LEZIONE 14

Equazioni goniometriche di riepilogo

$\operatorname{sen} x = t$

$$2\operatorname{sen}^2 x - \sqrt{2}\operatorname{sen} x = 0$$

$$2t^2 - \sqrt{2}t = 0 \qquad t(2t - \sqrt{2}) = 0$$

$$\boxed{t = 0} \qquad 2t - \sqrt{2} = 0 \Rightarrow \boxed{t = \frac{\sqrt{2}}{2}}$$

$$\operatorname{sen} x = \frac{\sqrt{2}}{2}$$

$$\operatorname{sen} x = 0 \Rightarrow x = \arcsin 0 + k\pi \qquad x = \arcsin \frac{\sqrt{2}}{2} + 2k\pi \quad k \in \mathbb{Z}$$

$$\boxed{x = k\pi, \quad k \in \mathbb{Z}} \qquad \boxed{x = \frac{\pi}{4} + 2k\pi \qquad x = \frac{3}{4}\pi + 2k\pi}$$

$$\cos^2 x - \frac{\sqrt{2}}{2}\cos x = 0 \qquad (*) \qquad \cos x = t$$

$$t^2 - \frac{\sqrt{2}}{2}t = 0 \Rightarrow t\left(t - \frac{\sqrt{2}}{2}\right) = 0$$

$$t = 0 \qquad\qquad t = \frac{\sqrt{2}}{2}$$

$\frac{\cancel{45}^{1}}{\cancel{180}_{4}}$

$\frac{\cancel{2}}{\cancel{4}}$ $\frac{\cancel{315}}{\cancel{180}}$ $\frac{\cancel{63}}{\cancel{36}}$

$$\cos x = 0 \qquad \boxed{x = \frac{\pi}{2} + k\pi}$$

$$x = \arccos 0 \qquad k \in \mathbb{Z}$$

$$\cos x = \frac{\sqrt{2}}{2}$$

$$x = \arccos\frac{\sqrt{2}}{2} = \frac{\pi}{4}$$

$$\boxed{x = \pm\frac{\pi}{4} + 2k\pi} \qquad \left(\frac{\pi}{4} + 2k\pi\right)$$

$tg^2 x + tg\, x = 0$

$t^2 + t = 0$

$t(t+1) = 0$

$t = 0 \qquad t = -1$

$\underline{tg\, x = 0} \qquad tg\, x = -1$

$\cancel{\frac{135}{180}}$ 45 15 3

$\cancel{60}$ $\cancel{204}$

$tg\, x = t$

$tg\, x = 0$

$x = arctg\, 0$

$x_1 = 0 + k\pi \qquad x_2 = \pi + k\pi$

$\boxed{x = k\pi} \qquad k \in \mathbb{Z}$

$tg\, x = -1$

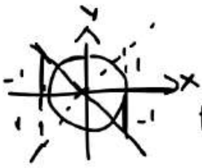
$x = arctg(-1)$

$\boxed{x = \frac{3}{4}\pi + k\pi}$ $\left(-\frac{\pi}{4} + k\pi\right)$ ORARIO

ANTIORARIO

$$\operatorname{cotg}^2 x + \operatorname{cotg} x = 0 \qquad \operatorname{cotg} x = \frac{1}{\operatorname{tg} x}$$

$$\frac{1}{\operatorname{tg}^2 x} + \frac{1}{\operatorname{tg} x} = 0$$



$$\operatorname{tg} x = t$$

$$\frac{1}{t^2} + \frac{1}{t} = 0$$

$t \neq 0$ $\quad \operatorname{tg} x \neq 0$ $\quad$ C.E.

$$\boxed{x \neq k\pi,\ k \in \mathbb{Z}}$$

$$\cancel{t^2} \cdot \frac{1 + t}{\cancel{t^2}} = 0 \cdot t^2$$

$$1 + t = 0 \Rightarrow \boxed{t = -1}$$

$$\operatorname{tg} x = -1 \qquad x = \operatorname{arctg}(-1) \qquad \boxed{x = \frac{3}{4}\pi + k\pi,\ k \in \mathbb{Z}}$$

Equazioni lineari in seno e coseno

Uso le formule parametriche (già viste e dimostrate) per seno e coseno, sfruttando una loro scrittura in funzione di tg(x/2)

$$\begin{cases} \operatorname{sen} x = \dfrac{2t}{1+t^2} \\ \cos x = \dfrac{1-t^2}{1+t^2} \end{cases}$$

$$t = \operatorname{tg}\frac{x}{2} = \frac{\operatorname{sen}\frac{x}{2}}{\cos\frac{x}{2}}$$

$$\cos\frac{x}{2} \neq 0 \implies \frac{x}{2} \neq \text{arccos}\,0$$

$$\frac{x}{2} \neq \frac{\pi}{2} + k\pi \quad \boxed{x \neq \pi + 2k\pi}$$

$$\operatorname{sen} x - \cos x = 0 \qquad \operatorname{sen} x = \cos x$$

$$\begin{cases} \operatorname{sen} x = \dfrac{2t}{1+t^2} \\ \cos x = \dfrac{1-t^2}{1+t^2} \end{cases}$$

$$\boxed{t = \operatorname{tg}\frac{x}{2} \qquad x \neq \pi + 2k\pi}$$

$$\frac{2t}{1+t^2} - \frac{(1-t^2)}{1+t^2} = 0$$

$$\cancel{1+t^2} \cdot \frac{2t - (1-t^2)}{\cancel{1+t^2}} = 0 \quad \cdot 1+t^2$$

$$1+t^2 \neq 0$$

$$t^2 \neq -1$$

$$\forall t \in \mathbb{R}$$

$$2t - 1 + t^2 = 0$$

$$t^2 + 2t - 1 = 0$$

$$t^2+2t-1=0$$

$$\Delta = b^2-4ac = 2^2-4(1)(-1)=4+4=8$$

$$t_1,t_2 = -\frac{b \pm \sqrt{\Delta}}{2a} = \frac{-2\pm\sqrt{8}}{2} \to \frac{-2-\sqrt{8}}{2},\quad \frac{-2+\sqrt{8}}{2}$$

$$\boxed{\sqrt{8}=\sqrt{2^3}=\sqrt{2^2\cdot 2}=2\sqrt{2}}$$

$$t_1 = \frac{-2-2\sqrt{2}}{2} = \frac{\cancel{2}(-1+\sqrt{2})}{\cancel{2}} \qquad t_2 = \frac{-2+2\sqrt{2}}{2} = \frac{\cancel{2}(-1+\sqrt{2})}{\cancel{2}}$$

$$\boxed{t_1=-1-\sqrt{2} \qquad t_2=-1+\sqrt{2}}$$

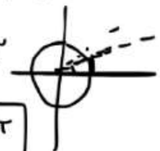
$t_1 = -1-\sqrt{2}$ $\qquad$ $t_2 = -1+\sqrt{2}$ $\qquad$ $\boxed{t = \operatorname{tg}\frac{x}{2}}$

$\operatorname{tg}\frac{x}{2} = -1-\sqrt{2}$ $\qquad$ $\operatorname{tg}\frac{x}{2} = -1+\sqrt{2}$

$\frac{x}{2} = \operatorname{arctg}(-1-\sqrt{2})$ $\qquad$ $\frac{x}{2} = \operatorname{arctg}(-1+\sqrt{2})$

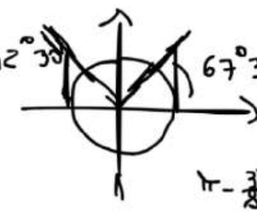
$-(1+\sqrt{2})$

$$\begin{array}{r} 180°60' - \\ 67°30' = \\ \hline 112°30' \end{array}$$



$\frac{x}{2} = \frac{\pi}{8} + k\pi$

$\boxed{x = \frac{\pi}{4} + 2k\pi}$



$112°30'$

$67°30' = \frac{3\pi}{8}$

$\operatorname{tg}\frac{3\pi}{8} = 1+\sqrt{2}$

$\pi - \frac{3}{8}\pi = \frac{8\pi - 3\pi}{8} = \frac{5}{8}\pi$

$\frac{x}{2} = \frac{5}{8}\pi + k\pi$

$\boxed{x = \frac{5}{4}\pi + 2k\pi}$

$x_1 = \frac{\pi}{4} + 2k\pi$

$x_2 = \frac{5}{4} + 2k\pi$

$\boxed{x = \frac{\pi}{4} + k\pi \quad k \in \mathbb{Z}}$

$$3\,\operatorname{sen}x-\sqrt{3}\cos x=0$$

$$3\cdot\left(\frac{2t}{1+t^2}\right)-\sqrt{3}\left(\frac{1-t^2}{1+t^2}\right)=0$$

$$\frac{6t}{1+t^2}-\frac{\sqrt{3}(1-t^2)}{1+t^2}=0$$

$$\frac{6t-\sqrt{3}+\sqrt{3}t^2}{\cancel{1+t^2}}=0$$

$$\boxed{\sqrt{3}t^2+6t-\sqrt{3}=0}$$

$$\boxed{\begin{array}{l} t^2\neq -1 \\ \forall t\in\mathbb{R}\end{array}}$$

$$\boxed{\begin{array}{l} \operatorname{sen}x=\dfrac{2t}{1+t^2} \\ \cos x=\dfrac{1-t^2}{1+t^2} \\ t=\operatorname{tg}\dfrac{x}{2} \\ x\neq\pi+2k\pi\end{array}}$$

$$\sqrt{3}\,t^2+6t-\sqrt{3}=0$$

$$\Delta = b^2-4ac = 36-4(\sqrt{3})(-\sqrt{3}) = 36+12=\underline{48}$$

$$t_1,t_2 = \frac{-b\pm\sqrt{\Delta}}{2a} = \frac{-6\pm\sqrt{48}}{2\sqrt{3}} = \frac{-6\pm4\sqrt{3}}{2\sqrt{3}}$$

$$\begin{array}{r|l} 48 & 2 \\ 24 & 2 \\ 12 & 2 \\ 6 & 2 \\ 3 & 3 \end{array}$$

$48=2^4\cdot 3$

$$\boxed{\sqrt{48}=\sqrt{2^4\cdot 3}=4\sqrt{3}}$$

$$t_1,t_2 = \frac{\cancel{2}(-3\pm2\sqrt{3})}{\cancel{2}\sqrt{3}} = \frac{-3\pm2\sqrt{3}}{\sqrt{3}}$$

$$t_1 = -\frac{3-2\sqrt{3}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{-3\sqrt{3}-2\cdot 3}{3} = \frac{-3\sqrt{3}-6}{3}$$

$$t_2 = \frac{-3+2\sqrt{3}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{-3\sqrt{3}+2\cdot 3}{3} = \frac{-3\sqrt{3}+6}{3}$$

$$t_1 = \frac{\cancel{3}(-\sqrt{3}-2)}{\cancel{3}} \Rightarrow \boxed{\begin{aligned} t_1 &= -\sqrt{3}-2 \\ t_2 &= -\sqrt{3}+2 \end{aligned}}$$

$$t_2 = \frac{\cancel{3}(-\sqrt{3}+2)}{\cancel{3}}$$

$t_1 = -\sqrt{3}-2$

$\begin{array}{r} 180- \\ 75= \\ \hline 105 \end{array}$

$t_2 = -\sqrt{3}+2$

$t = tg\frac{x}{2}$

$tg\frac{x}{2} = -\sqrt{3}-2$

$\frac{x}{2} = arctg(-\sqrt{3}-2)$

$-(2+\sqrt{3})$

$tg\frac{x}{2} = -\sqrt{3}+2$

$\frac{x}{2} = arctg(-\sqrt{3}+2)$

$15° = \frac{\pi}{12}$

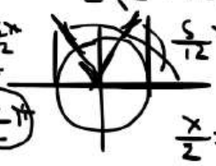
$\frac{x}{2} = \frac{\pi}{12} + K\pi$

$\boxed{x = \frac{\pi}{6} + 2K\pi}$

$\pi - \frac{5\pi}{12}$

$\frac{12\pi-5\pi}{12}$

$= \frac{7}{12}\pi$



$\frac{5}{12}\pi$

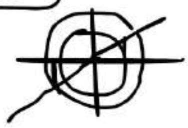
$\boxed{tg\frac{5}{12}\pi = 2+\sqrt{3}}$

$\frac{x}{2} = \frac{7}{12}\pi + K\pi \Rightarrow \boxed{x = \frac{7}{6}\pi + 2K\pi}$

$$x_1 = \frac{\pi}{6} + 2k\pi = 30^\circ + k \cdot 360^\circ$$

$$x_2 = \frac{7}{6}\pi + 2k = 210^\circ + k \cdot 360^\circ$$

$210^\circ = 30^\circ + 180^\circ$



$$\boxed{x = \frac{\pi}{6} + k\pi \quad k \in \mathbb{Z}}$$

$$\cos x + \operatorname{sen} x + \sqrt{2} = 0$$

$$\frac{1-t^2}{1+t^2} + \frac{2t}{1+t^2} + \sqrt{2} = 0$$

$$\frac{1-t^2+2t+\sqrt{2}(1+t^2)}{1+t^2} = 0$$

$$\frac{1-t^2+2t+\sqrt{2}+\sqrt{2}t^2}{\cancel{1+t^2}} = 0$$

$$\sqrt{2}t^2 - t^2 + 2t + 1 + \sqrt{2} = 0$$

$$\underbrace{(\sqrt{2}-1)}_{a}t^2 + \underbrace{2}_{b}t + \underbrace{\sqrt{2}+1}_{c} = 0$$

$$\Delta = b^2 - 4ac = 4 - 4(\sqrt{2}-1)(\sqrt{2}+1) =$$
$$= 4 - 4(2-1) = 4 - 4(2-1) = 4 - 4 = 0$$

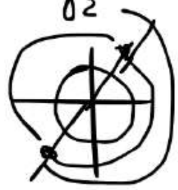
$$t_1 = t_2 = -\frac{b}{2a} = -\frac{\cancel{2}}{\cancel{2}(\sqrt{2}-1)} = -\frac{1}{\sqrt{2}-1} = \frac{1}{1-\sqrt{2}}$$

$$t_1 = t_2 = \frac{1}{1-\sqrt{2}} \cdot \frac{1+\sqrt{2}}{1+\sqrt{2}} = \frac{1+\sqrt{2}}{1-2} = \frac{1+\sqrt{2}}{-1} = -1-\sqrt{2}$$

$$\boxed{t_1 = t_2 = -1-\sqrt{2}}$$

$$\boxed{\begin{array}{l} \cos x = \dfrac{1-t^2}{1+t^2} \\ \operatorname{sen} x = \dfrac{2t}{1+t^2} \\ t = \operatorname{tg}\dfrac{x}{2} \\ x \neq \pi + 2k\pi \end{array}}$$

$t_1 = t_2 = -1 - \sqrt{2}$ $\qquad$ $t = \operatorname{tg} \frac{x}{2}$



$\operatorname{tg} \frac{x}{2} = -1 - \sqrt{2}$

$\frac{x}{2} = \operatorname{arctg}(-1 - \sqrt{2}) = \operatorname{arctg}(-(1+\sqrt{2}))$

$\operatorname{tg} \frac{3}{8}\pi = 1 + \sqrt{2}$

$\frac{x}{2} = \frac{5}{8}\pi + k\pi$ $\qquad$ $\boxed{x = \frac{5}{4}\pi + k\pi}$ $= \frac{\pi}{4} + k\pi$ $\qquad$ $\pi - \frac{3}{8}\pi = \frac{8\pi - 3\pi}{8} = \frac{5}{8}\pi \; (112^\circ 30')$

$$\sqrt{3}\cos x + \operatorname{sen} x = 2$$

$$\sqrt{3}\cdot\frac{(1-t^2)}{1+t^2} + \frac{2t}{1+t^2} = \frac{2}{1}$$

$$\frac{\sqrt{3}(1-t^2) + 2t}{\cancel{1+t^2}} = \frac{2(1+t^2)}{\cancel{1+t^2}}$$

$$\sqrt{3} - \sqrt{3}t^2 + 2t = 2 + 2t^2$$

$$-\sqrt{3}t^2 - 2t^2 + 2t + \sqrt{3} - 2 = 0$$

$$\sqrt{3}t^2 + 2t^2 - 2t - \sqrt{3} + 2 = 0$$

$$\boxed{\begin{gathered}\operatorname{sen} x = \frac{2t}{1+t^2} \\ \cos x = \frac{1-t^2}{1+t^2} \\ t = \operatorname{tg}\frac{x}{2} \\ x \neq \pi + 2k\pi\end{gathered}}$$

$$\boxed{(\sqrt{3}+2)t^2 - 2t + 2 - \sqrt{3} = 0}$$

$$(\sqrt{3}+2)t^2 - 2t + 2 - \sqrt{3} = 0$$

$$\Delta = b^2 - 4ac = (-2)^2 - 4\underbrace{(2+\sqrt{3})(2-\sqrt{3})}_{} =$$

$$= 4 - 4(4-3) = 4 - 4 = 0$$

$$t_1 = t_2 = -\frac{b}{2a} = \frac{\cancel{2}}{\cancel{2}(2+\sqrt{3})} = \frac{1}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{2-\sqrt{3}}{4-3}$$

$$\boxed{t_1 = t_2 = 2 - \sqrt{3}}$$

$t_1 = t_2 = 2 - \sqrt{3}$  $\frac{15}{180}$ 12 $\quad t = tg\frac{x}{2}$

$tg\frac{x}{2} = 2 - \sqrt{3}$

$\frac{x}{2} = arctg(2 - \sqrt{3})$

$\frac{x}{2} = \frac{\pi}{12} + k\pi$

$\boxed{x = \frac{\pi}{6} + 2k\pi,\ k \in \mathbb{Z}}$