Lezione 14

Equazioni goniometriche

Sono equazioni in cui sono presenti funzioni goniometriche dell'angolo (non noto) x e quindi:
senx, cosx e tgx

Equazioni riducibili ad equazioni elementari.

Sono le più facili fra le tipologie di equazioni goniometriche e l'obiettivo è ricondurle a un'equazione elementare del tipo: senx = 1, cosx = 0.

$$\text{sen}^2 x + \cos^2 x = 1$$

$$\boxed{\cos^2 x = 1 - \text{sen}^2 x}$$

$$8 \cos^2 x - 2\,\text{sen}\,x = 5$$

$$8 \cdot (1 - \text{sen}^2 x) - 2\,\text{sen}\,x = 5$$

$$8 - 8\,\text{sen}^2 x - 2\,\text{sen}\,x = 5$$

$$8\,\text{sen}^2 x + 2\,\text{sen}\,x - 8 = -5 \Longrightarrow \boxed{8\,\text{sen}^2 x + 2\,\text{sen}\,x - 3 = 0}$$

$$8\,\text{sen}^2x + 2\,\text{sen}\,x - 3 = 0 \qquad \boxed{\text{sen}\,x = t}$$

$$\boxed{8t^2 + 2t - 3 = 0}$$

$$\Delta = b^2 - 4ac = 2^2 - 4(8)(-3) = 4 + 96 = 100$$

$$t_1, t_2 = -\frac{b \pm \sqrt{\Delta}}{2a} = \frac{-2 \pm \sqrt{100}}{16} \quad \begin{cases} \dfrac{-2-10}{16} = -\dfrac{\cancel{12}^{\,3}}{\cancel{16}_{\,4}} \\[2ex] \dfrac{-2+10}{16} = \dfrac{\cancel{8}^{\,1}}{\cancel{16}_{\,2}} = \dfrac{1}{2} \end{cases}$$

$$t_1 = -\frac{3}{4} \ \vee\ t_2 = \frac{1}{2}$$

A $\boxed{\text{sen}\,x = -\frac{3}{4}}$ B $\boxed{\text{sen}\,x = \frac{1}{2}}$

(A) $\operatorname{sen} x = -\frac{3}{4} \implies \boxed{x = \operatorname{arcsen}\left(-\frac{3}{4}\right)} \approx -48^\circ\, 59'$

CIRCA

$k \in \mathbb{Z}$

(B) $\operatorname{sen} x = \frac{1}{2} \implies x = \operatorname{arcsen}\left(\frac{1}{2}\right) = \frac{\pi}{6} + 2k\pi$

$$\boxed{\begin{array}{ll} x_1 = \frac{\pi}{6} + 2k\pi & (30^\circ) \\ & k \in \mathbb{Z} \\ x_2 = \frac{5}{6}\pi + 2k\pi & (150^\circ) \end{array}}$$

$k = \pm 1, \pm 2, \ldots$

$\frac{150^\circ}{180^\circ} = \frac{\cancel{15}^{5}}{\cancel{18}_{6}}\pi$

$$\boxed{sen^2 x = 1 - \cos^2 x}$$

$$2\, sen^2 x = 1 + \cos x$$

$$2(1 - \cos^2 x) = 1 + \cos x$$

$$2 - 2\cos^2 x = 1 + \cos x$$

$$-2\cos^2 x - \cos x + 1 = 0 \Rightarrow \boxed{2\cos^2 x + \cos x - 1 = 0}$$

$$\boxed{\cos x = t}$$

$$2t^2 + t - 1 = 0$$

$$\Delta = b^2 - 4ac = 1 - 4(2)(-1) = 1 + 8 = 9$$

$$t_1, t_2 = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-1 \pm \sqrt{9}}{4}$$

$$\frac{-1-3}{4} = -\frac{\cancel{4}}{\cancel{4}} = -1$$

$$\frac{-1+3}{4} = \frac{\cancel{2}^{1}}{\cancel{4}_{2}} = \frac{1}{2}$$

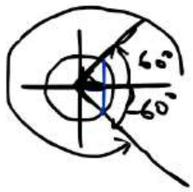
$$t_1 = -1 \lor t_2 = \frac{1}{2}$$

(A) $\boxed{\cos x = -1}$

(B) $\boxed{\cos x = \frac{1}{2}}$

A $\cos x = -1 \Longrightarrow x = \arccos(-1) = \pi + 2K\pi$

$$\boxed{x = \pi + 2K\pi, \quad K \in \mathbb{Z}}$$



B $\cos x = \frac{1}{2} \Longrightarrow x = \arccos\left(\frac{1}{2}\right)$

$x_1 = \frac{\pi}{3} + 2K\pi$

$x_2 = -\frac{\pi}{3} + 2K\pi$

$$\boxed{x = \pm\frac{\pi}{3} + 2K\pi, \quad K \in \mathbb{Z}}$$

$$\mathrm{tg}\,x + 2\,\mathrm{cotg}\,x = 3$$

$$\boxed{\mathrm{cotg}\,x = \frac{1}{\mathrm{tg}\,x}}$$

$$\mathrm{tg}\,x + 2 \cdot \frac{1}{\mathrm{tg}\,x} = 3$$

$$\boxed{\mathrm{tg}\,x = t} \qquad t \neq 0$$

$$\boxed{\begin{array}{ll} \text{C.E.}\ \ \mathrm{tg}\,x \neq 0 & \\ x \neq \mathrm{arctg}\,0 & \\ x \neq k\pi & k \in \mathbb{Z} \\ k=0 & k=1 \\ x \neq 0 & x \neq \pi \end{array}}$$

$$\frac{t}{1} + \frac{2}{t} = \frac{3}{1}$$

$$\cancel{t}\ \frac{t^2 + 2}{\cancel{t}} = \frac{3t}{\cancel{t}}\ \cancel{t}$$

$$t^2 + 2 = 3t$$

$$t^2 - 3t + 2 = 0$$

$$\Delta = b^2 - 4ac = (-3)^2 - 4(1)(2) = 9 - 8 = 1$$

$$t_1, t_2 = -\frac{b \pm \sqrt{\Delta}}{2a} = \frac{-(-3) \pm \sqrt{1}}{2} = \frac{3 \pm 1}{2}$$

$$\frac{3-1}{2} = \frac{\cancel{2}}{\cancel{2}} = 1$$

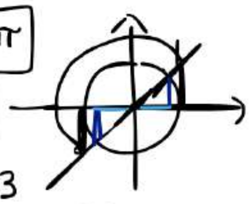
$$\frac{3+1}{2} = \frac{\cancel{4}}{\cancel{2}} = 2$$

$$t_1 = 1 \quad \text{și} \quad t_2 = 2$$

A $\boxed{\mathrm{tg}\,x = 1}$

B $\boxed{\mathrm{tg}\,x = 2}$

(A) $\boxed{tg\,x = 1 \Rightarrow x = arctg(1) = \frac{\pi}{4} + k\pi \quad k \in \mathbb{Z}}$



(B) $tg\,x = 2 \Rightarrow x = arctg(2) \simeq 63{,}43^\circ$

$$\boxed{x = arctg(2) + k\pi \;,\; k \in \mathbb{Z}}$$

$$\boxed{x_1 = \frac{\pi}{4} + k\pi \;\wedge\; x_2 = arctg\,2 + k\pi \;;\; k \in \mathbb{Z}}$$

$$1+\sqrt{2}\,\text{sen}x-\cos x-\sqrt{2}\cos x\,\text{sen}x=0$$

$$(1+\sqrt{2}\,\text{sen}x)-\cos x(1+\sqrt{2}\,\text{sen}x)=0$$

$$(1+\sqrt{2}\,\text{sen}x)\cdot(1-\cos x)=0$$

A B

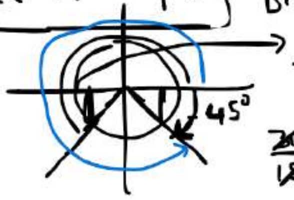
A $1+\sqrt{2}\,\text{sen}x=0 \Rightarrow \sqrt{2}\,\text{sen}x=-1 \Rightarrow \text{sen}x=-\frac{1}{\sqrt{2}}$

$$\text{sen}x=-\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} \Rightarrow \boxed{\text{sen}x=-\frac{\sqrt{2}}{2}}$$

$$\operatorname{sen} x = -\frac{\sqrt{2}}{2} \Longrightarrow x = \operatorname{arcsen}\left(-\frac{\sqrt{2}}{2}\right) = -\operatorname{arcsen}\left(\frac{\sqrt{2}}{2}\right)$$

$$\boxed{f(-x) = -f(x)}$$ FUNZ. DISPARI

$$x = -\operatorname{arc\,sen}\left(\frac{\sqrt{2}}{2}\right)$$



$$\boxed{\begin{array}{l} x_2 = -\frac{\pi}{4} + 2k\pi \quad \left(\frac{7}{4}\pi + 2k\pi\right) \\ x_1 = \frac{5}{4}\pi + 2k\pi \qquad k \in \mathbb{Z} \end{array}}$$

$$\frac{225}{180} \qquad \frac{225}{180}$$

$$1 - \cos x = 0 \Longrightarrow -\cos x = -1$$

$$\cos x = 1 \Rightarrow x = \arccos(1)$$

$$\boxed{x = 0 + 2k\pi = 2k\pi \qquad k \in \mathbb{Z}}$$

$$2\cos^2 x + 4\,\text{sen}^2 x = \frac{1}{2} + \text{tg}^2 x$$

$$\text{tg}\,x = \frac{\text{sen}\,x}{\cos x}$$

C.E.

$$\cos^2 x \neq 0$$
$$\cos x \neq 0$$
$$x \neq \frac{\pi}{2} + k\pi \qquad k \in \mathbb{Z}$$

$$2\cos^2 x + 4\,\text{sen}^2 x = \frac{1}{2} + \frac{\text{sen}^2 x}{\cos^2 x}$$

$$\frac{4\cos^4 x + 8\,\text{sen}^2 x\cos^2 x}{2\cancel{\cos^2 x}} = \frac{\cos^2 x + 2\,\text{sen}^2 x}{2\cancel{\cos^2 x}}$$

$$4\cos^4 x + 8\,\text{sen}^2 x\cos^2 x = \cos^2 x + 2\,\text{sen}^2 x$$

$$4\cos^4 x + 8\,\text{sen}^2 \cos^2 x - \cos^2 x - 2\,\text{sen}^2 x = 0$$

$$\cos^2 x\left(4\cos^2 x - 1\right) + 2\,\text{sen}^2 x\left(4\cos^2 x - 1\right) = 0$$

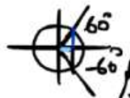
$(4\cos^2 x - 1)(\cos^2 x + 2\,\text{sen}^2 x) = 0$

$\qquad$ A $\qquad\qquad\qquad$ B

A $\qquad 4\cos^2 x - 1 = 0$ $\qquad\qquad \cos x = t$

$4t^2 - 1 = 0 \Longrightarrow t^2 = \frac{1}{4} \Longrightarrow \boxed{t = \pm\frac{1}{2}}$

$\cos x = \frac{1}{2}$ $\qquad\qquad\qquad\qquad \cos x = -\frac{1}{2}$



$x = \arccos\left(\frac{1}{2}\right)$ $\qquad \boxed{x = \pm\pi + k\pi}$ $\qquad x = \arccos\left(-\frac{1}{2}\right)$

$\pm\frac{\pi}{3} + 2k\pi$

$\boxed{x = \pm\frac{\pi}{3} + 2k\pi}$

$x_1 = \frac{\pi}{3} + k\pi$

$x_2 = -\frac{\pi}{3} + k\pi$

$\boxed{\begin{array}{l} x = \frac{4}{3}\pi + 2k\pi \\ x = \frac{2}{3}\pi + 2k\pi \end{array}}$

$\frac{240}{180}$ $= \frac{24}{18}$

$\frac{120}{180} = \frac{12}{18}$

B

$\cos^2 x + 2\,\text{sen}^2 x = 0$ $\qquad\qquad \cos^2 x = 1 - \text{sen}^2 x$

$1 - \text{sen}^2 x + 2\,\text{sen}^2 x = 0$

$\text{sen}^2 x = -1 \Longrightarrow$?!? $\qquad \text{sen}\, x = \pm\sqrt{-1} \rightarrow \nexists\, x \in \mathbb{R}$

$$\boxed{x = \pm\frac{\pi}{3} + k\pi,\ k \in \mathbb{Z}}$$