LEZIONE 15

Equazioni goniometriche riconducibili ad equazioni elementari

$$\boxed{\cot g\, x = \frac{1}{tg\, x}}$$

$$\cot g^2 x + \cot g\, x = 0$$

$$\left(\frac{1}{tg\, x}\right)^2 + \left(\frac{1}{tg\, x}\right) = 0$$

$$\frac{1}{tg^2 x} + \frac{1}{tg\, x} = 0$$

$$\cancel{tg^2 x}\ \frac{1 + tg\, x}{\cancel{tg^2 x}} = 0 \cdot tg^2 x$$

$$1 + tg\, x = 0 \Rightarrow \boxed{tg\, x = -1}$$

$$x = arctg(-1)$$

$$\boxed{x = \frac{3}{4}\pi + k\pi;\ k \in \mathbb{Z}}$$

C.E.

$$tg^2 x \neq 0$$

$$tg\, x \neq 0$$

$$x \neq 0 + k\pi \qquad k \in \mathbb{Z}$$

$$\boxed{x \neq k\pi}$$

$\cancel{45}\ 3$

$\frac{1\cancel{35}}{1\cancel{80}}$

$\cancel{60}\ 4$

$$\left(-\frac{\pi}{4} + k\pi\right)$$

$$\boxed{a\,\operatorname{sen}x + b\cos x + c = 0}$$
$a, b, c \in \mathbb{R}$

Equazioni lineari in seno e coseno

Sono equazioni in cui figurano solo seno e coseno come funzioni goniometriche, ma elevate al primo grado. In questo caso il miglior modo per risolvere questo tipo equazioni è utilizzare le formule parametriche, in cui seno e coseno di un angolo sono espressi in funzione della tangente dell'angolo dimezzato

$$\operatorname{sen}x = \frac{2t}{1+t^2}$$
$$\cos x = \frac{1-t^2}{1+t^2}$$

$$\boxed{\text{FORMULE PARAMETRICHE}}$$

$$t = \operatorname{tg}\frac{x}{2} = \frac{\operatorname{sen}\frac{x}{2}}{\cos\frac{x}{2}}$$

$$\cos\frac{x}{2} \neq 0 \Rightarrow \frac{x}{2} \neq \frac{\pi}{2} + k\pi \quad k \in \mathbb{Z}$$

$$x \neq \pi + 2k\pi, \; k \in \mathbb{Z}$$

$\operatorname{sen} 2\alpha = 2 \operatorname{sen} \alpha \cos \alpha$ | $\cos 2\alpha = \cos^2 \alpha - \operatorname{sen}^2 \alpha$

$\operatorname{sen} \alpha = 2 \operatorname{sen} \frac{\alpha}{2} \cos \frac{\alpha}{2}$ | $\cos \alpha = \cos^2 \frac{\alpha}{2} - \operatorname{sen}^2 \frac{\alpha}{2}$

$$\boxed{\cos^2 \frac{\alpha}{2} + \operatorname{sen}^2 \frac{\alpha}{2} = 1}$$

$$\operatorname{sen} \alpha = \frac{2 \operatorname{sen} \frac{\alpha}{2} \cos \frac{\alpha}{2}}{\cos^2 \frac{\alpha}{2} + \operatorname{sen}^2 \frac{\alpha}{2}}$$

$$\cos \alpha = \frac{\cos^2 \frac{\alpha}{2} - \operatorname{sen}^2 \frac{\alpha}{2}}{\cos^2 \frac{\alpha}{2} + \operatorname{sen}^2 \frac{\alpha}{2}}$$

DIVIDO TUTTO PER $\cos^2 \frac{\alpha}{2}$

$$\operatorname{sen}\alpha = \frac{\dfrac{2\operatorname{sen}\frac{\alpha}{2}\cos\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}}{\dfrac{\operatorname{sen}^2\frac{\alpha}{2}+\cos^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}} \qquad \cos\alpha = \frac{\dfrac{\cos^2\frac{\alpha}{2}-\operatorname{sen}^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}}{\dfrac{\operatorname{sen}^2\frac{\alpha}{2}+\cos^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}}$$

$$\operatorname{sen}\alpha = \frac{2\dfrac{\operatorname{sen}\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}}{1+\dfrac{\operatorname{sen}^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}} \qquad \cos\alpha = \frac{1-\dfrac{\operatorname{sen}^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}}{1+\dfrac{\operatorname{sen}^2\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2}}}$$

$$\operatorname{sen}\alpha = \frac{2\operatorname{tg}\frac{\alpha}{2}}{1+\operatorname{tg}^2\frac{\alpha}{2}} \qquad \cos\alpha = \frac{1-\operatorname{tg}^2\frac{\alpha}{2}}{1+\operatorname{tg}^2\frac{\alpha}{2}}$$

$$\operatorname{tg}\frac{\alpha}{2} = t \qquad \boxed{\operatorname{sen}\alpha = \frac{2t}{1+t^2}\ ;\ \cos\alpha = \frac{1-t^2}{1+t^2}}$$

$$\frac{\operatorname{sen}\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \qquad \frac{\alpha}{2} \neq K\frac{\pi}{2} + K\pi \implies \underline{\alpha \neq K\pi + 2K\pi}\ ,\ K \in \mathbb{Z}$$

c.q.d

$$\operatorname{sen} x - \cos x = 0$$

$$\frac{2t}{1+t^2} - \frac{(1-t^2)}{1+t^2} = 0$$

$$\cancel{1+t^2}\ \frac{2t-(1-t^2)}{\cancel{1+t^2}} = 0 \cdot 1+t^2$$

$$2t - 1 + t^2 = 0$$

$$t^2 + 2t - 1 = 0$$

$$\Delta = b^2 - 4ac = 2^2 - 4(1)(-1) = 4 + 4 = 8$$

$$t_1, t_2 = -\frac{b \pm \sqrt{\Delta}}{2a} = \frac{-2 \pm \sqrt{8}}{2}$$

$1 + t^2 \neq 0$

$t^2 \neq -1$

$\forall t \in \mathbb{R}$

$$\boxed{\begin{array}{l} \operatorname{sen} x = \dfrac{2t}{1+t^2} \\ \cos x = \dfrac{1-t^2}{1+t^2} \\ t = tg\,\dfrac{x}{2} \\ x \neq k\pi + 2k\pi \end{array}}$$

$$t_1, t_2 = \frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm 2\sqrt{2}}{2}$$

$$t_1, t_2 = \frac{\cancel{2}(-1 \pm \sqrt{2})}{\cancel{2}}$$

$$t_1, t_2 = -1 \pm \sqrt{2}$$

$$\underline{t_1 = -1 - \sqrt{2}} \ ; \ \underline{t_2 = -1 + \sqrt{2}}$$

$\sqrt{8} = ?$

$\sqrt{8} = \sqrt{2^3} =$

$= \sqrt{2 \cdot 2^2} =$

$= 2\sqrt{2}$

$$\begin{array}{r|l} 8 & 2 \\ 4 & 2 \\ 2 & 2 \\ 1 & \end{array}$$

$180° - 67°30' = 112°30'$ $\qquad \operatorname{tg}\frac{x}{2} = t$

$\pi - \frac{3}{8}\pi = \frac{8\pi - 3\pi}{8} = \frac{5}{8}\pi$

$\operatorname{tg}\frac{x}{2} = -1 - \sqrt{2}$

$\frac{x}{2} = \operatorname{arctg}(-1 - \sqrt{2}) = \operatorname{arctg}\left[-(1+\sqrt{2})\right]$

$112°30' = \frac{5}{8}\pi \qquad 67°30' = \frac{3}{8}\pi$

$\frac{x}{2} = \frac{5}{8}\pi + k\pi \qquad \boxed{x = \frac{5}{4}\pi + 2k\pi}$

$\operatorname{tg}\frac{x}{2} = -1 + \sqrt{2}$

$\frac{x}{2} = \operatorname{arctg}(\sqrt{2} - 1)$

$\frac{x}{2} = \frac{\pi}{8} + k\pi$

$\boxed{x = \frac{\pi}{4} + 2k\pi}$

$x = \frac{5}{4}\pi + 2k\pi \qquad x = \frac{\pi}{4} + 2k\pi$

$\boxed{x = \frac{\pi}{4} + k\pi \quad k \in \mathbb{Z}}$

$$3 \operatorname{sen} x - \sqrt{3} \cos x = 0$$

$$3 \cdot \frac{2t}{1+t^2} - \sqrt{3}\,\frac{(1-t^2)}{1+t^2} = 0$$

$$\cancel{(1+t^2)}\ \frac{6t - \sqrt{3} + \sqrt{3}\,t^2}{\cancel{1+t^2}} = 0 \cdot (1+t^2)$$

$$\sqrt{3}\,t^2 + 6t - \sqrt{3} = 0$$

$$\Delta = b^2 - 4ac = 6^2 - 4(\sqrt{3})(-\sqrt{3}) = 36 + 12 = 48$$

$$\boxed{\begin{array}{l} \operatorname{sen} x = \dfrac{2t}{1+t^2} \\ \cos x = \dfrac{1-t^2}{1+t^2} \\ t = \operatorname{tg} \dfrac{x}{2} \\ x \neq \pi + 2k\pi \end{array}}$$

$\Delta = 48$

$t_1, t_2 = \dfrac{-\cancel{b} \pm \sqrt{\Delta}}{2a} = \dfrac{-6 \pm \sqrt{48}}{2\sqrt{3}}$

$\boxed{\sqrt{48} = \sqrt{2^4 \cdot 3} = 4\sqrt{3}}$

$= -\dfrac{6 \pm 4\sqrt{3}}{2\sqrt{3}} = \dfrac{\cancel{2}(-3 \pm 2\sqrt{3})}{\cancel{2}\sqrt{3}} = \dfrac{-3 \pm 2\sqrt{3}}{\sqrt{3}}$

48	2
24	2
12	2
6	2
3	3
1	

$$t_1 = \frac{-3 - 2\sqrt{3}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$t_2 = \frac{-3 + 2\sqrt{3}}{\sqrt{3}}$$

$$t_1 = \frac{-3\sqrt{3} - 6}{3} = \boxed{\frac{\cancel{3}(-\sqrt{3}-2)}{\cancel{3}}}$$

$$\boxed{t_2 = -\sqrt{3} + 2}$$

$$t_1 = -\sqrt{3} - 2$$

$$t_2 = -\sqrt{3} + 2$$

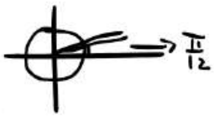
$\operatorname{tg}\frac{x}{2} = -2-\sqrt{3}$

$\frac{x}{2} = \operatorname{arctg}(-2-\sqrt{3})$

$= \operatorname{arctg}\left(-(2+\sqrt{3})\right)$

$180° - 75° = 105°$

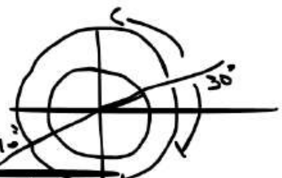
$\operatorname{tg}\frac{x}{2} = +2+\sqrt{3}$



$\frac{x}{2} = \frac{\pi}{12} + k\pi$

$\boxed{x = \frac{\pi}{6} + 2k\pi}$

$\boxed{x = \frac{7}{6}\pi + 2k\pi}$



$x = \frac{7}{6}\pi + 2k\pi \quad k \in \mathbb{Z}$

$x = \frac{\pi}{6} + 2k\pi \quad k \in \mathbb{Z} \longrightarrow \boxed{x = \frac{\pi}{6} + k\pi,\ k \in \mathbb{Z}}$