$$\lim_{x \to +\infty} \boxed{\left(\frac{1}{x}\right)}^{\boxed{\frac{1}{1+\ln x}}} =$$

$$= \lim_{x \to +\infty} e^{\frac{1}{1+\ln x} \log_e\left(\frac{1}{x}\right)} =$$

$$= \lim_{x \to +\infty} e^{\frac{\ln\left(\frac{1}{x}\right)}{1+\ln x}} =$$

$$= \lim_{x \to +\infty} e^{\frac{\overbrace{\ln 1}^{0} - \ln x}{1+\ln x}} =$$

$$= e^{-\lim_{x \to +\infty} \frac{\ln x}{1+\ln x}} =$$

$$= e^{-\lim_{x \to +\infty} \frac{\cancel{\ln x}}{\cancel{\ln x}\left(1+\underbrace{\frac{1}{\ln x}}_{+\infty}\right)}} = e^{-1} = e$$

$\log_e = \ln$

$$f(x)^{g(x)} =$$

$$= e^{g(x) \log_e f(x)}$$

$$f(x) = \frac{1}{x}$$

$$g(x) = \frac{1}{1+\ln x}$$

$$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$$

$\log_e = \ln$

$$\lim_{x \to +\infty} \boxed{\left(\frac{1}{x}\right)^{\frac{1}{1+\ln x}}} =$$

$$f(x)^{g(x)} = e^{g(x) \log_e f(x)}$$

$$= \lim_{x \to +\infty} e^{-\frac{1}{1+\ln x} \log_e\left(\frac{1}{x}\right)} =$$

$$= \lim_{x \to +\infty} e^{-\frac{\ln\left(\frac{1}{x}\right)}{1+\ln x}} =$$

$$= \lim_{x \to +\infty} e^{\frac{\ln 1 - \ln x}{1+\ln x}} =$$

$\ln 1 \to 0$

$$= e^{-\lim_{x \to +\infty} \frac{\ln x}{1+\ln x}} =$$

$$= e^{\lim_{x \to +\infty} \frac{\ln x}{1+\ln x}} =$$

$$= e^{\lim_{x \to +\infty} \frac{\cancel{\ln x}}{\cancel{\ln x}\left(1+\frac{1}{\ln x}\right)}} = e^1 = e$$

$\ln x \to +\infty$, $\to 1$

$f(x) = \frac{1}{x}$

$g(x) = \frac{1}{1+\ln x}$

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

LEZIONE 15

$\log_e = \ln$

$$\lim_{x \to +\infty} \boxed{\left(\frac{1}{x}\right)^{\frac{1}{1+\ln x}}} =$$

$$f(x)^{g(x)} = e^{g(x) \log_e f(x)}$$

$$= \lim_{x \to +\infty} e^{-\frac{1}{1+\ln x} \log_e\left(\frac{1}{x}\right)} =$$

$$= \lim_{x \to +\infty} e^{-\frac{\ln\left(\frac{1}{x}\right)}{1+\ln x}} =$$

$$= \lim_{x \to +\infty} e^{\frac{\ln 1 - \ln x}{1+\ln x}} =$$

$\ln 1 \to 0$

$$= e^{-\lim_{x \to +\infty} \frac{\ln x}{1+\ln x}} =$$

$$= e^{\lim_{x \to +\infty} \frac{\ln x}{1+\ln x}} =$$

$$= e^{\lim_{x \to +\infty} \frac{\cancel{\ln x}}{\cancel{\ln x}\left(1+\frac{1}{\ln x}\right)}} = e^1 = e$$

$\ln x \to +\infty$, $\to 1$

$f(x) = \frac{1}{x}$

$g(x) = \frac{1}{1+\ln x}$

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

$$\lim_{x\to+\infty} \boxed{\left(\frac{1}{x}\right)}^{\boxed{\frac{1}{1+\ln x}}} =$$

$$= \lim_{x\to+\infty} e^{\frac{1}{1+\ln x}\log_e\left(\frac{1}{x}\right)} =$$

$$= \lim_{x\to+\infty} e^{-\frac{\ln\left(\frac{1}{x}\right)}{1+\ln x}} =$$

$$= \lim_{x\to+\infty} e^{\frac{\ln 1 - \ln x}{1+\ln x}} =$$

$$= e^{-\lim_{x\to+\infty}\frac{\ln x}{1+\ln x}} = \to 1$$

$$= e^{\lim_{x\to+\infty}\frac{\cancel{\ln x}}{\cancel{\ln x}\left(1+\frac{1}{\ln x}\right)}} = e^{1} = e$$

$+\infty$

$\log_e = \ln$

$$f(x)^{g(x)} =$$

$$= e^{g(x)\log_e f(x)}$$

$$f(x) = \frac{1}{x}$$

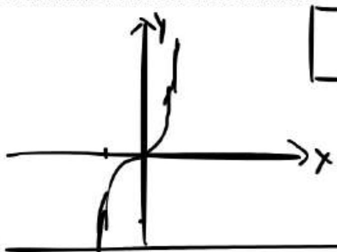
$$g(x) = \frac{1}{1+\ln x}$$

$$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$$

LIMITE INFINITO DI UNA FUNZIONE PER X CHE TENDE A INFINITO

$$y = f(x) = x^3$$

x	y
-10	-10^3
10	10^3
100	100^3
1000	1000^3



$$\boxed{\lim_{x \to \infty} f(x) = \infty}$$

$$\boxed{\forall M > 0 \; \exists I(\infty) : \forall x \in I(\infty) \Rightarrow |f(x)| > M}$$

$$\lim_{x \to \infty} \frac{x^2+4}{5x} = \infty \qquad \text{Dom} f \ \mathbb{R} \setminus \{0\} \qquad |f(x)| > K$$

$$f(x) < -K$$

$$\forall M > 0 \ \exists \ I(\infty); \forall x \in I(\infty) \Rightarrow \underbrace{\left| \frac{x^2+4}{5x} \right| > M} \qquad \overset{\vee}{f(x)} > K$$

$$\text{(A)} \ \boxed{\frac{x^2+4}{5x} < -M} \ \vee \ \boxed{\frac{x^2+4}{5x} > M} \ \text{(B)}$$

$$\frac{x^2+4}{5x} < -M \implies \frac{x^2+4}{5x} + \frac{M}{1} < 0$$

$D_{5x>0}$

$\boxed{x>0}$

$$\frac{x^2+4+5Mx}{5x} < 0$$

N

$$x^2 + 5Mx + 4 > 0$$

$$\Delta = b^2 - 4ac = (5M)^2 - 4 \cdot 4 = $$

$$= 25M^2 - 16$$

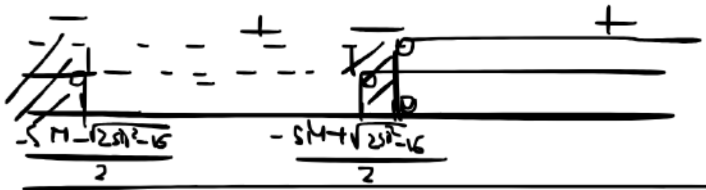
$$x_1, x_2 = -\frac{b \pm \sqrt{\Delta}}{2a} = -\frac{5M \pm \sqrt{25M^2-16}}{2}$$

$$\frac{-50-\sqrt{2500-16}}{2} = -\frac{50-49,83}{2} = \frac{-49,95}{1}$$

$$x_1 = \frac{-5M-\sqrt{25M^2-16}}{2} \qquad x_2 = \frac{-5M+\sqrt{25M^2-16}}{2}$$

$-9{,}085$

$$\boxed{x < \frac{-5M-\sqrt{25M^2-16}}{2} \vee x > \frac{-5M+\sqrt{25M^2-16}}{2}}$$



A

$$\boxed{x < -\frac{5M-\sqrt{25M^2-16}}{2} \quad \vee \quad -\frac{5M+\sqrt{25M^2-16}}{2} < x < 0}$$

(B) $\frac{x^2+4}{5x} > M \Rightarrow \frac{x^2+4}{5x} - M > 0$

$\frac{x^2+4-5Mx}{5x} > 0$

N $x^2 - 5Mx + 4 > 0$

$\Delta = (-5M)^2 - 4 \cdot 4 = 25M^2 - 16$

$x_1, x_2 = \frac{5M \pm \sqrt{25M^2-16}}{2}$

$- - - \quad - \quad + \quad - \quad +$

$0 \quad \frac{5M-\sqrt{25M^2-16}}{2} \quad \frac{5M+\sqrt{25M^2-16}}{2}$

$\frac{5M-\sqrt{25M^2-16}}{2} \cong 0{,}08 \quad \vee \quad x > \frac{5M+\sqrt{25M^2-16}}{2} \cong 49{,}915$

(B)

$0 < x < \frac{5M-\sqrt{25M^2-16}}{2} \quad \vee \quad x > \frac{5M+\sqrt{25M^2-16}}{2}$

(A) $\cup$ (B)

$x < -\frac{5M-\sqrt{25M^2-16}}{2} \quad \vee \quad -\frac{5M+\sqrt{25M^2-16}}{2} < x < 0$ (A)

$0 < x < \frac{5M-\sqrt{25M^2-16}}{2} \quad \vee \quad x > \frac{5M+\sqrt{25M^2-16}}{2}$ (B)

$\underbrace{x < -\frac{5M-\sqrt{25M^2-16}}{2}}_{I(-\infty)} \quad \vee \quad \underbrace{x > \frac{5M+\sqrt{25M^2-16}}{2}}_{I(+\infty)}$

$\left]-\infty; -\frac{5M-\sqrt{25M^2-16}}{2}\right[\cup \left]\frac{5M+\sqrt{25M^2-16}}{2}; +\infty\right[$

$I(\infty)$