Lezione 15

Equazioni lineari in seno e coseno

$$\text{sen}x - \cos x = 0 \qquad \underline{\text{sen}x = \cos x}$$

$$\frac{2t}{1+t^2} - \frac{(1-t^2)}{1+t^2} = 0$$

$$\frac{2t - 1(1-t^2)}{1+t^2} = 0$$

$$\cancel{(1+t^2)}\ \frac{2t-1+t^2}{\cancel{1+t^2}} = 0 \cdot 1+t^2$$

$$\boxed{t^2 + 2t - 1 = 0}$$

$$\boxed{\begin{array}{l} \text{sen}x = \dfrac{2t}{1+t^2} \\ \cos x = \dfrac{1-t^2}{1+t^2} \\ t = \text{tg}\,\dfrac{x}{2} \\ x \neq \pi + 2k\pi \quad k \in \mathbb{Z} \end{array}}$$

$t^2 + 2t - 1 = 0$

$\Delta = b^2 - 4ac = 2^2 - 4(1)(-1) = 4 + 4 = 8$

$$\begin{array}{r|l} 8 & 2 \\ 4 & 2 \\ 2 & 2 \\ 1 & \end{array}$$

$t_1, t_2 = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm \sqrt{2^3}}{2} =$

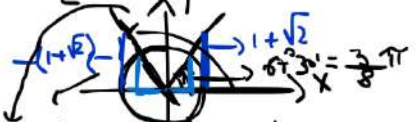
$= \frac{-2 \pm \sqrt{2^2 \cdot 2}}{2} = \frac{-2 \pm 2\sqrt{2}}{2} = \frac{\cancel{2}(-1 \pm \sqrt{2})}{\cancel{2}}$

$\boxed{t_1 = -1 - \sqrt{2}\ ;\ t_2 = -1 + \sqrt{2}}$

$t_1 = -1 - \sqrt{2}$ $\qquad$ $t = \operatorname{tg}\frac{x}{2}$

$\operatorname{tg}\frac{x}{2} = -1 - \sqrt{2}$

$\frac{x}{2} = \operatorname{arctg}(-1-\sqrt{2}) = \operatorname{arctg}\left(-\underline{(1+\sqrt{2})}\right)$



$\boxed{\frac{5}{8}\pi}$

$180° - 67°30' = 112°30'$

$\pi - \frac{3}{8}\pi = \frac{8\pi - 3\pi}{8} = \frac{5}{8}\pi$

$\frac{x}{2} = \frac{5}{8}\pi + k\pi \qquad k \in \mathbb{Z}$

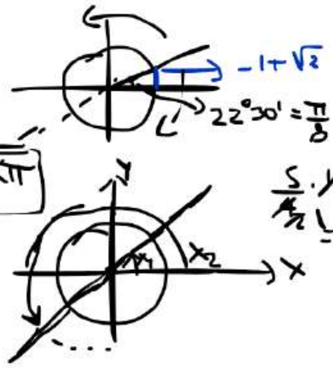
$\boxed{x = \frac{5}{4}\pi + 2k\pi} \qquad k \in \mathbb{Z}$

$t_2 = -1+\sqrt{2}$

$tg\,\frac{x}{2} = -1+\sqrt{2}$

$\frac{x}{2} = arctg\,(-1+\sqrt{2})$

$\frac{x}{2} = \frac{\pi}{8} + k\pi \Rightarrow \boxed{x = \frac{\pi}{4} + 2k\pi}$



$\frac{5 \cdot 180}{4} = 225$

$x_2 = \frac{5}{4}\pi + 2k\pi \quad x_1 = \frac{\pi}{4} + 2k\pi$

$\boxed{x = \frac{\pi}{4} + k\pi,\ k \in \mathbb{Z}}$

$$3 \operatorname{sen} x - \sqrt{3} \cos x = 0$$

$$3 \cdot \frac{2t}{1+t^2} - \sqrt{3} \cdot \frac{(1-t^2)}{1+t^2} = 0$$

$$\frac{6t - \sqrt{3} + \sqrt{3}t^2}{\cancel{1+t^2}} = 0 \cdot \cancel{(1+t^2)}$$

$$\sqrt{3}t^2 + 6t - \sqrt{3} = 0$$

$$\Delta = b^2 - 4ac = 36 - 4(\sqrt{3})(-\sqrt{3}) =$$

$$= 36 + 12 = \underline{48}$$

$$\boxed{\begin{array}{l} \operatorname{sen} x = \dfrac{2t}{1+t^2} \\ \cos x = \dfrac{1-t^2}{1+t^2} \\ t = \operatorname{tg}\dfrac{x}{2} = \dfrac{\operatorname{sen}\frac{x}{2}}{\cos\frac{x}{2}} \\ x \neq \pi + 2k\pi,\ k \in \mathbb{Z} \\ \cos\dfrac{x}{2} \neq 0 \end{array}}$$

$$t_1, t_2 = -\frac{b \pm \sqrt{\Delta}}{2a} = \frac{-6 \pm \sqrt{48}}{2\sqrt{3}} =$$

$$\begin{array}{r|l} 48 & 2 \\ 24 & 2 \\ 12 & 2 \\ 6 & 2 \\ 3 & 3 \\ 1 & \end{array}$$

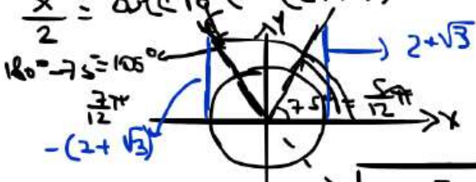
$$= \frac{-6 \pm \sqrt{2^4 \cdot 3}}{2\sqrt{3}} = \frac{-6 \pm 4\sqrt{3}}{2\sqrt{3}} = \frac{\cancel{2}(-3 \pm 2\sqrt{3})}{\cancel{2}\sqrt{3}}$$

$$t_1, t_2 = \frac{-3 \pm 2\sqrt{3}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{-3\sqrt{3} \pm 6}{3} = \frac{\cancel{3}(-\sqrt{3} \pm 2)}{\cancel{3}}$$

$$\boxed{t_1 = -2 - \sqrt{3} \; ; \; t_2 = 2 - \sqrt{3}}$$

$\operatorname{tg}\frac{x}{2} = -2-\sqrt{3} = -(2+\sqrt{3})$

$\frac{x}{2} = \operatorname{arctg}(-(2+\sqrt{3}))$



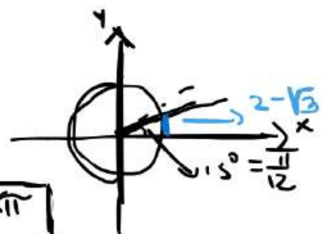
$\frac{x}{2} = \frac{7}{12}\pi + k\pi \Rightarrow \boxed{x = \frac{7}{6}\pi + 2k\pi}$

$t = \operatorname{tg}\frac{x}{2}$

$\cancel{75}$ 7
$\cancel{105}$
$\cancel{180}$ $\cancel{60}$ 12

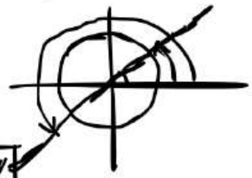
$$\operatorname{tg}\frac{x}{2} = 2-\sqrt{3}$$

$$\frac{x}{2} = \operatorname{arctg}(2-\sqrt{3})$$



$$\frac{x}{2} = \frac{\pi}{12} + k\pi \qquad \boxed{x = \frac{\pi}{6} + 2k\pi}$$

$$\frac{7}{\cancel{6}_{1}}\cdot\cancel{180}^{30}$$



$$x_1 = \frac{\pi}{6} + 2k\pi \qquad x_2 = \frac{7}{6}\pi + 2k\pi$$

30° $\qquad$ 210°

$$\boxed{x = \frac{\pi}{6} + k\pi,\; k\in\mathbb{Z}}$$

$$\operatorname{sen} x - (2+\sqrt{3})\cos x = 0$$

$$\frac{2t}{1+t^2} - (2+\sqrt{3})\frac{(1-t^2)}{1+t^2} = 0$$

$$2t - (2+\sqrt{3})(1-t^2) = 0$$

$$2t - 2(1-t^2) - \sqrt{3}(1-t^2) = 0$$

$$2t - 2 + 2t^2 - \sqrt{3} + \sqrt{3}t^2 = 0$$

$$(\sqrt{3}+2)t^2 + 2t - (2+\sqrt{3}) = 0$$

$$\operatorname{sen} x = \frac{2t}{1+t^2}$$

$$\cos x = \frac{1-t^2}{1+t^2}$$

$$t = \operatorname{tg}\frac{x}{2}$$

$$x \neq \pi + 2k\pi$$
$$k \in \mathbb{Z}$$

$$(\sqrt{3}+2)t^2+2t-(2+\sqrt{3})=0$$

$$\Delta = b^2-4ac = 4+4(2+\sqrt{3})^2 =$$

$$= 4+4(4+3+4\sqrt{3}) =$$

$$= 4+4(7+4\sqrt{3}) = 4+28+16\sqrt{3} = \underline{32+16\sqrt{3}}$$

$$t_1,t_2 = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{-2 \pm \sqrt{32+16\sqrt{3}}}{2(2+\sqrt{3})}$$

$$t_1,t_2 = \frac{-2 \pm \sqrt{16(2+\sqrt{3})}}{2(2+\sqrt{3})} = \frac{-2 \pm 4\sqrt{2+\sqrt{3}}}{2(2+\sqrt{3})} = \frac{\cancel{2}(-1 \pm 2\sqrt{2+\sqrt{3}})}{\cancel{2}(2+\sqrt{3})}$$

$$\boxed{t_1,t_2 = \frac{-1 \pm 2\sqrt{2+\sqrt{3}}}{2+\sqrt{3}}}$$

$$\sqrt{a+\sqrt{b}} = \sqrt{\frac{a+\sqrt{a^2-b}}{2}} + \sqrt{\frac{a-\sqrt{a^2-b}}{2}}$$

$$\sqrt{2+\sqrt{3}} = \sqrt{\frac{2+\sqrt{4-3}}{2}} + \sqrt{\frac{2-\sqrt{4-3}}{2}} =$$

$$= \sqrt{\frac{2+1}{2}} + \sqrt{\frac{2-1}{2}} = \sqrt{\frac{3}{2}} + \sqrt{\frac{1}{2}}$$

$$= \frac{\sqrt{3}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} + \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}$$

$$t_1,t_2 = \frac{-1 \pm 2\left(\frac{\sqrt{6}}{2}+\frac{\sqrt{2}}{2}\right)}{2+\sqrt{3}}$$

$$t_1,t_2 = \frac{-1 \pm \cancel{2}\left(\frac{\sqrt{6}+\sqrt{2}}{\cancel{2}}\right)}{2+\sqrt{3}} = \frac{-1 \pm (\sqrt{6}+\sqrt{2})}{2+\sqrt{3}}$$

$$t_1 = \frac{-1-\sqrt{6}-\sqrt{2}}{2+\sqrt{3}} \cdot \frac{(2-\sqrt{3})}{(2-\sqrt{3})} \qquad t_2 = \frac{-1+\sqrt{6}+\sqrt{2}}{2+\sqrt{3}}$$

$$t_1 = \frac{-2+\sqrt{3}-2\sqrt{6}+\sqrt{18}-2\sqrt{2}+\sqrt{6}}{4-3}$$

$$t_1 = -2+\sqrt{3}-2\sqrt{6}+3\sqrt{2}-2\sqrt{2}+\sqrt{6}$$

$$\boxed{t_1 = \sqrt{2}-\sqrt{6}+\sqrt{3}-2}$$

$$\operatorname{tg}\frac{x}{2} = \sqrt{2}-\sqrt{6}+\sqrt{3}-2$$

$$\frac{x}{2} = \operatorname{arctg}(\sqrt{2}-\sqrt{6}+\sqrt{3}-2)$$

$$\frac{x}{2} = -\frac{7}{24}\pi + k\pi$$

$$\frac{7}{\cancel{12}}\cdot\cancel{180}^{15}$$

$$\boxed{-\frac{7}{24}\pi = -52{,}5}$$

$$\boxed{x = -\frac{7}{12}\pi + 2k\pi}$$

$$t_2 = \frac{-1+\sqrt{6}+\sqrt{2}}{2+\sqrt{3}} \cdot \frac{(2-\sqrt{3})}{2-\sqrt{3}}$$

$\sqrt{18} = 3\sqrt{2}$

$$t_2 = \frac{-2+\sqrt{3}+2\sqrt{6}-3\sqrt{2}+2\sqrt{2}-\sqrt{6}}{4-3}$$

$$t_2 = -\sqrt{2}+\sqrt{6}+\sqrt{3}-2$$

$\frac{x}{2} = 37°5$

$$tg\,\frac{x}{2} = -\sqrt{2}+\sqrt{6}+\sqrt{3}-2$$

$$\frac{x}{2} = arctg(-\sqrt{2}+\sqrt{6}+\sqrt{3}-2)$$

$$\frac{x}{2} = \frac{5}{24}\pi + K\pi$$

$$\boxed{x = \frac{5}{12}\pi + K\pi}$$

$$x_1 = \frac{5}{12}\pi + K\pi \qquad x_2 = -\frac{7}{12}\pi + K\pi$$

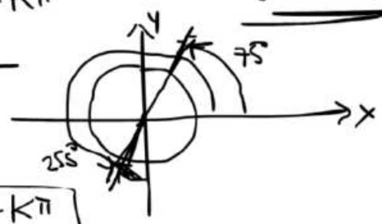
$\frac{7\pi}{12} = 105°$

$-105°$

360

105

255



$$\boxed{x = \frac{5}{12}\pi + K\pi}$$