LEZIONE 12

ESERCITAZIONE CONCLUSIVA SU TUTTA LA GONIOMETRIA DI BASE

$$\operatorname{sen}\frac{11}{6}\pi + \cos\frac{\pi}{3} + 5\operatorname{tg}\frac{7}{4}\pi =$$

$$= \underline{\operatorname{sen}330^\circ} + \cos 60^\circ + \underline{5\operatorname{tg}315^\circ} =$$

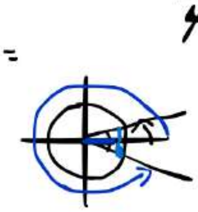
$$= \operatorname{sen}(360^\circ - 30^\circ) + \frac{1}{2} + 5\operatorname{tg}(360^\circ - 45^\circ) =$$

$$= -\operatorname{sen}30^\circ + \frac{1}{2} - 5\operatorname{tg}45^\circ =$$

$$= -\cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - 5 \cdot 1 = \underline{-5}$$

$$\frac{11}{\cancel{6}_1} \cdot \cancel{180^\circ}^{30} = 330^\circ$$

$$\frac{7}{\cancel{4}_1} \cdot \cancel{180^\circ}^{45} = 315$$



$$\frac{3\,\mathrm{tg}\frac{\pi}{6} + \mathrm{tg}\frac{\pi}{3}}{\mathrm{tg}\frac{\pi}{4}} = \frac{3\,\mathrm{tg}\,30^\circ + \mathrm{tg}\,60^\circ}{\mathrm{tg}\,45^\circ} = \frac{\cancel{3}\frac{\sqrt{3}}{\cancel{3}} + \sqrt{3}}{1} =$$

$$= \sqrt{3} + \sqrt{3} = \underline{\underline{2\sqrt{3}}}$$

$$\frac{1-\cos 420^\circ}{1+\cos 420^\circ} - \operatorname{ctg}^2 390^\circ =$$

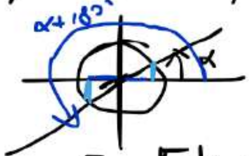
$$\begin{array}{r} 420- \\ 360 \\ \hline 60^\circ \end{array}$$

$$= \frac{1-\cos 60^\circ}{1+\cos 60^\circ} - \operatorname{ctg}^2 30^\circ =$$

$$= \frac{1-\frac{1}{2}}{1+\frac{1}{2}} - \left(\sqrt{3}\right)^2 = \frac{\frac{2-1}{2}}{\frac{2+1}{2}} - 3 = \boxed{-\frac{8}{3}}$$

$$= \frac{\frac{1}{2}}{\frac{3}{2}} - 3 = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} - 3 = \frac{1}{3} - \frac{3}{1} = \frac{1-9}{3}$$

$$\operatorname{sen}(180^\circ+\alpha)\cdot\cos(180^\circ+\alpha)\cdot\left[\operatorname{tg}\alpha+\operatorname{cotg}(180^\circ+\alpha)\right]$$



$$\left[-\operatorname{sen}\alpha\right]\cdot\left[-\cos\alpha\right]\cdot\left[\operatorname{tg}\alpha+\operatorname{cotg}\alpha\right]=$$

$$=\operatorname{sen}\alpha\cos\alpha\cdot\left[\frac{\operatorname{sen}\alpha}{\cos\alpha}+\frac{\cos\alpha}{\operatorname{sen}\alpha}\right]=$$

$$=\cancel{\operatorname{sen}\alpha}\,\cancel{\cos\alpha}\cdot\left[\frac{\underbrace{\operatorname{sen}^2\alpha+\cos^2\alpha}_{1}}{\cancel{\operatorname{sen}\alpha}\,\cancel{\cos\alpha}}\right]=1$$

$$\frac{sen^2(\pi+\alpha) - sen(\pi+\alpha)}{cos(\pi+\alpha)} + \frac{1}{cotg\,\alpha}$$

$\pi = 180^\circ$

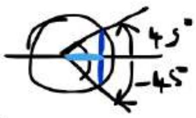
$$= \frac{[-sen\,\alpha]^2 - [-sen\,\alpha]}{-cos\,\alpha} + tg\,\alpha$$

$$= \frac{sen^2\alpha + sen\,\alpha}{-cos\,\alpha} + \frac{sen\,\alpha}{cos\,\alpha} = \qquad = -tg\,\alpha \cdot sen\,\alpha$$

$$= \frac{-sen^2\alpha - \cancel{sen\,\alpha} + \cancel{sen\,\alpha}}{\cancel{cos\,\alpha}} = -\frac{sen^2\alpha}{cos\,\alpha} = -\boxed{\frac{sen\,\alpha}{cos\,\alpha}}\ sen\,\alpha$$

$$\left(1-\cos(-45^\circ)+\operatorname{sen}(-30^\circ)\right)\cdot tg^2(-60^\circ)=$$

$$=\left(1-\cos 45^\circ-\operatorname{sen}30^\circ\right)\cdot\left[-tg(60^\circ)\right]^2=$$



$$=\left(1-\frac{\sqrt{2}}{2}-\frac{1}{2}\right)\cdot\left[-\sqrt{3}\right]^2=$$

$$=\left(\frac{2-\sqrt{2}-1}{2}\right)\cdot 3 \qquad =\frac{\left(1-\sqrt{2}\right)}{2}\cdot 3$$

$$\frac{\operatorname{sen}(\alpha+\beta)}{\operatorname{sen}(\alpha-\beta)} = \frac{\operatorname{sen}\alpha\cos\beta+\operatorname{sen}\beta\cos\alpha}{\operatorname{sen}\alpha\cos\beta-\operatorname{sen}\beta\cos\alpha} =$$

PER DIVIDO $\boxed{\cos\alpha\cos\beta}$

$$= \frac{\dfrac{\operatorname{sen}\alpha\cancel{\cos\beta}}{\cos\alpha\cancel{\cos\beta}}+\dfrac{\operatorname{sen}\beta\cancel{\cos\alpha}}{\cancel{\cos\alpha}\cos\beta}}{\dfrac{\operatorname{sen}\alpha\cancel{\cos\beta}}{\cos\alpha\cancel{\cos\beta}}-\dfrac{\operatorname{sen}\beta\cancel{\cos\alpha}}{\cancel{\cos\alpha}\cos\beta}} = \boxed{\frac{\operatorname{tg}\alpha+\operatorname{tg}\beta}{\operatorname{tg}\alpha-\operatorname{tg}\beta}}$$

$$\frac{\operatorname{tg}(\alpha-\beta)+\operatorname{tg}\beta}{\operatorname{tg}(\alpha+\beta)-\operatorname{tg}\beta}=\frac{\dfrac{\operatorname{tg}\alpha-\operatorname{tg}\beta}{1+\operatorname{tg}\alpha\operatorname{tg}\beta}+\dfrac{\operatorname{tg}\beta}{1}}{\dfrac{\operatorname{tg}\alpha+\operatorname{tg}\beta}{1-\operatorname{tg}\alpha\operatorname{tg}\beta}-\dfrac{\operatorname{tg}\beta}{1}}=$$

$$=\frac{\dfrac{\operatorname{tg}\alpha-\operatorname{tg}\beta+\operatorname{tg}\beta(1+\operatorname{tg}\alpha\operatorname{tg}\beta)}{1+\operatorname{tg}\alpha\operatorname{tg}\beta}}{\dfrac{\operatorname{tg}\alpha+\operatorname{tg}\beta-\operatorname{tg}\beta(1-\operatorname{tg}\alpha\operatorname{tg}\beta)}{1-\operatorname{tg}\alpha\operatorname{tg}\beta}}=\frac{\dfrac{\operatorname{tg}\alpha-\cancel{\operatorname{tg}\beta}+\cancel{\operatorname{tg}\beta}+\operatorname{tg}\alpha\operatorname{tg}^2\beta}{1+\operatorname{tg}\alpha\operatorname{tg}\beta}}{\dfrac{\operatorname{tg}\alpha+\cancel{\operatorname{tg}\beta}-\cancel{\operatorname{tg}\beta}+\operatorname{tg}\alpha\operatorname{tg}^2\beta}{1-\operatorname{tg}\alpha\operatorname{tg}\beta}}$$

$$\frac{\cancel{tg\alpha + tg\alpha\, tg^2\beta}}{1 + tg\alpha\, tg\beta} \cdot \frac{1 - tg\alpha\, tg\beta}{\cancel{tg\alpha + tg\alpha\, tg^2\beta}} =$$

$$= \frac{1 - tg\alpha\, tg\beta}{1 + tg\alpha\, tg\beta} = \frac{1 - \frac{sen\alpha}{cos\alpha} \cdot \frac{sen\beta}{cos\beta}}{1 + \frac{sen\alpha}{cos\alpha} \cdot \frac{sen\beta}{cos\beta}} = \frac{\frac{cos\alpha\, cos\beta - sen\alpha\, sen\beta}{\cancel{cos\alpha\, cos\beta}}}{\frac{cos\alpha\, cos\beta + sen\alpha\, sen\beta}{\cancel{cos\alpha\, cos\beta}}}$$

$$= \frac{cos\alpha\, cos\beta - sen\alpha\, sen\beta}{cos\alpha\, cos\beta + sen\alpha\, sen\beta} = \boxed{\frac{cos\,(\alpha+\beta)}{cos\,(\alpha-\beta)}}$$

$$\boxed{\underbrace{\operatorname{sen} 2\alpha}\cdot\left(\operatorname{tg}\alpha+\operatorname{cotg}\alpha\right)} =$$

$$\boxed{\operatorname{sen}^2\alpha+\cos^2\alpha=1}$$

$$= 2\operatorname{sen}\alpha\cos\alpha\cdot\left[\frac{\operatorname{sen}\alpha}{\cos\alpha}+\frac{\cos\alpha}{\operatorname{sen}\alpha}\right] =$$

$$= 2\operatorname{sen}\alpha\cos\alpha\cdot\left[\frac{\operatorname{sen}^2\alpha+\cos^2\alpha}{\operatorname{sen}\alpha\cos\alpha}\right]$$

$$= 2\,\cancel{\operatorname{sen}}\alpha\,\cancel{\cos}\alpha\cdot\frac{1}{\cancel{\operatorname{sen}}\alpha\,\cancel{\cos}\alpha} = 2$$

$$\frac{1}{2}\underbrace{tg\,2\alpha}\cdot(1+tg\,\alpha)$$

$$(A^2-B^2)=(A+B)(A-B)$$

$$\frac{1}{\cancel{2}}\cdot\left[\frac{\cancel{2}tg\,\alpha}{1-tg^2\alpha}\right](1+tg\,\alpha)=\frac{tg\,\alpha}{(1-tg^2\alpha)}\cdot(1+tg\,\alpha)=$$

$$=\frac{tg\,\alpha}{\cancel{(1+tg\,\alpha)}\cdot(1-tg\,\alpha)}\cdot\cancel{(1+tg\,\alpha)}=\boxed{\frac{tg\,\alpha}{1-tg\,\alpha}}$$

$$\frac{1}{1-\operatorname{tg}^2\frac{\alpha}{2}}-\frac{1}{1-\operatorname{ctg}^2\frac{\alpha}{2}}=$$

$$\boxed{\begin{gathered}\operatorname{tg}\frac{\alpha}{2}=\pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}\\ \operatorname{ctg}\frac{\alpha}{2}=\pm\sqrt{\frac{1+\cos\alpha}{1-\cos\alpha}}\end{gathered}}$$

$$=\frac{1}{\frac{1}{1}-\left(\frac{1-\cos\alpha}{1+\cos\alpha}\right)}-\frac{1}{1-\left(\frac{1+\cos\alpha}{1-\cos\alpha}\right)}=$$

$$=\frac{1}{\frac{1+\cos\alpha-(1-\cos\alpha)}{1+\cos\alpha}}-\frac{1}{\frac{1-\cos\alpha-(1+\cos\alpha)}{1-\cos\alpha}}=\frac{1}{\frac{\cancel{1}\cos\alpha\cancel{-1}+\cos\alpha}{1+\cos\alpha}}-\frac{1}{\frac{\cancel{1}-\cos\alpha\cancel{-1}-\cos\alpha}{1-\cos\alpha}}$$

$$=\frac{1+\cos\alpha}{2\cos\alpha}-\frac{1-\cos\alpha}{-2\cos\alpha}=\frac{1+\cancel{\cos\alpha}+1-\cancel{\cos\alpha}}{2\cos\alpha}=\frac{\cancel{2}}{\cancel{2}\cos\alpha}=\frac{1}{\cos\alpha}$$