Lezione 14

RIPRESA DELLA 2° TIPOLOGIA DI LIMITE

$\varepsilon = 0{,}1;\ 0{,}01$

$|f(x) - l| < \varepsilon$

$$\boxed{\lim_{x \to \infty} \frac{1}{x} = 0}$$

$\frac{\varepsilon}{1}$

$$\forall \varepsilon > 0\ \exists I(\infty) : \forall x \in I(\infty) \Rightarrow \left| \frac{1}{x} - 0 \right| < \varepsilon$$

$\left| \frac{1}{x} \right| < \varepsilon$ PASSAGGIO AGLI INVERSI $K \in \mathbb{R}$

$\boxed{|x| > \frac{1}{\varepsilon}}$

$\boxed{\begin{array}{l} |f(x)| > K \\ f(x) < -K \vee f(x) > K \end{array}}$

$$\boxed{|x| > \frac{1}{\varepsilon} \Rightarrow x < -\frac{1}{\varepsilon} \vee x > \frac{1}{\varepsilon}}$$

$$\underbrace{\left] -\infty ; -\frac{1}{\varepsilon} \right[}_{I(-\infty)} \cup \underbrace{\left] \frac{1}{\varepsilon} ; +\infty \right[}_{I(+\infty)} = \underline{I(\infty)} \ !!!$$

$$\lim_{x\to\infty} \frac{x^2+1}{x^2} = 1^+$$

$-\varepsilon < \frac{x^2+1}{x^2} - 1 < \varepsilon$ $\left|\frac{x^2+1}{x^2} - 1\right| < \varepsilon$

$\forall \varepsilon > 0 \ \exists I(\infty) : \forall x \in I(\infty) \Longrightarrow$ $\boxed{0 \le \frac{x^2+1}{x^2} - 1 < \varepsilon}$

$$0 \le \frac{x^2+1}{x^2} - 1 < \varepsilon \Longrightarrow 0+1 \le \frac{x^2+1}{x^2} \cancel{-1} \cancel{+1} < \varepsilon + 1$$

$\Longleftrightarrow$ $\boxed{1 \le \frac{x^2+1}{x^2} < 1+\varepsilon}$ $\boxed{\frac{x^2+1}{x^2} = \cancel{\frac{x^2}{x^2}} + \frac{1}{x^2}}$

$$\begin{cases} 1 \le \frac{x^2+1}{x^2} \Longrightarrow \frac{x^2+1}{x^2} \ge 1 \\ \frac{x^2+1}{x^2} < 1+\varepsilon \end{cases}$$

$$\begin{cases} \cancel{1} + \frac{1}{x^2} \ge \cancel{1} \Longrightarrow \frac{1}{x^2} \ge 0 \\ \cancel{1} + \frac{1}{x^2} < \cancel{1} + \varepsilon \Longrightarrow \frac{1}{x^2} < \varepsilon \end{cases}$$

$\forall x \in \mathbb{R} \setminus \{0\}$

$$\begin{cases} \frac{1}{x^2} \ge 0 \quad (A) \\ \frac{1}{x^2} < \varepsilon \quad (B) \end{cases}$$

(A)

$\forall x \in \mathbb{R}\setminus\{0\}$

N $1 \ge 0$ $\forall x \in \mathbb{R}$

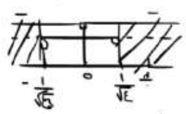
D $x^2 > 0$ $\forall x \in \mathbb{R}\setminus\{0\}$

$$\frac{1}{x^2} < \varepsilon \Rightarrow \frac{1}{x^2} - \varepsilon < 0 \Rightarrow \frac{1 - \varepsilon x^2}{x^2} < 0$$

N $1 - \varepsilon x^2 > 0$

$\varepsilon x^2 - 1 < 0$

$\boxed{x^2 < \frac{1}{\varepsilon}}$



$x^2 = \frac{1}{\varepsilon}$

$x = \pm\frac{1}{\sqrt{\varepsilon}}$

$-\frac{1}{\sqrt{\varepsilon}} < x < \frac{1}{\sqrt{\varepsilon}}$

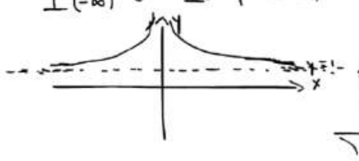
D $x^2 > 0$ $\forall x \in \mathbb{R}\setminus\{0\}$

$$x < -\frac{1}{\sqrt{\varepsilon}} \quad \vee \quad x > \frac{1}{\sqrt{\varepsilon}}$$

$$\underbrace{\left]-\infty; -\frac{1}{\sqrt{\varepsilon}}\right[}_{I(-\infty)} \cup \underbrace{\left]\frac{1}{\sqrt{\varepsilon}}; +\infty\right[}_{I(+\infty)} = I(\infty)$$

$\pm\frac{1}{\sqrt{\varepsilon}} \simeq \pm 3{,}16$

$\varepsilon = 0{,}1$



asintoto orizzontale

Limite infinito di una funzione con x che tende ad un valore finito

ES. $y = f(x) = \frac{1}{x-1}$ Dom f $x - 1 \neq 0$ $\boxed{x \neq 1}$

$\mathbb{R} \setminus \{1\}$

x	f(x)
0,9	-10
0,99	-100
0,999	-1000
1,1	10
1,01	100
1,001	1000

$\to 0$ f(x) DIVENTA PICCOLISSIMA $\quad x \to 1^-$ $f(x) \to -\infty$

$x \to 1^+$ $f(x) \to +\infty$

o f(x) DIVENTA GRANDISSIMO

$$\boxed{\lim_{x \to 1} \frac{1}{x-1} = \infty}$$

$\boxed{M = 10, 100, 1000 \text{ ARBITR. GRANDE}}$

$$\lim_{x \to 1^-} \frac{1}{x-1} = -\infty \qquad \lim_{x \to 1^+} \frac{1}{x-1} = +\infty$$

$$\forall M > 0 \ \exists I(1) : \forall x \in I(1) \Longrightarrow \frac{1}{x-1} < -M \vee \frac{1}{x-1} > M$$

$$\boxed{\left|\frac{1}{x-1}\right| > M}$$

$$\lim_{x \to c} f(x) = \infty$$

$$\forall \underline{M} > 0 \ \exists I(c) : \forall x \in I(c) \Longrightarrow |f(x)| > M$$

$$\boxed{\lim_{x \to 1} \frac{2x-1}{x-1} = \infty}$$

Dom f $\mathbb{R} \setminus \{1\}$

$\forall M > 0 \ \exists I(1) : \forall x \in I(1) \setminus \{1\} \Rightarrow \left| \frac{2x-1}{x-1} \right| > M$

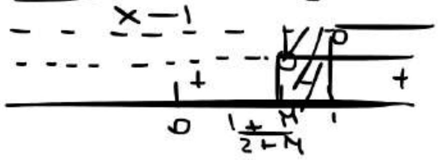
B

A $\frac{2x-1}{x-1} < -M$ $\vee$ $\frac{2x-1}{x-1} > M$

(A) $\dfrac{2x-1}{x-1} < -M \implies \dfrac{2x-1}{x-1} + \dfrac{M}{1} < 0$

$\dfrac{2x-1+M(x-1)}{x-1} < 0$

$\dfrac{2x-1+Mx-M}{x-1} < 0$



(A) $\boxed{\dfrac{1+M}{2+M} < x < 1}$

N $2x-1+Mx-M > 0$

$(2+M)x > 1+M$

$\boxed{x > \dfrac{1+M}{2+M}} \approx 0{,}9$

D $x-1>0 \implies \boxed{x>1}$

(B) $\frac{2x-1}{x-1} > M \Longrightarrow \frac{2x-1}{x-1} - \frac{M}{1} > 0$

$\frac{2x-1-M(x-1)}{x-1} > 0$

N

$2x-1-Mx+M>0$

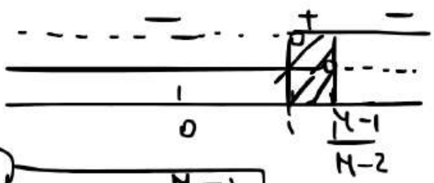
$(2-M)x > 1-M$

$(M-2)x < M-1$

$x < \frac{M-1}{M-2}$

$\simeq 1,125$

D $x-1>0$ $x>1$

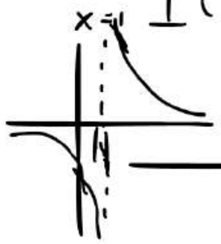


(B) $\boxed{1<x<\frac{M-1}{M-2}}$

(A) $\frac{1+M}{2+M} < x < 1 \quad \vee \quad 1 < x < \frac{M-1}{M-2}$

$$I(1) = \left]\frac{1+M}{2+M}; \frac{M-1}{M-2}\right[\setminus \{1\}$$

$$I(1) = \left]\underset{\approx 0,91}{\frac{1+M}{2+M}}; 1\right[\cup \left]1; \underset{\approx 1,125}{\frac{M-1}{M-2}}\right[$$



$\longrightarrow$ ASINTOTO VERTICALE