Lezione 13
Ultima esercitazione goniometria elementare

$p = 60° + \alpha$
$q = 60° - \alpha$

$\boxed{\text{PROSTAFERESI}}$

$$\cos(\underbrace{60° + \alpha}_{p}) + \cos(\underbrace{60° - \alpha}_{q}) =$$

$$= \cos p + \cos q = 2 \cos \frac{p+q}{2} \cos \frac{p-q}{2} =$$

$$= 2 \cos\left(\frac{60° + \cancel{\alpha} + 60° - \cancel{\alpha}}{2}\right) \cos\left(\frac{\cancel{60°} + \alpha - (\cancel{60°} - \alpha)}{2}\right) =$$

$$= 2 \cos \frac{120°}{2} \cdot \cos \frac{\cancel{2}\alpha}{\cancel{2}} = 2 \cos 60° \cos \alpha = \cancel{2} \cdot \frac{1}{\cancel{2}} \cos \alpha$$

$$= \boxed{\cos \alpha}$$

$\underline{\cos(60° + \alpha)} + \underline{\cos(60° - \alpha)} =$ FORMULE DI ADDIZ. E SOTTRAZ. COSENO

$= \cos 60° \cos\alpha \underline{\cancel{- \text{sen}\, 60° \,\text{sen}\,\alpha}} + \cos 60° \cos\alpha + \underline{\cancel{\text{sen}\, 60° \,\text{sen}\,\alpha}} =$

$= 2\cos 60° \cos\alpha = \cancel{2} \cdot \frac{1}{\cancel{2}} \cos\alpha = \boxed{\cos\alpha}$

$$\boxed{= \text{cotg}\,\underbrace{(45^\circ+\alpha)}_{A} + \text{cotg}\,\underbrace{(45^\circ-\alpha)}_{B} =} \quad \text{cotg}\,A + \text{cotg}\,B$$

$$\boxed{\begin{aligned} A &= 45^\circ+\alpha \\ B &= 45^\circ-\alpha \end{aligned}}$$

$$\text{cotg}\,A + \text{cotg}\,B = \frac{\cos A}{\text{sen}\,A} + \frac{\cos B}{\text{sen}\,B} = \frac{\text{sen}\,B\cos A + \text{sen}\,A\cos B}{\text{sen}\,A\cdot\text{sen}\,B} =$$

$$= \frac{\text{sen}\,(A+B)}{\text{sen}\,A\cdot\text{sen}\,B} = \frac{\text{sen}\,(45^\circ+\cancel{\alpha}+45^\circ-\cancel{\alpha})}{\text{sen}(45^\circ+\alpha)\,\text{sen}\,(45^\circ-\alpha)} = \frac{\text{sen}\,(90^\circ)}{\text{sen}(45^\circ+\alpha)\,\text{sen}(45^\circ-\alpha)} =$$

$$\frac{1}{\text{sen}(45^\circ+\alpha)\,\text{sen}(45^\circ-\alpha)} =$$

$$= \frac{1}{\operatorname{sen}(45^\circ+\alpha)\operatorname{sen}(45^\circ-\alpha)} =$$

$\operatorname{sen}(45^\circ+\alpha) = \operatorname{sen}45^\circ\cos\alpha + \operatorname{sen}\alpha\cos 45^\circ$

$\operatorname{sen}(45^\circ-\alpha) = \operatorname{sen}45^\circ\cos\alpha - \operatorname{sen}\alpha\cos 45^\circ$

$$= \frac{1}{(\underbrace{\operatorname{sen}45^\circ\cos\alpha}+\underbrace{\operatorname{sen}\alpha\cos 45^\circ})(\underbrace{\operatorname{sen}45^\circ\cos\alpha}-\underbrace{\operatorname{sen}\alpha\cos 45^\circ})} =$$

$$= \frac{1}{\operatorname{sen}^2 45^\circ\cos^2\alpha - \operatorname{sen}^2\alpha\cos^2 45^\circ} =$$

$$= \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2\cos^2\alpha - \left(\frac{\sqrt{2}}{2}\right)^2\operatorname{sen}^2\alpha} =$$

$$= \frac{1}{\frac{1}{2}\cos^2\alpha - \frac{1}{2}\operatorname{sen}^2\alpha} = \frac{1}{\frac{\cos^2\alpha-\operatorname{sen}^2\alpha}{2}} =$$

$$= \frac{2}{\cos^2\alpha-\operatorname{sen}^2\alpha} = \frac{2}{\cos^2\alpha-(1-\cos^2\alpha)} =$$

$\operatorname{sen}^2\alpha = 1-\cos^2\alpha$

$\cos 2\alpha = 2\cos^2\alpha - 1$

$$= \frac{2}{2\cos^2\alpha - 1} = \boxed{\frac{2}{\cos 2\alpha}}$$

$$\operatorname{tg} 7\alpha + \operatorname{tg} 3\alpha = \frac{\operatorname{sen} 7\alpha}{\cos 7\alpha} + \frac{\operatorname{sen} 3\alpha}{\cos 3\alpha} =$$

$$\boxed{\operatorname{tg} x = \frac{\operatorname{sen} x}{\cos x}}$$

$$= \frac{\operatorname{sen} 7\alpha \cdot \cos 3\alpha + \cos 7\alpha \operatorname{sen} 3\alpha}{\cos 7\alpha \cdot \cos 3\alpha} =$$

$$= \frac{\operatorname{sen}(7\alpha + 3\alpha)}{\cos 7\alpha \cdot \cos 3\alpha} = \boxed{\frac{\operatorname{sen} 10\alpha}{\cos 7\alpha \cdot \cos 3\alpha}}$$

$$\overbrace{\operatorname{sen}\alpha\cdot\operatorname{sen}\beta}^{\text{WERNER}}-\overbrace{\frac{1}{2}\cos(\alpha-\beta)}^{\text{SOTTRAZ. DEL COSENO}}=$$

$$=\frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]-\frac{1}{2}\cos(\alpha-\beta)=$$

$$=\cancel{\frac{1}{2}\cos(\alpha-\beta)}-\frac{1}{2}\cos(\alpha+\beta)-\cancel{\frac{1}{2}\cos(\alpha-\beta)}=$$

$$=-\frac{1}{2}\cos(\alpha+\beta)$$

$$\frac{\operatorname{sen}\alpha - \operatorname{sen}\beta}{\cos\alpha + \cos\beta} = \frac{\cancel{2}\cancel{\cos\left(\frac{\alpha+\beta}{2}\right)} \cdot \operatorname{sen}\frac{\alpha-\beta}{2}}{\cancel{2}\cancel{\cos\left(\frac{\alpha+\beta}{2}\right)} \cdot \cos\frac{\alpha-\beta}{2}} =$$

$$= \frac{\operatorname{sen}\left(\frac{\alpha-\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)} = \boxed{\operatorname{tg}\left(\frac{\alpha-\beta}{2}\right)}$$

$$\frac{\cos\left(\alpha-\frac{\pi}{4}\right)\operatorname{sen}\left(\frac{3}{4}\pi+\alpha\right)}{\cos\alpha+\operatorname{sen}\alpha} = \qquad A=\alpha-\frac{\pi}{4}$$

WERNER

$$B=\frac{3}{4}\pi+\alpha$$

$$= \frac{\cos A \operatorname{sen} B}{\cos\alpha+\operatorname{sen}\alpha} = \frac{\operatorname{sen}B\cos A}{\cos\alpha+\operatorname{sen}\alpha} = \frac{\frac{1}{2}\left[\operatorname{sen}(B+A)+\operatorname{sen}(B-A)\right]}{\cos\alpha+\operatorname{sen}\alpha} =$$

$$= \frac{\frac{1}{2}\left[\operatorname{sen}\left(\frac{3}{4}\pi+\alpha+\alpha-\frac{\pi}{4}\right)+\operatorname{sen}\left(\frac{3}{4}\pi+\cancel{\alpha}-\cancel{\alpha}+\frac{\pi}{4}\right)\right]}{\cos\alpha+\operatorname{sen}\alpha} =$$

$$= \frac{\frac{1}{2}\left[\operatorname{sen}\left(\frac{\pi}{2}+2\alpha\right)+\cancel{\operatorname{sen}(\pi)}\right]}{\cos\alpha+\operatorname{sen}\alpha} = \frac{\frac{1}{2}\operatorname{sen}\left(\frac{\pi}{2}+2\alpha\right)}{\cos\alpha+\operatorname{sen}\alpha} =$$

$$= \frac{\frac{1}{2}\left[\underbrace{\overset{=1}{\operatorname{sen}\frac{\pi}{2}}}\cos 2\alpha+\cancel{\operatorname{sen}2\alpha\cos\frac{\pi}{2}}\right]}{(\cos\alpha+\operatorname{sen}\alpha)} = \frac{\frac{1}{2}\cos 2\alpha}{(\cos\alpha+\operatorname{sen}\alpha)}$$

$$= \frac{\frac{1}{2}\left(\cos^2\alpha-\operatorname{sen}^2\alpha\right)}{(\cos\alpha+\operatorname{sen}\alpha)} = \frac{1}{2}\,\frac{\cancel{(\cos\alpha+\operatorname{sen}\alpha)}(\cos\alpha-\operatorname{sen}\alpha)}{\cancel{(\cos\alpha+\operatorname{sen}\alpha)}}$$

$$= \boxed{\frac{1}{2}(\cos\alpha-\operatorname{sen}\alpha)}$$

$$\frac{\operatorname{sen}(3\alpha+\beta)\cdot\operatorname{sen}(3\alpha-\beta)-\operatorname{sen}(\alpha+\beta)\cdot\operatorname{sen}(\alpha-\beta)}{\operatorname{sen}4\alpha\cdot\operatorname{sen}2\alpha} =$$

$A = 3\alpha+\beta$

$B = 3\alpha-\beta$

$$\operatorname{sen}(3\alpha+\beta)\operatorname{sen}(3\alpha-\beta) =$$

$$= \operatorname{sen}A\operatorname{sen}B = \frac{1}{2}\left[\cos(A-B)-\cos(A+B)\right] =$$

$$= \frac{1}{2}\left[\cos(3\cancel{\alpha}+\beta-\cancel{3}\alpha+\beta)-\cos(3\alpha+\cancel{\beta}+3\alpha-\cancel{\beta})\right] =$$

$$= \boxed{\frac{1}{2}\left[\cos 2\beta-\cos(6\alpha)\right]}$$

$$\operatorname{sen}(\alpha+\beta)\cdot\operatorname{sen}(\alpha-\beta)= \qquad A=\alpha+\beta$$

$$=\operatorname{sen}A\,\operatorname{sen}B=\frac{1}{2}\left[\cos(A-B)-\cos(A+B)\right]= \qquad B=\alpha-\beta$$

$$=\frac{1}{2}\left[\cos(\cancel{\alpha}+\beta-\cancel{\alpha}+\beta)-\cos(\alpha+\cancel{\beta}+\alpha-\cancel{\beta})\right]=$$

$$=\boxed{\frac{1}{2}\left[\cos 2\beta-\cos 2\alpha\right]}$$

$$\text{sen}\, 4\alpha \,\text{sen}\, 2\alpha =$$

$A = 4\alpha$
$B = 2\alpha$

$$= \text{sen}\, A \,\text{sen}\, B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right] =$$

$$= \frac{1}{2}\left[\cos(4\alpha - 2\alpha) - \cos(4\alpha + 2\alpha)\right] =$$

$$= \boxed{\frac{1}{2}\left[\cos 2\alpha - \cos 6\alpha\right]}$$

$$\frac{\frac{1}{2}\left[\cos 2\beta - \cos 6\alpha\right] - \frac{1}{2}\left[\cos 2\beta - \cos 2\alpha\right]}{\frac{1}{2}\left[\cos 2\alpha - \cos 6\alpha\right]} =$$

$$= \frac{\cancel{\frac{1}{2}}\left\{(\cos 2\beta - \cos 6\alpha) - (\cos 2\beta - \cos 2\alpha)\right\}}{\cancel{\frac{1}{2}}\left[\cos 2\alpha - \cos 6\alpha\right]}$$

$$\frac{\cancel{\cos 2\beta} - \cos 6\alpha - \cancel{\cos 2\beta} + \cos 2\alpha}{\cos 2\alpha - \cos 6\alpha} = \frac{\cancel{(\cos 2\alpha - \cos 6\alpha)}}{\cancel{(\cos 2\alpha - \cos 6\alpha)}} = 1$$

$$\frac{\text{sen}^2\alpha - \text{sen}^2\beta}{\cos^2\alpha - \cos^2\beta} =$$

$$\boxed{\begin{array}{l}\text{sen}^2\alpha = 1 - \cos^2\alpha \\ \text{sen}^2\beta = 1 - \cos^2\beta\end{array}}$$

$$= \frac{1 - \cos^2\alpha - (1 - \cos^2\beta)}{\cos^2\alpha - \cos^2\beta} = \frac{\cancel{1} - \cos^2\alpha - \cancel{1} + \cos^2\beta}{\cos^2\alpha - \cos^2\beta} =$$

$$= \frac{\cos^2\beta - \cos^2\alpha}{\cos^2\alpha - \cos^2\beta} = \frac{-1\cancel{(\cos^2\alpha - \cos^2\beta)}}{\cancel{\cos^2\alpha - \cos^2\beta}} = -1$$

$$\frac{2\,tg\left(\frac{\pi}{2}-\alpha\right)}{1+\left[cotg\left(\pi-\alpha\right)\right]^2}-\left[\cos\left(\frac{\pi}{2}-\alpha\right)+sen\left(\frac{\pi}{2}-\alpha\right)\right]$$

$$\frac{2\,cotg\,\alpha}{1+\left(-cotg\,\alpha\right)^2}-\left[\cos\left(\frac{\pi}{2}-\alpha\right)+sen\left(\frac{\pi}{2}-\alpha\right)\right]$$

COMPLEMENTARI

$sen\left(\frac{\pi}{2}-\alpha\right)=\cos\alpha$

$\cos\left(\frac{\pi}{2}-\alpha\right)=sen\,\alpha$

$tg\left(\frac{\pi}{2}-\alpha\right)=cotg\,\alpha$

$$=\frac{2\,cotg\,\alpha}{1+cotg^2\alpha}-\left[sen\,\alpha+\cos\alpha\right]=$$

$sen(\pi-\alpha)=sen\,\alpha$

$\cos(\pi-\alpha)=-\cos\alpha$

SUPPLEMENTARI

$tg(\pi-\alpha)=-tg\,\alpha$ $\quad cotg(\pi-\alpha)=-cotg\,\alpha$

$$=\frac{2\,\frac{\cos\alpha}{sen\,\alpha}}{\frac{1}{1}+\frac{\cos^2\alpha}{sen^2\alpha}}-\left[sen\,\alpha+\cos\alpha\right]=$$

$cotg\,\alpha=\frac{\cos\alpha}{sen\,\alpha}$

$$=\frac{2\,\frac{\cos\alpha}{sen\,\alpha}}{\frac{sen^2\alpha+\cos^2\alpha}{sen^2\alpha}}-sen\,\alpha-\cos\alpha=$$

$\boxed{sen^2\alpha+\cos^2\alpha=1}$

$$=\frac{2\,\frac{\cos\alpha}{sen\,\alpha}}{\frac{1}{sen^2\alpha}}-sen\,\alpha-\cos\alpha=$$

$$=2\,\frac{\cos\alpha}{\cancel{sen\,\alpha}}\cdot sen^{\cancel{2}}\alpha-sen\,\alpha-\cos\alpha=$$

$$=2\,sen\,\alpha\cos\alpha-sen\,\alpha-\cos\alpha$$

$$=\boxed{sen\,2\alpha-sen\,\alpha-\cos\alpha}$$