Lezione 13

Equazioni goniometriche

Risolvere un'equazione goniometrica corrisponde a trovare sempre la x, (in questo caso l'angolo) relativo a funzioni goniometriche già studiate: senx, cosx, tgx, cotgx,....

Equazioni goniometriche riducibili ed equazioni elementari

$$\boxed{\cos^2 x = 1 - \operatorname{sen}^2 x}$$

$$8\cos^2 x - 2\operatorname{sen} x = 5$$

$$8(1 - \operatorname{sen}^2 x) - 2\operatorname{sen} x = 5$$

$$8 - 8\operatorname{sen}^2 x - 2\operatorname{sen} x = 5$$

$$-8\operatorname{sen}^2 x - 2\operatorname{sen} x + 3 = 0 \Longrightarrow \boxed{8\operatorname{sen}^2 x + 2\operatorname{sen} x - 3 = 0}$$

$8\,\text{sen}^2 x + 2\,\text{sen}\,x - 3 = 0$ $\qquad\qquad$ $\text{sen}\,x = t$

$8t^2 + 2t - 3 = 0$

$\Delta = b^2 - 4ac = 2^2 - 4(8)(-3) = 4 + 96 = \underline{100}$

$$t_1, t_2 = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-2 \pm \sqrt{100}}{16} \begin{cases} \dfrac{-2-10}{16} = -\dfrac{\cancel{12}^{\,3}}{\cancel{16}_{\,4}} \\[2ex] \dfrac{-2+10}{16} = \dfrac{\cancel{8}^{\,1}}{\cancel{16}_{\,2}} \end{cases}$$

$$\boxed{t_1 = -\frac{3}{4}\ ;\ t_2 = \frac{1}{2}}$$

$\operatorname{sen} x = -\frac{3}{4}$

$x = \operatorname{arcsen}\left(-\frac{3}{4}\right)$

$\Downarrow$

$\boxed{x = -\operatorname{arcsen}\frac{3}{4}}$

FUNZ. DISPARI

$\boxed{f(-x) = -f(x)}$

$\operatorname{sen} x = \frac{1}{2}$

$x = \operatorname{arcsen}\left(\frac{1}{2}\right)$

$x = \frac{\pi}{6} \qquad x = \frac{5}{6}\pi$

$\boxed{x^2 = 4 \quad x = \pm\sqrt{4} = \pm 2}$

$$\boxed{\begin{aligned} x_1 &= \frac{\pi}{6} + 2k\pi \\ x_2 &= \frac{5}{6}\pi + 2k\pi \end{aligned}}$$

$\boxed{k \in \mathbb{Z}}$

$K = \pm 1, \pm 2, \pm 3, \dots$

ES 2

$$2\sin^2 x = 1 + \cos x \qquad \sin^2 x = 1 - \cos^2 x$$

$$2(1-\cos^2 x) = 1 + \cos x \qquad \boxed{\cos x = t}$$

$$2 - 2\cos^2 x = 1 + \cos x$$

$$-2\cos^2 x - \cos x + 1 = 0 \Longrightarrow 2\cos^2 x + \cos x - 1 = 0$$

$$\Delta = b^2 - 4ac = (1)^2 - 4(2)(-1) = 1 + 8 = 9$$

$$t_{1,2} = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-1 \pm \sqrt{9}}{4} \begin{cases} \frac{-1-3}{4} = -\frac{\cancel{4}}{\cancel{4}} = -1 \\ \frac{-1+3}{4} = \frac{\cancel{2}}{\cancel{4}} = \frac{1}{2} \end{cases}$$

$$\boxed{t_1 = -1 \; ; \; t_2 = \frac{1}{2}}$$

$\cos x = -1$

$x = \arccos(-1)$

$$\boxed{x = \pi + 2k\pi}$$

$k \in \mathbb{Z}$

$k = \pm 1 ; \pm 2 ; \ldots$

$\cos x = \frac{1}{2}$

$x = \arccos\left(\frac{1}{2}\right)$

$x_1 = \frac{\pi}{3} + 2k\pi$

$\underbrace{x_2 = -\frac{\pi}{3} + 2k\pi}_{\text{VERS. ORARIO}} \qquad \left(\underbrace{x_2 = \frac{5}{3}\pi + 2k\pi}_{\text{VERS. ANTIOR.}}\right)$

$$\boxed{x = \pm \frac{\pi}{3} + 2k\pi}$$

ES. 3 $\quad tg\,x + 2\,ctg\,x = 3$

$ctg\,x = \frac{1}{tg\,x}$

$tg\,x + \frac{2}{tg\,x} = 3$

$tg\,x = t$

C.E $\quad t \neq 0$

$\Downarrow$

$tg\,x \neq 0$

$x \neq 0 + k\pi$

$\boxed{x \neq k\pi}$ $k \in \mathbb{Z}$

$\frac{t}{1} + \frac{2}{t} = 3$

$\cancel{t}\ \frac{t^2 + 2}{\cancel{t}} = \frac{3t}{\cancel{t}}\ \cancel{t}$

$\boxed{t^2 - 3t + 2 = 0}$

$\Delta = b^2 - 4ac = (-3)^2 - 4(1)(2) = 9 - 8 = 1$

$t_1, t_2 = -\frac{b \pm \sqrt{\Delta}}{2a} = \frac{3 \pm 1}{2}$

$t_1 = \frac{3-1}{2} = \frac{\cancel{2}^{1}}{\cancel{2}_1} = 1$

$t_2 = \frac{3+1}{2} = \frac{\cancel{4}^{2}}{\cancel{2}_1} = 2$

$tg\,x = 1$

$x = arctg\,1$

$\boxed{x = \frac{\pi}{4} + k\pi}$

$\frac{\pi}{4} + \pi = \frac{5}{4}\pi\ (225°)$

$tg\,x = 2$

$\boxed{x = arctg\,2}$

$x \simeq 63°,43$

$\boxed{\sqrt{2}} \simeq 1,414\ldots$

ES. 4

$$1+\sqrt{2}\,\mathrm{sen}x-\cos x-\sqrt{2}\cos x\,\mathrm{sen}x=0$$

$$1\cdot(1+\sqrt{2}\,\mathrm{sen}x)-\cos x(1+\sqrt{2}\,\mathrm{sen}x)=0$$

$$(1+\sqrt{2}\,\mathrm{sen}x)(1-\cos x)=0$$

$$1+\sqrt{2}\,\mathrm{sen}x=0 \qquad\qquad 1-\cos x=0$$

$$\sqrt{2}\,\mathrm{sen}x=-1 \qquad\qquad \cos x=1$$

$$\mathrm{sen}x=-\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=-\frac{\sqrt{2}}{2}$$

A $\boxed{\mathrm{sen}x=-\frac{\sqrt{2}}{2}}$ B $\boxed{\cos x=1}$

$$x=\mathrm{arcsen}\left(-\frac{\sqrt{2}}{2}\right)=-\mathrm{arcsen}\left(\frac{\sqrt{2}}{2}\right)$$

$$\boxed{x_1=-\frac{\pi}{4}+2K\pi=\frac{7}{4}\pi+2K\pi}$$

$$\boxed{x_2=\frac{5}{4}\pi+2K\pi}$$

$K\in\mathbb{Z}$

$$\cos x=1\Rightarrow x=\arccos 1\Rightarrow x=0+2K\pi=2K\pi$$

$$\boxed{x=2K\pi} \qquad K\in\mathbb{Z}$$

ES. 5 $\quad 2\cos^2 x + 4\,\text{sen}^2 x = \frac{1}{2} + \text{tg}^2 x$

$$\boxed{\text{tg}\,x = \frac{\text{sen}\,x}{\cos x}}$$

$$2\cos^2 x + 4\,\text{sen}^2 x = \frac{1}{2} + \frac{\text{sen}^2 x}{\cos^2 x}$$

$\cancel{2\cos^2 x}$

$$\frac{\cancel{4}\cos^4 x + 8\,\text{sen}^2 x \cos^2 x}{2\cancel{\cos^2 x}} = \frac{\cos^2 x + 2\,\text{sen}^2 x}{2\cancel{\cos^2 x}} \qquad 2\cancel{\cos^2 x}$$

$\cos^2 x \neq 0$

$\cos x \neq 0$

$x \neq \frac{\pi}{2} + k\pi$

$k \in \mathbb{Z}$

$$4\cos^4 x + 8\,\text{sen}^2 x \cos^2 x = \cos^2 x + 2\,\text{sen}^2 x$$

$$4\cos^4 x + 8\,\text{sen}^2 x \cos^2 x - \cos^2 x - 2\,\text{sen}^2 x = 0$$

$$\underbrace{4\cos^4 x}_{1} + \underbrace{8\,\text{sen}^2 x \cos^2 x}_{2} \underbrace{- \cos^2 x}_{3} \underbrace{- 2\,\text{sen}^2 x}_{4} = 0$$

$$\cos^2 x\,(4\cos^2 x - 1) + 2\,\text{sen}^2 x\,(4\cos^2 x - 1) = 0$$

$$(4\cos^2 x - 1)\,(\cos^2 x + 2\,\text{sen}^2 x) = 0$$

A B

A $\quad 4\cos^2 x - 1 = 0 \implies \cos^2 x = \frac{1}{4} \implies \cos x = \pm\sqrt{\frac{1}{4}}$

$\cos x = \pm \frac{1}{2}$

$\cos x = \frac{1}{2}$ $\quad \boxed{x = \arccos \frac{1}{2}}$

$\boxed{x_1 = \pm \frac{\pi}{3} + 2k\pi}$ $\quad \boxed{k \in \mathbb{Z}}$

$\cos x = -\frac{1}{2}$

$\boxed{x = \arccos\left(-\frac{1}{2}\right)}$

$\boxed{x = \pm \frac{5}{6}\pi + 2k\pi \quad k \in \mathbb{Z}}$

Ⓑ $\cos^2 x + 2\,\text{sen}^2 x = 0$ $\cos^2 x = 1 - \text{sen}^2 x$

$1 - \text{sen}^2 x + 2\,\text{sen}^2 x = 0$

$\text{sen}^2 x = -1$?!? $\text{sen}\,x = \pm\sqrt{-1}$?!?

$\underline{\nexists x \in \mathbb{R}}$

ES. 6 $\underline{sen(x+45^\circ)} - \underline{sen(x-45^\circ)} = 1$ $45^\circ \Rightarrow \frac{\pi}{4}$

$$\boxed{sen(\alpha \pm \beta) = sen\alpha\cos\beta \pm \cos\alpha\, sen\beta}$$

$sen x \cos 45^\circ + sen 45^\circ \cos x +$
$- [sen x \cos 45^\circ - sen 45^\circ \cos x] = 1$

$\underline{\cancel{sen x \cos 45^\circ}} + sen 45^\circ \cos x - \underline{\cancel{sen x \cos 45^\circ}} + sen 45^\circ \cos x = 1$

$2\, sen 45^\circ \cos x = 1 \Rightarrow \cancel{2} \cdot \frac{\sqrt{2}}{\cancel{2}} \cos x = 1$

$\sqrt{2}\cos x = 1 \Rightarrow \cos x = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \Rightarrow \boxed{\cos x = \frac{\sqrt{2}}{2}}$

$x = \pm\frac{\pi}{4} + 2k\pi$
$k \in \mathbb{Z}$