12$
61	$(2m - 1)(m - 2)$	$2m^2 - 5m + 2$
62	$(a - 2)(a + 6)$	$a^2 + 4a - 12$
63	$\left(a + \frac{1}{2}\right)(2b - 6)$	$2ab - 6a + b - 3$
64	$(2a^2 + b^3)(a^3 + 2b^2)$	$2a^5 + 4a^2b^2 + a^3b^3 + 2b^5$
65	$\left(x - \frac{1}{3}\right)(3y - 6)$	$3xy - 6x - y + 2$
66	$(a - 2)(a + 2)$	$a^2 - 4$
67	$(a + b + c)(a + b - c)$	$a^2 + 2ab + b^2 - c^2$
68	$(x - y)(x^2 + xy + y^2)$	$x^3 - y^3$
69	$(a + b)(a^2 - ab + b^2)$	$a^3 + b^3$

70	$(5a - b + 3c)(5a + b - 3c)$	$25a^2 - b^2 + 6bc - 9c^2$
71	$(x + 1)(x - 2)(x + 3)$	$x^3 + 2x^2 - 5x - 6$
72	$(a + b)(a^2 - ab + b^2)(a^3 - b^3)$	$a^6 - b^6$
73	$(1 - a)(1 + a + a^2)(1 + a^3 + a^6)$	$1 - a^9$
74	$(2x - 1)(2x + 1) + (-2x - 1)(2x + 1) - 2(-2x + 1)$	$-4$
75	$(x - y)(1 + x) + (y - 1)(x - 2y) - y(1 - 2y)$	$x^2$
76	$3x^2y(xy - 1) + (xy - 1)(xy + 1) - x^2y^2(3x + 1) + 1$	$-3x^2y$
77	$(m + 3)(m + 2)(m + 1) - (m - 3)(m - 2)(m - 1) - 6(m^2 + 2)$	$6m^2$
78	$(3b - a^2)(a^3 - 4b^3) - (3b - a^2)(a^3 + 2b^3) - 6b(b^3 + a^2b^2)$	$-24b^4$
79	$3x^2(3 + x^2) - (x^2 - 1)(x^2 - 2) - 2[(x^2 + 1)(x^2 + 2) - 3]$	$6x^2$
80	$(1 + x - x^3)(5 + x^3) - (1 - x^2)(1 + x + x^2)(1 - x + x^2) - 4(1 - x^3) - 5x(1 - x^3)$	$6x^4$
81	$2(3x + 1)(2x - 1) - 2(6x + 1)(x + 2) + (-3x)^2 - 5(4x + 1)$	$9x^2 - 48x - 11$
82	$(3 + m)(1 - m)(m + 2) + (m^2 - 2m + 1)(m + 3)$	$-3m^2 - 6m + 9$
83	$5a(a^2x) - ax^2(-14a - 9x) + (0,3a + 0,6x)(a + x)(-15ax)$	$0$

84	$(a + 2) \left\{ \left[ 6a^2b - 3ab \left( 2a - \frac{1}{3}b \right) + b^2 \right] b^2 - 3ab^3 \left( \frac{1}{3}ab + b \right) \right.$	$2b^4$
85	$\left[ \left( \frac{3}{2}ab^2 - 0,2a^2b \right) \left( 2a + \frac{20}{3}b \right) - (10b^2 - 0,4a^2)ab + \frac{4}{3}a^2b^2 \right]^2$	$9a^4b^4$

## 4. divisione di un polinomio per un monomio



86	$(6x^2 - 10x):2$	$3x^2 - 5x$
87	$(a^2 + 2a):a$	$a + 2$
88	$(4x^3 + 6x^2):2x$	$2x^2 + 6x$
89	$(12a^4 - 15a^2):3a^2$	$4a^2 - 5$
90	$(20x^5 + 12x^3):4x^2$	$5x^3 + 3x$
91	$(-18a^6 - 24a^4):(-6a^2)$	$3a^4 + 4a^2$
92	$(3x^3 - 2x^2 - x):(x)$	$3x^2 - 2x - 1$
93	$(a^2b - 2ab^2 - 3a^2b^2):(ab)$	$a - 2b - 3ab$
94	$(2a^2b - 4ab^2 - 8a^2b^2):(2ab)$	$a - 2b - 4ab$
95	$(x^2y - 2xy^2 - 3x^2y^2):(2xy)$	$\frac{1}{2}x - y - \frac{3}{2}xy$
96	$(12a^4y^5 - 4a^3y^2 + 8ay^4):(-4ay^2)$	$-3a^3y^3 + a^2 - 2y^2$
97	$(8x^2y^3 - 6xy^2 + 4xy):(-2xy)$	$-4xy^2 + 3y - 2$

98	$\left(\frac{6}{5}m^2 - \frac{7}{3}m^3 + 4m^5 - \frac{1}{2}m^6\right) : \left(\frac{3}{2}m^2\right)$	$-\frac{1}{3}m^4 + \frac{8}{3}m^3 - \frac{14}{9}m^2 + \frac{4}{5}m$
99	$\left(\frac{3}{2}p^6q - 6p^5q - 4p^3pq^4 + \frac{5}{6}p^5q^2 - \frac{3}{8}p^3q^3\right) : \left(\frac{1}{4}p^3q\right)$	$6p^3 - 24p^2 - 16pq^3 + \frac{10}{3}p^2q - \frac{3}{2}q^2$
100	$[(2x + z^2)(x^3z - z^2) : z + z^3] : x - 2x(x^3 + z) - z(x^2z - 2 - 2x)$	$2x^3 - 2x^4$
101	$\{(x + y)[(x + y) + (x - y)] - 2xy\} : [(-2)(-x)^2]$	-1
102	$[(1 - a)(1 + a + a^2)(1 + a^3) - 1] : (-a)^5$	$a$
103	$\left\{2a^4 \left[ b \left( \frac{1}{8}b + a \right) + 16a \left( \frac{1}{8}a - \frac{1}{16}b \right) \right] : a^2 + a^2 \left( a^2 - \frac{1}{4}b^2 \right) \right\} : (-5a^4)$	-1
104	$\left\{ \left[ \left( 2m^2n^3 - \frac{1}{4}m^5 \right) (-8m) + 2(1 + 2mn)(1 - 2mn + 4m^2n^2) \right] : 2 \right\} : \frac{1}{2}$	$2m^6 + 2$

## 5. esercizi di riepilogo



105	$\left[ 2 \left( 3 + x + \frac{1}{2}y \right) (x + 3y - 4) - \left( y + \frac{14}{3} \right) 3y + 2x(1 - x) - 4 \right] : 7$	$xy - 4$
106	$\left[ 4(2a - b) \left( a - \frac{1}{3}b \right) + 2(b - 2a + 1) \left( 2a - \frac{2}{3}b - 1 \right) + 2 \left( \frac{5}{3}b - 4a + 1 \right) \right]^3 - 4$	-4
107	$(m^2 + n^2)[(1,5m - 0,6n)(0,6m + 1,5n) - 1,805mn] - (-m^2)^2$	$-n^4$
108	$\frac{a}{2} \left( \frac{a}{3} + \frac{b}{2} \right) - \frac{2}{3}b \left( \frac{a}{4} + \frac{3}{2}a \right) - \left( \frac{1}{3}a^2 - \frac{2}{3}b^2 - \frac{11}{6}ab \right) \cdot \frac{1}{2}$	$\frac{1}{3}b^2$
109	$\left\{ \left[ \left( \frac{1}{2}x + y \right) \left( \frac{y}{3} - x \right) 2 + \frac{4}{3}xy \right] \left( \frac{2}{3}y^2 + x^2 \right) + \left( \frac{5}{9}y^4 + x^4 + \frac{1}{3}x^3y \right) \right\} : 2(-y)^3$	$\frac{1}{9}x - \frac{1}{2}y$
110	$\left\{ \left[ \left( x + \frac{1}{2}y \right) \left( b + \frac{2}{3}a \right) - bx - \frac{1}{3}ay \right] (ax - 2by) 6 + 5by(ax + by) \right\} : \left( -\frac{1}{2} \right)^2$	$16a^2x^2 - 4b^2y^2$
111	$2 \left( a^3 + \frac{1}{4} \right) \left( \frac{1}{16} - a^3 \right) + \left[ \left( a - \frac{1}{2} \right) \left( a^4 + \frac{1}{2}a^3 + \frac{1}{4}a^2 \right) \left( a^3 + \frac{1}{8} \right) \right] : \left( \frac{1}{2}a^2 \right)$	$-\frac{3}{8}a^3$

112	$\left[ \left(-\frac{2}{3}ax\right)^2 + \frac{1}{2}ax\left(\frac{4}{3}a + \frac{5}{2}x\right) + a\left(\frac{2}{3}ax - \frac{5}{4}x^2\right) \right] : (-ax)$	$-\frac{4}{9}ax - \frac{4}{3}$
113	$[x(x - 2y) - 2x(y - 2z) + x^2](2y - 1) - (y - 1)(x - 3y + 2z) \cdot 2x$	$2x^2y - 2xy^2 + 4xyz - 2xy$
114	$\{[(m + 2)(m - 1) + 2](m^2 + m - 1) + m\} \left(-\frac{1}{2}m\right)^2 - \left(-\frac{1}{2}m^3\right)^2$	$\frac{1}{2}m^5$
115	$\left[\left(\frac{1}{2}x + \frac{2}{3}y^2\right)(6x - 18y^2) - 4\left(x^2 - \frac{39}{16}y^4\right) + \left(x + \frac{9}{2}y^2\right)\left(x + \frac{1}{2}y^2\right)\right] : xy$	0