

195

LEZIONE 12

Domini di funzioni trascendenti

$$\ln = \log_e$$

ARG. DEL LOGARITMO

DENOM. ESPONENTE

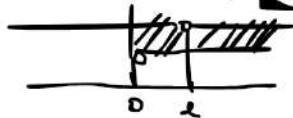
$$y = 3x - \frac{1}{1 - \ln x}$$

$$\begin{cases} x > 0 \\ 1 - \ln x \neq 0 \end{cases}$$

$$1 - \ln x = 0$$

$$- \ln x = -1 \Rightarrow \ln x = 1 \Rightarrow \ln x = \ln(e^1) \Rightarrow x = e$$

$$\begin{cases} x > 0 \\ x \neq e \end{cases}$$



$$\text{Dom } f: 0 < x < e \vee x > e \\]0, e[\vee]e, +\infty[$$

$$\log_e e = 1$$

$$\log_e e^{\frac{1}{2}} = \frac{1}{2} \cdot \log_e e$$

$$e \approx 2,718 \dots$$

196

$$y = e^{\frac{3}{\ln x - 2}}$$

$$\begin{cases} x > 0 \\ \ln x - 2 \neq 0 \Rightarrow x \neq e^2 \end{cases}$$

$$2^3 = 8 \\ \log_2 8 = 3$$

$$\ln x - 2 = 0$$

$$\ln x = 2 \Rightarrow \ln \emptyset = \ln(e^2) \Rightarrow x = e^2$$

$$\log_e e^2 = 2 \\ = 2 \log_e e$$

$$\begin{cases} x > 0 \\ x \neq e^2 \end{cases}$$



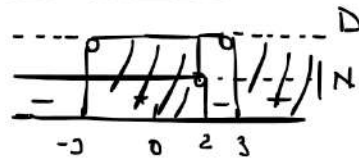
Dom f

$$0 < x < e^2 \vee x > e^2$$

$$]0; e^2[\cup]e^2; +\infty[$$

197

$$y = \log_{\frac{1}{2}} \left(\frac{2-x}{9-x^2} \right)$$



Dom f

$$-3 < x < 2 \vee x > 3$$

$$\boxed{]-3; 2[\cup]3; +\infty[}$$

$$\frac{2-x}{9-x^2} > 0$$

$$N \quad 2-x > 0$$

$$-x > -2$$

$$\boxed{x < 2}$$

$$x^2 = 9$$

$$x = \pm\sqrt{9} = \pm 3$$

$$x_1 = -3 \quad x_2 = 3$$

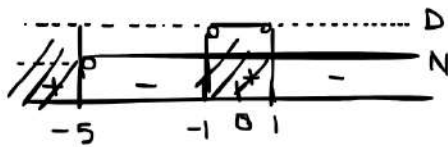
$$D \quad 9-x^2 > 0$$

$$-x^2 > -9$$

$$x^2 < 9$$

$$\boxed{-3 < x < 3}$$

198 $y = \log_2 \frac{x+5}{1-x^2}$



Def $x < -5 \vee -1 < x < 1$
 $] -\infty; -5[\cup] -1; 1[$

$\frac{x+5}{1-x^2} > 0$

N $x+5 > 0$

$x > -5$

D $1-x^2 > 0$

$x^2 < 1$ $x = \pm 1$

$x_1 = -1$ $x_2 = 1$

$-1 < x < 1$

199

$$y = 5^{\frac{x}{2-x^2}}$$

$$2 - x^2 \neq 0$$

$$2 - x^2 = 0 \Rightarrow -x^2 = -2 \Rightarrow x^2 = 2 \Rightarrow \boxed{x = \pm\sqrt{2}}$$

$$\boxed{x \neq \pm\sqrt{2}}$$

$$\text{Dom } f: \mathbb{R} \setminus \{\pm\sqrt{2}\}$$

$$x \neq -\sqrt{2} \vee x \neq \sqrt{2}$$

200

$$y = 2^{\frac{x+1}{x^2-4}}$$

$$x^2 - 4 \neq 0 \Rightarrow x^2 \neq 4 \Rightarrow x \neq \pm \sqrt{4}$$

$$\text{Dom } f \quad \mathbb{R} \setminus \{\pm 2\}$$

$$x \neq \pm 2$$

$$\boxed{x \neq \pm 2}$$

201

$$y = e^{\frac{\sqrt{x^2 - 7x + 12}}{x-5}} \quad \begin{cases} x^2 - 7x + 12 \geq 0 \\ x - 5 \neq 0 \end{cases}$$

$$x^2 - 7x + 12 \geq 0$$

$$\Delta = b^2 - 4ac = (-7)^2 - 4(1)(12) = 49 - 48 = 1$$

$$x_1, x_2 = -\frac{b \pm \sqrt{\Delta}}{2a} = -\frac{(-7) \pm 1}{2} = \frac{7 \pm 1}{2}$$

$$\begin{aligned} \frac{8}{2} &= 4 \\ \frac{6}{2} &= 3 \end{aligned}$$

$$x_1 = 3 \quad x_2 = 4 \quad x \leq 3 \vee x \geq 4$$

$$\begin{cases} x \leq 3 \vee x \geq 4 \\ x \neq 5 \end{cases}$$



$$\boxed{x \leq 3 \vee 4 \leq x < 5 \vee x > 5}$$

$$\text{Dom } f =]-\infty; 3] \cup [4; 5[\cup]5; +\infty[$$

202

$$y = \left(\frac{3}{4}\right) \sqrt{2-x^2}$$

Dom f $-\sqrt{2} \leq x \leq \sqrt{2}$
 $[-\sqrt{2}; \sqrt{2}]$

$$2-x^2 \geq 0$$

$$-x^2 \geq -2$$

$$x^2 \leq 2$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

$$x_1 = -\sqrt{2}; x_2 = \sqrt{2}$$

$$\boxed{-\sqrt{2} \leq x \leq \sqrt{2}}$$

203 PERSONALIZATIO

$$y = \sqrt{\left(\frac{1}{2}\right)^{\frac{x-3}{x-5}} - \frac{1}{8}}$$

$$\begin{cases} x-5 \neq 0 \\ \left(\frac{1}{2}\right)^{\frac{x-3}{x-5}} - \frac{1}{8} \geq 0 \end{cases}$$

$$\left(\frac{1}{2}\right)^{\frac{x-3}{x-5}} \geq \left(\frac{1}{8}\right) \Rightarrow \left(\frac{1}{2}\right)^{\frac{x-3}{x-5}} \geq \left(\frac{1}{2}\right)^3 \quad \left(\frac{1}{2} < 1\right) \quad \frac{1}{8} \left(\frac{1}{2}\right)^3$$

$$\frac{x-3}{x-5} \leq 3$$

$$\left(\frac{1}{2}\right)^3 > \left(\frac{1}{2}\right)^2 \Rightarrow 3 < 4$$

$$\begin{matrix} 2^3 > 2^2 > 2^1 \\ \left(\frac{1}{2}\right)^3 < \left(\frac{1}{2}\right)^2 < \left(\frac{1}{2}\right)^1 \end{matrix}$$

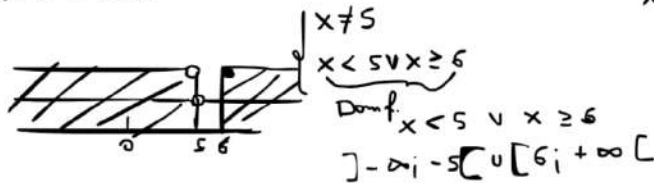
$$\frac{x-3}{x-5} - \frac{3}{1} \leq 0 \Rightarrow \frac{x-3-3(x-5)}{(x-5)} \leq 0$$

$$\frac{x-3-3x+15}{x-5} \leq 0 \quad \frac{12-2x}{x-5} \leq 0$$



$$x < 5 \vee x \geq 6$$

$$\begin{aligned} N \quad 12-2x &\geq 0 \\ -2x &\geq -12 \\ 2x &\leq 12 \\ x &\leq 6 \\ D \quad x-5 &> 0 \\ x &> 5 \end{aligned}$$



209 $y = \ln[\ln(x-2)]$

$$\begin{cases} x-2 > 0 \\ \ln(x-2) > 0 \end{cases}$$

ARG. DEL LOGARITMO PIU' INTERNO
AND. DEL LOGARITMO PIU' ESTERNO

$\begin{cases} x > 2 \\ \ln(x-2) > \ln 1 \end{cases} \Rightarrow x-2 > 1 \Rightarrow x > 3$ $\ln = \log_e$ $\ln 1 = 0$

$\boxed{\text{Dom }]3; +\infty[}$ $\begin{cases} x > 2 \\ x > 3 \end{cases}$