

LEZIONE 15

$$\begin{aligned}\sin x - \sqrt{3} \cos x &= 2 \\ \frac{2t}{1+t^2} - \sqrt{3} \frac{(1-t^2)}{1+t^2} &= 2 \\ 2t - \sqrt{3}(1-t^2) &= 2(1+t^2) \\ 2t - \sqrt{3} + \sqrt{3}t^2 &= 2 + 2t^2 \\ (\sqrt{3}-2)t^2 + 2t - \sqrt{3} - 2 &= 0\end{aligned}$$

$$\begin{aligned}\sin x &= \frac{2t}{1+t^2} \\ \cos x &= \frac{1-t^2}{1+t^2} \\ t &= \sqrt{\frac{x}{2}} \\ x &\neq \pi + k\pi\end{aligned}$$

$$\begin{aligned}(\sqrt{3}-2)t^2 + 2t + (-\sqrt{3}-2) &= 0 \\ \Delta &= b^2 - 4ac = 4 - 4(\sqrt{3}-2)(-\sqrt{3}-2) = 0 \\ \Delta &= 4 - 4(-3+4) = 4 - 4(1) = 4 - 4 = 0 \\ t_1 = t_2 &= -\frac{b \pm \sqrt{\Delta}}{2a} = \frac{-2}{2(\sqrt{3}-2)} = \frac{1}{2-\sqrt{3}}\end{aligned}$$

$$t_1 = t_2 = \frac{1}{\underbrace{2 - \sqrt{3}}_{(A-B)}} \cdot \frac{2 + \sqrt{3}}{\underbrace{2 + \sqrt{3}}_{(A+B)}} = \frac{2 + \sqrt{3}}{4 - 3} = 2 + \sqrt{3}$$

$$\boxed{t_1 = t_2 = 2 + \sqrt{3}}$$

$$\operatorname{tg} \frac{x}{2} = 2 + \sqrt{3}$$

$$\frac{x}{2} = \operatorname{arctg}(2 + \sqrt{3})$$

$$\frac{x}{2} = \frac{5}{12} \pi + k\pi$$

$$\boxed{x = \frac{5}{6} \pi + 2k\pi, k \in \mathbb{Z}}$$

$$\operatorname{tg} \frac{x}{2} = t$$

$$\frac{5}{12} \cdot 2$$

$$\frac{5}{6} \cdot \frac{20}{180} = 150$$

$$\sqrt{3} \sin x + \cos x + 1 = 0$$

$$\sqrt{3} \left(\frac{2t}{1+t^2} \right) + \frac{1-t^2}{1+t^2} + 1 = 0$$

$$\frac{2\sqrt{3}t + 1 - t^2 + 1 + t^2}{1+t^2} = 0$$

$$\cancel{(1+t^2)} \frac{2\sqrt{3}t + 2}{\cancel{1+t^2}} = 0 \cdot (1+t^2) \quad 2\sqrt{3}t + 2 = 0 \Rightarrow 2(\sqrt{3}t + 1) = 0$$

$$x \neq \pi + 2K\pi$$

$$K \in \mathbb{Z}$$

$$\operatorname{tg} \frac{x}{2} = t$$

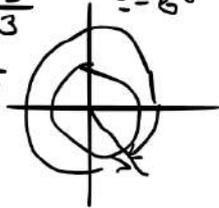
$$\sqrt{3}t + 1 = 0$$

$$\sqrt{3}t = -1 \Rightarrow t = -\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$t = -\frac{\sqrt{3}}{3} \quad t = \operatorname{tg} \frac{x}{2} \Rightarrow \operatorname{tg} \frac{x}{2} = -\frac{\sqrt{3}}{3}$$

$$\frac{x}{2} = \operatorname{arctg} \left(-\frac{\sqrt{3}}{3} \right)$$

$$\frac{x}{2} = \frac{5\pi}{6} + k\pi$$
$$\boxed{x = \frac{5\pi}{3} + 2k\pi}$$



$\left(-\frac{\pi}{2} \right)$

$\frac{5\pi}{6}$
 $= 30^\circ$
 $\therefore -60^\circ$

$$\sin x - \cos x - 1 = 0$$

$$\frac{2t}{1+t^2} - \frac{(1-t^2)}{1+t^2} - 1 = 0$$

$$\frac{2t - (1-t^2) - (1+t^2)}{1+t^2} = 0 \cdot (1+t^2)$$

$$2t - 1 + t^2 - 1 - t^2 = 0$$

$$2t - 2 = 0$$

$$2(t-1) = 0$$

$$t-1 = 0 \Rightarrow \boxed{t=1}$$

$$\sin x = \frac{2t}{1+t^2}$$

$$\cos x = \frac{1-t^2}{1+t^2}$$

$$x \neq \pi + 2k\pi$$

$$t = \tan \frac{x}{2}$$

$$\frac{x}{2} = \arctan 1 \Rightarrow \frac{x}{2} = \frac{\pi}{4} + k\pi$$

$$x = \frac{\pi}{2} + 2k\pi$$

$$3 \operatorname{Sen} x + \sqrt{3} \operatorname{Cos} x + \sqrt{3} = 0$$

$$3 \cdot \left(\frac{2t}{1+t^2} \right) + \sqrt{3} \left(\frac{1-t^2}{1+t^2} \right) + \sqrt{3} = 0$$

$$\frac{6t + \sqrt{3} - \sqrt{3}t^2 + \sqrt{3}(1+t^2)}{(1+t^2)} = 0$$

$$6t + \sqrt{3} - \sqrt{3}t^2 + \sqrt{3} + \sqrt{3}t^2 = 0$$

$$6t + 2\sqrt{3} = 0 \quad 6t = -2\sqrt{3} \Rightarrow t = -\frac{2\sqrt{3}}{6}$$

$$\operatorname{sen} x = \frac{2t}{1+t^2}$$

$$\operatorname{cos} x = \frac{1-t^2}{1+t^2}$$

$$t = \frac{\operatorname{tg} x}{2}$$

$$x \neq \pi + 2k\pi$$

$$k \in \mathbb{Z}$$

$$t = -\frac{\sqrt{3}}{3} \qquad t = \operatorname{tg} \frac{x}{2}$$

$$\operatorname{tg} \frac{x}{2} = -\frac{\sqrt{3}}{3} \Rightarrow \frac{x}{2} = \operatorname{arctg} \left(-\frac{\sqrt{3}}{3} \right) \rightarrow \left(-\frac{\pi}{6} \right)$$

$$\frac{x}{2} = \frac{5}{6}\pi + k\pi \Rightarrow x = \frac{5}{3}\pi + 2k\pi$$

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EQUAZIONI OMOGENEE DI 2° GRADO IN SENO E COSENO

$a, b, c \in \mathbb{R}$

$$a \sin^2 x + b \sin x \cos x + c \cos^2 x = 0$$

Generalmente si divide tutto per coseno al quadrato

$$a \frac{\sin^2 x}{\cos^2 x} + b \frac{\sin x \cancel{\cos x}}{\cancel{\cos^2 x}} + c \frac{\cancel{\cos^2 x}}{\cancel{\cos^2 x}} = 0$$

$$\frac{\sin x}{\cos x} = \tan x$$

$$\cos x \neq 0$$

$$x \neq \frac{\pi}{2} + k\pi$$

$$a \tan^2 x + b \tan x + c = 0$$

Equazione di secondo grado in tangente di x

$$\sqrt{3} \cos^2 x + 3 \cos x \sin x = 0$$

$$\sqrt{3} + 3 \operatorname{tg} x = 0$$

$$3 \operatorname{tg} x = -\sqrt{3} \Rightarrow$$

$$\operatorname{tg} x = \left(-\frac{\sqrt{3}}{3}\right)$$

$$x = \operatorname{arctg}\left(-\frac{\sqrt{3}}{3}\right) \Rightarrow$$

$$x = \frac{5\pi}{6} + k\pi$$

$$\cos^2 x \neq 0$$

$$\cos x \neq 0$$

$$x \neq \frac{\pi}{2} + k\pi$$

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$$\sqrt{3} \cos^2 x + 3 \cos x \sin x = 0$$

$$\cos x (\sqrt{3} \cos x + 3 \sin x) = 0$$

$$\boxed{\cos x = 0}$$

$$\boxed{\sqrt{3} \cos x + 3 \sin x = 0}$$

$$\sin^2 x - (1 + \sqrt{3}) \sin x \cos x + \sqrt{3} \cos^2 x = 0$$

$$\operatorname{tg}^2 x - (1 + \sqrt{3}) \operatorname{tg} x + \sqrt{3} = 0$$

$$\Delta = [-(1 + \sqrt{3})]^2 - 4(1)(\sqrt{3}) =$$

$$= 1 + 3 + 2\sqrt{3} - 4\sqrt{3} = 4 - 2\sqrt{3}$$

$$(\operatorname{tg} x)_1, (\operatorname{tg} x)_2 = \frac{1 + \sqrt{3} \pm \sqrt{4 - 2\sqrt{3}}}{2}$$

$$\frac{\cos^2 x \neq 0}{\cos x \neq 0}$$
$$\boxed{x \neq \frac{\pi}{2} + k\pi}$$

$$(\operatorname{tg} x)_1, (\operatorname{tg} x)_2 = \frac{1 + \sqrt{3} \pm \sqrt{4 - 2\sqrt{3}}}{2} \quad \sqrt{4 - \sqrt{12}}$$

$$\sqrt{a - \sqrt{b}} = \sqrt{\frac{a + \sqrt{a^2 - b}}{2}} - \sqrt{\frac{a - \sqrt{a^2 - b}}{2}}$$

$$\sqrt{4 - \sqrt{12}} = \sqrt{\frac{4 + \sqrt{16 - 12}}{2}} - \sqrt{\frac{4 - \sqrt{16 - 12}}{2}} =$$

$$= \sqrt{\frac{4 + 2}{2}} - \sqrt{\frac{4 - 2}{2}} = \sqrt{\frac{6}{2}} - \sqrt{\frac{2}{2}}$$

$$= \sqrt{3} - \sqrt{1} = (\sqrt{3} - 1)$$

$$(\operatorname{tg} x)_1, (\operatorname{tg} x)_2 = \frac{1 + \sqrt{3} \pm (\sqrt{3} - 1)}{2} \begin{cases} \rightarrow \frac{1 + \sqrt{3} - \sqrt{3} + 1 - 2}{2} \\ \rightarrow \frac{1 + \sqrt{3} + \sqrt{3} + 1 - 2\sqrt{3}}{2} = \sqrt{3} \end{cases}$$

$$\operatorname{tg} x = 1$$

$$x = \arctg 1$$

$$x = \frac{\pi}{4} + k\pi$$

$$\operatorname{tg} x = \sqrt{3}$$

$$x = \arctg \sqrt{3}$$

$$x = \frac{\pi}{3} + k\pi$$

$$k \in \mathbb{Z}$$

$$\sqrt{3} \sin x \cos x + \sin^2 x = 0$$

$$\sqrt{3} \operatorname{tg} x + \operatorname{tg}^2 x = 0$$

$$\operatorname{tg} x (\operatorname{tg} x + \sqrt{3}) = 0$$

$$\operatorname{tg} x = 0$$

$$x = K\pi, K \in \mathbb{Z}$$

$$\operatorname{tg} x = -\sqrt{3}$$

$$x = \arctg(-\sqrt{3})$$

$$x = \frac{2}{3}\pi + K\pi, K \in \mathbb{Z}$$

$$\cos^2 x = 0$$

$$\cos x = 0$$

$$x = \frac{\pi}{2} + K\pi$$

