

Identità e Espressioni Goniometriche

Esercizi svolti

prof. Francesco Zumbo

www.francescozumbo.it

Esercizi svolti di espressioni e identità goniometriche che coinvolgono tutte le più significative formule : archi associati, addizione e sottrazione, duplicazione, bisezione, Werner, prostaferesi.

Le tracce degli esercizi svolti sono state prese dal Libro :

Autori : Bergamini – Trifone – Barozzi

Titolo del libro: Matematica Blu 2.0

Casa Editrice : Zanichelli

Formulario di goniometria

Funzioni goniometriche di angoli particolari

Gradi	Radiani	Seno	Coseno	Tangente	Cotangente
0°	0	0	1	0	<i>non esiste</i>
15°	$\frac{\pi}{12}$	$\frac{\sqrt{6} - \sqrt{2}}{4}$	$\frac{\sqrt{6} + \sqrt{2}}{4}$	$2 - \sqrt{3}$	$2 + \sqrt{3}$
18°	$\frac{\pi}{10}$	$\frac{\sqrt{5} - 1}{4}$	$\frac{\sqrt{10 + 2\sqrt{5}}}{4}$	$\sqrt{\frac{5 - 2\sqrt{5}}{5}}$	$\sqrt{5 + 2\sqrt{5}}$
22°30'	$\frac{\pi}{8}$	$\frac{\sqrt{2} - \sqrt{2}}{2}$	$\frac{\sqrt{2} + \sqrt{2}}{2}$	$\sqrt{2} - 1$	$\sqrt{2} + 1$
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
36°	$\frac{\pi}{5}$	$\frac{\sqrt{10 - 2\sqrt{5}}}{4}$	$\frac{\sqrt{5} + 1}{4}$	$\sqrt{5 - 2\sqrt{5}}$	$\sqrt{\frac{5 + 2\sqrt{5}}{5}}$
45°	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
54°	$\frac{3}{10}\pi$	$\frac{\sqrt{5} + 1}{4}$	$\frac{\sqrt{10 - 2\sqrt{5}}}{4}$	$\sqrt{\frac{5 + 2\sqrt{5}}{5}}$	$\sqrt{5 - 2\sqrt{5}}$
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
67°30'	$\frac{3}{8}\pi$	$\frac{\sqrt{2} + \sqrt{2}}{2}$	$\frac{\sqrt{2} - \sqrt{2}}{2}$	$\sqrt{2} + 1$	$\sqrt{2} - 1$
72°	$\frac{2}{5}\pi$	$\frac{\sqrt{10 + 2\sqrt{5}}}{4}$	$\frac{\sqrt{5} - 1}{4}$	$\sqrt{5 + 2\sqrt{5}}$	$\sqrt{\frac{5 - 2\sqrt{5}}{5}}$
75°	$\frac{5}{12}\pi$	$\frac{\sqrt{6} + \sqrt{2}}{4}$	$\frac{\sqrt{6} - \sqrt{2}}{4}$	$2 + \sqrt{3}$	$2 - \sqrt{3}$
90°	$\frac{\pi}{2}$	1	0	<i>non esiste</i>	0
180°	π	0	-1	0	<i>non esiste</i>
270°	$\frac{3}{2}\pi$	-1	0	<i>non esiste</i>	0
360°	2π	0	1	0	<i>non esiste</i>

Relazioni fondamentali tra le funzioni goniometriche di uno stesso angolo

$$\sin^2 \alpha + \cos^2 \alpha = 1 \quad \tan \alpha = \frac{\sin \alpha}{\cos \alpha} \quad \cot \alpha = \frac{\cos \alpha}{\sin \alpha} \quad \operatorname{cosec} \alpha = \frac{1}{\sin \alpha} \quad \sec \alpha = \frac{1}{\cos \alpha}$$

Funzioni goniometriche di angoli associati

$\sin(-\alpha) = -\sin \alpha$	$\cos(-\alpha) = \cos \alpha$	$\tan(-\alpha) = -\tan \alpha$	$\cot(-\alpha) = -\cot \alpha$
$\sin(2\pi - \alpha) = -\sin \alpha$	$\cos(2\pi - \alpha) = \cos \alpha$	$\tan(2\pi - \alpha) = -\tan \alpha$	$\cot(2\pi - \alpha) = -\cot \alpha$
$\sin(\pi - \alpha) = \sin \alpha$	$\cos(\pi - \alpha) = -\cos \alpha$	$\tan(\pi - \alpha) = -\tan \alpha$	$\cot(\pi - \alpha) = -\cot \alpha$
$\sin(\pi + \alpha) = -\sin \alpha$	$\cos(\pi + \alpha) = -\cos \alpha$	$\tan(\pi + \alpha) = \tan \alpha$	$\cot(\pi + \alpha) = \cot \alpha$
$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos \alpha$	$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin \alpha$	$\tan\left(\frac{\pi}{2} - \alpha\right) = \cot \alpha$	$\cot\left(\frac{\pi}{2} - \alpha\right) = \tan \alpha$
$\sin\left(\frac{\pi}{2} + \alpha\right) = \cos \alpha$	$\cos\left(\frac{\pi}{2} + \alpha\right) = -\sin \alpha$	$\tan\left(\frac{\pi}{2} + \alpha\right) = -\cot \alpha$	$\cot\left(\frac{\pi}{2} + \alpha\right) = -\tan \alpha$

Formule di addizione e sottrazione

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \sin \beta \cos \alpha$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\left\{ \begin{array}{l} \tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} \\ \alpha + \beta, \alpha, \beta \neq \frac{\pi}{2} + k\pi \end{array} \right.$$

$$\left\{ \begin{array}{l} \tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta} \\ \alpha - \beta, \alpha, \beta \neq \frac{\pi}{2} + k\pi \end{array} \right.$$

Formule di duplicazione

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha = 1 - 2 \sin^2 \alpha = 2 \cos^2 \alpha - 1$$

$$\left\{ \begin{array}{l} \tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha} \\ \alpha \neq \frac{\pi}{4} + k\frac{\pi}{2} \wedge \alpha \neq \frac{\pi}{2} + k\pi \end{array} \right.$$

Formule di bisezione

$$\cos \frac{\alpha}{2} = \pm \sqrt{\frac{1 + \cos \alpha}{2}} \quad \sin \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{2}} \quad \tan \frac{\alpha}{2} = \pm \sqrt{\frac{1 - \cos \alpha}{1 + \cos \alpha}} \quad (\alpha \neq \pi + 2k\pi)$$

$$\tan \frac{\alpha}{2} = \frac{\sin \alpha}{1 + \cos \alpha} \quad \text{con } \alpha \neq \pi + 2k\pi$$

$$\tan \frac{\alpha}{2} = \frac{1 - \cos \alpha}{\sin \alpha} \quad \text{con } \alpha \neq k\pi$$

Formule parametriche

$$\sin \alpha = \frac{2t}{1 + t^2} \quad \cos \alpha = \frac{1 - t^2}{1 + t^2} \quad \left(t = \tan \frac{\alpha}{2}, \quad \alpha \neq \pi + 2k\pi \right)$$

Formule di prostaferesi

$$\sin p + \sin q = 2 \sin \frac{p + q}{2} \cos \frac{p - q}{2}$$

$$\cos p + \cos q = 2 \cos \frac{p + q}{2} \cos \frac{p - q}{2}$$

$$\sin p - \sin q = 2 \cos \frac{p + q}{2} \sin \frac{p - q}{2}$$

$$\cos p - \cos q = -2 \sin \frac{p + q}{2} \sin \frac{p - q}{2}$$

Formule di Werner

$$\sin \alpha \cos \beta = \frac{1}{2} [\sin(\alpha + \beta) + \sin(\alpha - \beta)]$$

$$\cos \alpha \cos \beta = \frac{1}{2} [\cos(\alpha + \beta) + \cos(\alpha - \beta)]$$

$$\sin \alpha \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

Esercizio pagina 736 n° 167

③

$$\cos\left(d - \frac{2}{3}\pi\right) + \frac{1}{2} \operatorname{sen} d + \cos\left(d - \frac{7}{6}\pi\right) - \operatorname{sen}\left(d - \frac{\pi}{6}\right) =$$

utilizziamo le formule di addizione e sottrazione

$$\begin{aligned} &= \cos d \cdot \cos\left(\frac{2}{3}\pi\right) + \operatorname{sen} d \cdot \operatorname{sen}\left(\frac{2}{3}\pi\right) + \frac{1}{2} \operatorname{sen} d + \\ &+ \cos d \cdot \cos\frac{7}{6}\pi + \operatorname{sen} d \cdot \operatorname{sen}\frac{7}{6}\pi - \left[\operatorname{sen} d \cdot \cos\left(\frac{\pi}{6}\right) - \right. \\ &\left. - \cos d \cdot \operatorname{sen}\left(\frac{\pi}{6}\right) \right] = \end{aligned}$$

Dalle tabelle dei valori sappiamo che

$$\begin{array}{l|l} \cos\frac{2}{3}\pi = -\frac{1}{2} & \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2} \\ \operatorname{sen}\frac{2}{3}\pi = \frac{\sqrt{3}}{2} & \operatorname{sen}\frac{\pi}{6} = \frac{1}{2} \\ \cos\frac{7}{6}\pi = -\frac{\sqrt{3}}{2} & \\ \operatorname{sen}\frac{7}{6}\pi = -\frac{1}{2} & \end{array}$$

da cui

$$\begin{aligned} &= -\frac{1}{2} \cos d + \frac{\sqrt{3}}{2} \cdot \operatorname{sen} d + \frac{1}{2} \operatorname{sen} d + \left(-\frac{\sqrt{3}}{2}\right) \cdot \cos d + \\ &+ \left(-\frac{1}{2}\right) \cdot \operatorname{sen} d - \left[\frac{\sqrt{3}}{2} \cdot \operatorname{sen} d - \frac{1}{2} \cdot \cos d \right] = \end{aligned}$$

(4)

$$= -\frac{1}{2} \cos d + \frac{\sqrt{3}}{2} \sin d + \frac{1}{2} \sin d - \frac{\sqrt{3}}{2} \cos d - \frac{1}{2} \sin d -$$
$$- \frac{\sqrt{3}}{2} \sin d + \frac{1}{2} \cos d = -\frac{\sqrt{3}}{2} \cos d$$

OK.

Exercício nº 257 pag. 742

(5)

$$\cos\left(\frac{3}{4}\pi + 2d\right) - \cos\left(2d + \frac{\pi}{4}\right) + \sqrt{2} \cos^2 d =$$

$$\cos \frac{3}{4}\pi \cdot \cos 2d - \sin \frac{3}{4}\pi \cdot \sin 2d - \left[\cos 2d \cdot \cos \frac{\pi}{4} - \sin 2d \cdot \sin \frac{\pi}{4} \right] +$$

$$+ \sqrt{2} \cos^2 d =$$

$$= -\frac{\sqrt{2}}{2} \cdot \cos 2d - \frac{\sqrt{2}}{2} \cdot \sin 2d - \left[\frac{\sqrt{2}}{2} \cdot \cos 2d - \frac{\sqrt{2}}{2} \sin 2d \right] + \sqrt{2} \cos^2 d =$$

$$= \underline{-\frac{\sqrt{2}}{2} \cdot \cos 2d} - \frac{\sqrt{2}}{2} \cdot \sin 2d - \underline{\frac{\sqrt{2}}{2} \cos 2d} + \frac{\sqrt{2}}{2} \sin 2d + \sqrt{2} \cos^2 d =$$

$$= -\cancel{2} \cdot \frac{\sqrt{2}}{\cancel{2}} \cos 2d + \sqrt{2} \cos^2 d = -\sqrt{2} \cos 2d + \sqrt{2} \cos^2 d =$$

$$= -\sqrt{2} (2 \cos^2 d - 1) + \sqrt{2} \cos^2 d = -2\sqrt{2} \cos^2 d + \sqrt{2} + \sqrt{2} \cos^2 d =$$

$$= \sqrt{2} - \sqrt{2} \cos^2 d = \sqrt{2} - \sqrt{2} (1 - \sin^2 d) = \sqrt{2} - \sqrt{2} + \sqrt{2} \sin^2 d =$$

$$= \sqrt{2} \sin^2 d \quad \text{OK}$$

Esercizio N° 417 prof 752

⑥

$$\operatorname{tg}\left(\frac{\pi}{4} + d\right) \cdot \operatorname{tg}\left(\frac{\pi}{4} - d\right) - 1$$

$$\operatorname{tg}\left(d + \frac{\pi}{4}\right) + \operatorname{tg}\left(\frac{3}{4}\pi + d\right) =$$

$$\frac{\operatorname{tg} \frac{\pi}{4} + \operatorname{tg} d}{1 - \operatorname{tg} \frac{\pi}{4} \cdot \operatorname{tg} d} \cdot \frac{\operatorname{tg} \frac{\pi}{4} - \operatorname{tg} d}{1 + \operatorname{tg} \frac{\pi}{4} \cdot \operatorname{tg} d} - 1$$

$$\frac{\operatorname{tg} d + \operatorname{tg} \frac{\pi}{4}}{1 - \operatorname{tg} d \cdot \operatorname{tg} \frac{\pi}{4}} + \frac{\operatorname{tg} \frac{3}{4}\pi + \operatorname{tg} d}{1 - \operatorname{tg} \frac{3}{4}\pi \cdot \operatorname{tg} d}$$

$$\frac{1 + \cancel{\operatorname{tg} d}}{1 - \cancel{\operatorname{tg} d}} \cdot \frac{1 - \cancel{\operatorname{tg} d}}{1 + \cancel{\operatorname{tg} d}} - 1$$

$$\frac{\operatorname{tg} d + 1}{1 - \operatorname{tg} d} + \frac{-1 + \operatorname{tg} d}{1 + \operatorname{tg} d}$$

$$\frac{(1+\operatorname{tg}d)^2 + (\operatorname{tg}d - 1)(1 - \operatorname{tg}d)}{(1 - \operatorname{tg}d)(1 + \operatorname{tg}d)} = 0$$

OK

Esercizio N° 442 pag 753

(9)

$$2 \cos\left(d - \frac{\beta}{2}\right) \cdot \operatorname{sen} \frac{\beta}{2} - 2 \cos^2 \frac{d}{2} \operatorname{sen} \beta + \operatorname{sen} d \cos \beta =$$

$$= 2 \left[\cos d \cdot \cos \frac{\beta}{2} + \operatorname{sen} d \operatorname{sen} \frac{\beta}{2} \right] \cdot \operatorname{sen} \frac{\beta}{2} -$$

$$- 2 \cdot \frac{1 + \cos d}{2} \cdot \operatorname{sen} \beta + \operatorname{sen} d \cos \beta =$$

$$= 2 \left[\cos d \cdot \cos \frac{\beta}{2} \cdot \operatorname{sen} \frac{\beta}{2} + \operatorname{sen} d \cdot \operatorname{sen} \frac{\beta}{2} \cdot \operatorname{sen} \frac{\beta}{2} \right] -$$

$$- (1 + \cos d) \cdot \operatorname{sen} \beta + \operatorname{sen} d \cos \beta =$$

$$= 2 \cos d \cos \frac{\beta}{2} \operatorname{sen} \frac{\beta}{2} + 2 \operatorname{sen} d \operatorname{sen} \frac{\beta}{2} \cdot \operatorname{sen} \frac{\beta}{2} -$$

$$- \operatorname{sen} \beta - \cos d \operatorname{sen} \beta + \operatorname{sen} d \cos \beta =$$

$$= 2 \cos d \cos \frac{\beta}{2} \operatorname{sen} \frac{\beta}{2} + 2 \operatorname{sen} d \cdot \operatorname{sen}^2 \frac{\beta}{2} - \operatorname{sen} \beta - \cos d \operatorname{sen} \beta +$$
$$+ \operatorname{sen} d \cos \beta =$$

Nel riepunto le

10

$$\sin \frac{\alpha}{2} = \cos \frac{\alpha}{2} = \frac{1}{2} \sin \alpha$$

$$= 2 \cos \left(\frac{1}{2} \sin \beta \right) + 2 \sin \left(\frac{1 - \cos \beta}{2} \right) - \sin \beta - \cos \sin \beta +$$
$$+ 2 \sin \cos \beta =$$

$$= \cancel{\cos \frac{\alpha}{2}} \sin \beta + \sin \alpha - \cancel{\sin \cos \beta} - \sin \beta - \cancel{\cos \frac{\alpha}{2}} \sin \beta +$$

$$+ 2 \sin \cos \beta =$$

$$= \sin \alpha - \cancel{\sin \cos \beta} - \sin \beta + \cancel{\sin \cos \beta} =$$

$$= \sin \alpha - \sin \beta$$

OK

$$\left[\cos^2\left(\frac{\alpha-\beta}{2}\right) - \sin^2\left(\frac{\alpha+\beta}{2}\right) \right] \cdot \frac{\sin(\pi+\beta)}{\sin 2\alpha \cdot \sin 2\beta} =$$

$$= \left[\cos\left(\frac{\alpha-\beta}{2}\right) \cdot \cos\left(\frac{\alpha-\beta}{2}\right) - \sin\left(\frac{\alpha+\beta}{2}\right) \cdot \sin\left(\frac{\alpha+\beta}{2}\right) \right] \cdot$$

$$\cdot \frac{\overset{0}{\nearrow} \sin \pi \cos \beta + \overset{(-1)}{\nearrow} \cos \pi \cdot \sin \beta}{2 \sin \alpha \cos \alpha \cdot 2 \sin \beta \cos \beta} =$$

Applichiamo le formule di Weierstrass adatte opportunamente

$$= \left[\frac{1}{2} \left(\cos\left(\frac{\alpha-\beta}{2} + \frac{\alpha-\beta}{2}\right) + \cos\left(\frac{\alpha-\beta}{2} - \frac{\alpha-\beta}{2}\right) \right) \right] =$$

$$- \frac{1}{2} \left[\cos\left(\frac{\alpha+\beta}{2} - \frac{\alpha+\beta}{2}\right) - \cos\left(\frac{\alpha+\beta}{2} + \frac{\alpha+\beta}{2}\right) \right] \quad (12)$$

$$\cdot \frac{-\cancel{\sin\beta}}{4 \cancel{\sin\beta} \cos\beta} =$$

$$= \frac{1}{2} \left[\cos\left(\frac{\alpha-\beta+\alpha-\beta}{2}\right) + \cos\left(\frac{\alpha-\beta-\alpha+\beta}{2}\right) \right] -$$

$$- \frac{1}{2} \left[\cos\left(\frac{\alpha+\beta-\alpha-\beta}{2}\right) - \cos\left(\frac{\alpha+\beta+\alpha+\beta}{2}\right) \right] \cdot \frac{-1}{4 \cancel{\sin\beta} \cos\beta} =$$

$$= \frac{1}{2} \left[\cos\left(\frac{2\alpha-2\beta}{2}\right) + \cos 0^\circ \right] - \frac{1}{2} \left[\cos 0^\circ - \cos\left(\frac{2\alpha+2\beta}{2}\right) \right] \cdot$$

-1

4 znd cos d cos β

$$= \frac{1}{2} \left[\cos\left(\frac{z(d-\beta)}{z}\right) + 1 \right] - \frac{1}{2} \left[1 - \cos\left(\frac{z(d+\beta)}{z}\right) \right]$$

-1

4 znd cos d cos β

$$= \frac{1}{2} \left[\cos(d-\beta) + 1 \right] - \frac{1}{2} \left[1 - \cos(d+\beta) \right] \cdot \frac{-1}{4 \text{ znd cos } d \text{ cos } \beta} =$$

$$= \left[\frac{1}{2} \cos(d-\beta) + \frac{1}{2} - \frac{1}{2} + \frac{1}{2} \cos(d+\beta) \right] \cdot \frac{-1}{4 \text{ znd cos } d \text{ cos } \beta} =$$

$$= \cancel{\cos(d-\beta)} \left[\frac{1}{2} \cos(d-\beta) + \frac{1}{2} \cos(d+\beta) \right]$$

$$\cdot \frac{-1}{4 \cancel{\cos d} \cos \beta}$$

$$= \frac{1}{2} \left[\cos(d-\beta) + \cos(d+\beta) \right] \cdot \frac{-1}{4 \cancel{\cos d} \cos \beta}$$

$$= \frac{1}{2} \left[\cancel{\cos d} \cos \beta + \cancel{\sin d} / \cancel{\sin \beta} + \cancel{\cos d} \cos \beta - \cancel{\sin d} / \cancel{\sin \beta} \right]$$

$$\cdot \frac{-1}{4 \cancel{\cos d} \cos \beta} = \frac{1}{2} \left(\cancel{2 \cos d} \cos \beta \right) \cdot \frac{-1}{4 \cancel{\cos d} \cos \beta}$$

$$= - \frac{1}{4 \cancel{\cos d}}$$

$$\frac{(\operatorname{sen} d + \operatorname{cos} d)^2 - \operatorname{sen} 2d}{(\operatorname{cosec} d + \operatorname{cotg} d)^2} = \frac{1 - \operatorname{cos} d}{1 + \operatorname{cos} d} =$$

$$= \frac{\operatorname{sen}^2 d + \operatorname{cos}^2 d + 2 \operatorname{sen} d \operatorname{cos} d - 2 \operatorname{sen} d \operatorname{cos} d}{\left(\frac{1}{\operatorname{sen} d} + \frac{\operatorname{cos} d}{\operatorname{sen} d}\right)^2} = \frac{1 - \operatorname{cos} d}{1 + \operatorname{cos} d}$$

$$= \frac{1 + 0}{\left(\frac{1 + \operatorname{cos} d}{\operatorname{sen} d}\right)^2} = \frac{1 - \operatorname{cos} d}{1 + \operatorname{cos} d} =$$

$$= \frac{1}{\frac{1 + \operatorname{cos}^2 d + 2 \operatorname{cos} d}{\operatorname{sen}^2 d}} = \frac{1 - \operatorname{cos} d}{1 + \operatorname{cos} d} =$$

$$= \frac{\operatorname{sen}^2 d}{1 + \operatorname{cos}^2 d + 2 \operatorname{cos} d} = \frac{1 - \operatorname{cos} d}{1 + \operatorname{cos} d} =$$

$$= \frac{\operatorname{sen}^2 d}{(1 + \operatorname{cos} d)^2} = \frac{1 - \operatorname{cos} d}{1 + \operatorname{cos} d} =$$

$$= \frac{\sin^2 d - (1 - \cos d)(1 + \cos d)}{(1 + \cos d)^2} =$$

$$= \frac{\sin^2 d - (1 - \cos^2 d)}{(1 + \cos d)^2} = \frac{\sin^2 d - 1 + \cos^2 d}{(1 + \cos d)^2} =$$

$$= \frac{1 - 1}{(1 + \cos d)^2} = 0 \quad \text{OK}$$

N° 446

pag 753

(17)

$$\frac{\cos 3d + \cos 4d + \cos 5d}{\sin 3d + \sin 4d + \sin 5d} =$$

applichiamo le formule di prostaferesi a
5d e 3d

$$= \frac{2 \cos \frac{3d+5d}{2} \cdot \cos \frac{5d-3d}{2} + \cos 4d}{2 \sin \frac{3d+5d}{2} \cdot \cos \frac{5d-3d}{2} + \sin 4d} =$$

$$= \frac{2 \cos 4d \cdot \cos d + \cos 4d}{2 \sin 4d \cdot \cos d + \sin 4d} =$$

$$= \frac{\cos 4d (2 \cancel{\cos d} + 1)}{\sin 4d (2 \cancel{\cos d} + 1)} = \cot 4d$$

OK

Esercizi presi dal libro:

(18)

G. Zwirner, L. Scaglianti

Funzioni in \mathbb{R}

1A

CASA editrice: CEDAM

$$\frac{\left(\operatorname{sen} \frac{d}{2} + \cos \frac{d}{2}\right)^2 - 1}{\operatorname{tg} \frac{d}{2} - \operatorname{sen} \frac{d}{2} \cdot \cos \frac{d}{2}} = 2 \operatorname{ctg}^2 \frac{d}{2}$$

"

Ricordiamo che dalle formule di duplicazione

$$\operatorname{sen} 2d = 2 \operatorname{sen} d \cos d. \text{ Sostituiamo } d \text{ con } \frac{d}{2} :$$

$$\operatorname{sen} 2 \cdot \frac{d}{2} = 2 \operatorname{sen} \frac{d}{2} \cdot \cos \frac{d}{2} ; \boxed{\operatorname{sen} d = 2 \operatorname{sen} \frac{d}{2} \cdot \cos \frac{d}{2}} \text{ inoltre}$$

$$\boxed{\frac{1}{2} \operatorname{sen} d = \operatorname{sen} \frac{d}{2} \cdot \cos \frac{d}{2}} \quad "$$

$$\frac{\overbrace{\operatorname{sen}^2 \frac{d}{2} + \cos^2 \frac{d}{2}}^{\rightarrow 1} + 2 \operatorname{sen} \frac{d}{2} \cos \frac{d}{2} - 1}{\operatorname{tg} \frac{d}{2} - \frac{1}{2} \operatorname{sen} d} = 2 \operatorname{ctg}^2 \frac{d}{2} ;$$

$$\frac{1 - 1 + 2 \cdot \frac{1}{2} \operatorname{sen} d}{\operatorname{tg} \frac{d}{2} - \frac{1}{2} \operatorname{sen} d} = 2 \operatorname{ctg}^2 \frac{d}{2} ;$$

$$\frac{\text{rand}}{\text{tg} \frac{d}{2} - \frac{1}{2} \text{rand}} = z \text{ctg}^2 \frac{d}{2}$$

$$\frac{\text{rand}}{z \text{tg} \frac{d}{2} - \text{rand}} = z \text{ctg}^2 \frac{d}{2} ;$$

$$\text{rand} \cdot \frac{z}{z \text{tg} \frac{d}{2} - \text{rand}} = z \text{ctg}^2 \frac{d}{2}$$

~~z \text{rand} = (z \text{ctg}^2 \frac{d}{2}) (z \text{tg} \frac{d}{2} - \text{rand})~~

$$z \text{rand} = (z \text{ctg}^2 \frac{d}{2}) (z \text{tg} \frac{d}{2} - \text{rand})$$

$$2r_{end} = 2 \left(\frac{1+w_d}{r_{end}} \right)^2 \left(2 \left(\frac{r_{end}}{1+w_d} \right) - r_{end} \right)$$

21

$$2r_{end} = 2 \frac{(1+w_d)^2}{r_{end}^2} \cdot \left(\frac{2r_{end}}{1+w_d} - r_{end} \right);$$

$$2r_{end} = 2 \cdot \frac{(1+w_d)^2}{r_{end}^2} \cdot \frac{2r_{end}}{1+w_d} - 2 \frac{(1+w_d)^2}{r_{end}^2} \cdot r_{end}$$

$$\cancel{2}r_{end} = 2 \cdot \frac{1+w_d}{r_{end}} \cdot \cancel{2} - \cancel{2} \frac{(1+w_d)^2}{r_{end}}$$

$$r_{end} = \frac{2(1+w_d)}{r_{end}} - \frac{(1+w_d)^2}{r_{end}}$$

$$r_{end} - \frac{2(1+w_d)}{r_{end}} + \frac{(1+w_d)^2}{r_{end}} = 0$$

$$\frac{r_{end}^2 - 2(1+w_d) + (1+w_d)^2}{r_{end}} = 0$$

$$\sin^2 d - 2(1 + \cos d) + (1 + \cos d)^2 = 0$$

$$\sin^2 d - 2 - 2\cos d + 1 + \cos^2 d + 2\cos d = 0$$

$$\sin^2 d + \cos^2 d - 2 + 1 = 0$$

$$1 - 2 + 1 = 0 \quad ; \quad 0 = 0 \quad \therefore$$

N170 pag 596 - ZWIRNER

(23)

$$2 \operatorname{rem} d \cdot \frac{(1 - \cos d)}{\sin^2 \frac{d}{2}} = \frac{4 \operatorname{rem} d + 2 \operatorname{rem}^2 d}{1 + \cos d}$$

$$2 \operatorname{rem} d \cdot \frac{1 - \cos d}{\frac{1 - \cos d}{2}} = \frac{4 \operatorname{rem} d + 2(2 \operatorname{rem} d \cos d)}{1 + \cos d}$$

$$2 \operatorname{rem} d \cdot (1 - \cancel{\cos d}) \cdot \frac{2}{(1 - \cancel{\cos d})} = \frac{4 \operatorname{rem} d + 4 \operatorname{rem} d \cos d}{1 + \cos d}$$

$$4 \operatorname{rem} d = \frac{4 \operatorname{rem} d + 4 \operatorname{rem} d \cos d}{1 + \cos d} ;$$

$$4 = \frac{4 + 4 \cos d}{1 + \cos d} ;$$

$$4(1 + \cos d) = 4(1 + \cos d) ;$$

$$1 + \cos d = 1 + \cos d \Rightarrow 0 = 0 \quad ;)$$