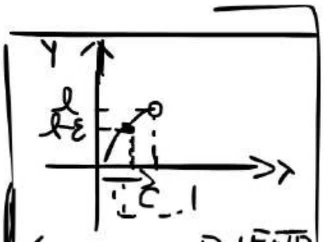
Lezione 13

Limite per difetto o per eccesso

Si dice limite per eccesso o per difetto e si definiscono nel seguente modo:

(A) $\lim_{x \to c} f(x) = l^+$ ECCESSO

(B) $\lim_{x \to c} f(x) = l^-$ DIFETTO



DIFETTO

$|f(x) - l| < \varepsilon \Rightarrow -\varepsilon < f(x) - l < \varepsilon$

$l - \varepsilon < f(x) < l + \varepsilon$

ECCESSO

(A) $\forall \varepsilon > 0 \ \exists I(c) : \forall x \in I(c) \Rightarrow l \leq f(x) < l + \varepsilon$

(B) $\forall \varepsilon > 0 \ \exists I(c) : \forall x \in I(c) \Rightarrow l - \varepsilon < f(x) \leq l$

Limite finito di una funzione per x che tende a un valore infinito.

$$y = f(x) = \frac{x-1}{x}$$

Dom f $\quad x \neq 0$

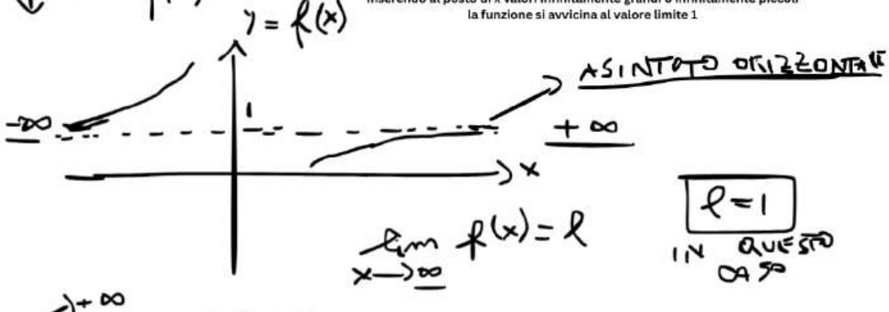
$\mathbb{R} \setminus \{0\}$

x	$f(x)$
10	0,9
100	0,99
1000	0,999
−10	+1,1
−100	+1,01
−1000	+1,001

$M = \boxed{10,\ 100,\ 1000,\ 10000 \ldots \text{ INFINIT. GRANDI}}$

$\boxed{-10, -100, -1000, -1000 \text{ INFINT. PICCOLI}}$

Inserendo al posto di x valori infinitamente grandi o infinitamente piccoli
la funzione si avvicina al valore limite 1



$$\lim_{x \to \infty} f(x) = l$$

$\boxed{l = 1}$ IN QUESTO CASO

$\infty \to +\infty$, $\infty \to -\infty$ CONTEMPOR.

$$|f(x)-l|<\varepsilon \Rightarrow \left|\frac{x-1}{x}-\frac{1}{1}\right|<\varepsilon$$

$$\left|\frac{\cancel{x}-1-\cancel{x}}{x}\right|<\varepsilon \Rightarrow \left|-\frac{1}{x}\right|<\varepsilon \Rightarrow \boxed{\left|\frac{1}{x}\right|<\varepsilon}$$

$$\boxed{\begin{array}{l} |f(x)|<K \quad K\in\mathbb{R} \\ -K<f(x)<K \end{array}}$$

$$|x|>\frac{1}{\varepsilon}$$

PASSAGGIO AGLI INVERSI

$$\boxed{x<-\frac{1}{\varepsilon} \vee x>\frac{1}{\varepsilon}}$$

$$|f(x)|>K$$

$$f(x)<-K \vee f(x)>K$$

$$\boxed{I(\infty)=\underbrace{\left]-\infty;-\frac{1}{\varepsilon}\right[}_{I(-\infty)} \cup \underbrace{\left]\frac{1}{\varepsilon};+\infty\right[}_{I(+\infty)}}$$

|||
...

$$\lim_{x\to\infty} f(x)=l$$

$$\boxed{\forall\varepsilon>0\ \exists I(\infty): \forall x\in I(\infty) \Longrightarrow |f(x)-l|<\varepsilon}$$

ES. $\boxed{\lim_{x \to \infty} \frac{x-1}{2x+3} = \frac{1}{2}}$

$$\forall \varepsilon > 0 \ \exists I(\infty) : \forall x \in I(\infty) \Rightarrow \boxed{\left| \frac{x-1}{2x+3} - \frac{1}{2} \right| < \varepsilon}$$

$$\left| \frac{2(x-1)-(2x+3)}{2(2x+3)} \right| < \varepsilon \Rightarrow \left| \frac{\cancel{2x}-2-\cancel{2x}-3}{2(2x+3)} \right| < \varepsilon$$

$$\left| \frac{-5}{2(2x+3)} \right| < \varepsilon \Rightarrow \left| \frac{5}{4x+6} \right| < \varepsilon$$

PASSAGGIO AGLI INVERSI

$$\frac{5}{|4x+6|} < \varepsilon \Rightarrow \cancel{5} \cdot \frac{|4x+6|}{\cancel{5}} > \frac{1}{\varepsilon} \cdot 5$$

$K \in \mathbb{R}^+$

$$\boxed{\begin{array}{l} |f(x)| > K \\ f(x) < -K \lor f(x) > K \end{array}}$$

$$|4x+6| > \frac{5}{\varepsilon}$$

$$4x+6 < -\frac{5}{\varepsilon} \quad \lor \quad 4x+6 > \frac{5}{\varepsilon}$$

$$4x < -6 - \frac{5}{\varepsilon} \quad \lor \quad 4x > -6 + \frac{5}{\varepsilon} \qquad \varepsilon = 0{,}1$$

$$x < -\frac{\cancel{6}^{\,3}}{\cancel{4}_{\,2}} - \frac{5}{4\varepsilon} \quad \lor \quad x > -\frac{\cancel{6}^{\,3}}{\cancel{4}_{\,2}} + \frac{5}{4\varepsilon}$$

$$\underbrace{x < -\frac{3}{2} - \overbrace{\frac{5}{4\varepsilon}}^{12{,}5}}_{-14} \quad \lor \quad x > \underbrace{-\frac{3}{2} + \overbrace{\frac{5}{4\varepsilon}}^{12{,}5}}_{11}$$

$$x \in \underbrace{\left]-\infty; \underbrace{-14}_{-\frac{3}{2}-\frac{5}{4\varepsilon}}\right[}_{I(-\infty)} \cup \underbrace{\left]\overbrace{11}^{-\frac{3}{2}+\frac{5}{4\varepsilon}}, +\infty\right[}_{I(+\infty)} = I(\infty)$$