$M = 10, 100, 1000, \ldots$

$M > 0$

$$\lim_{x \to +\infty} \sqrt{x+2} = +\infty$$

$$\forall M > 0 \ \exists I(+\infty) : \forall x \in I(+\infty) \Longrightarrow \boxed{\sqrt{x+2} > M}$$

$$\boxed{\sqrt{x+2} > M}$$

DIS. IRRAZIONALE

ELEVO TUTTO AL QUADRATO

$$\boxed{\sqrt{f(x)} > g(x)}$$

$$\cancel{\begin{cases} f(x) \geq 0 \\ M < 0 \end{cases}} \ \vee \ M \geq 0$$

$$x + 2 > M^2$$

$$\boxed{x > M^2 - 2} \quad]M^2 - 2; +\infty[\to I(+\infty)$$

$M^2 - 2 = 98$

$M = 10$

$$\lim_{x \to -\infty} \sqrt{2-4x} = +\infty$$

$$\forall M > 0 \ \exists I(-\infty) : \forall x \in I(-\infty) \Longrightarrow \sqrt{2-4x} > M$$

$$\sqrt{2-4x} > M$$

$$2 - 4x > M^2 \Longrightarrow -4x > -2 + M^2$$

$M = 10$

$$4x < 2 - M^2$$

$$\boxed{x < \frac{2 - M^2}{4}} \quad \frac{2-100}{4} = -\frac{98}{4} = -24{,}5$$

$$\boxed{I(-\infty) = \left]-\infty ; \frac{2-M^2}{4}\right[}$$

Prime considerazioni precedenti ai teoremi sui limiti

$$\lim f(x) = \ell \implies \lim f(x) - A = \ell - A$$

ES $\lim_{x \to 2} x^2 = 4 \Rightarrow \lim_{x \to 2} x^2 - 2 = 4 - 2 = 2$

Teorema di unicità del limite.

Ip $\exists \lim_{x \to c} f(x) = l$

Ts $\exists! \lim_{x \to c} f(x) = l$

↳ UNICO

Dim

$\boxed{\text{PER ASSURDO}}$

$\nexists! \lim_{x \to c} f(x) = l$

A $\boxed{\lim_{x \to c} f(x) = l}$ B $\boxed{\lim_{x \to c} f(x) = l'}$

A $\forall \frac{\varepsilon}{2} > 0 \ \exists I(c) : \forall x \in I(c) \Rightarrow |f(x) - l| < \frac{\varepsilon}{2}$

B $\forall \frac{\varepsilon}{2} > 0 \ \exists I'(c) : \forall x \in I'(c) \Rightarrow |f(x) - l'| < \frac{\varepsilon}{2}$

$I(c) \cap I'(c) = I(c)$

$$\begin{cases} |f(x) - l| < \frac{\varepsilon}{2} \\ |f(x) - l'| < \frac{\varepsilon}{2} \end{cases}$$

SOMMO MEMBRO A MEMBRO

$|f(x) - l| + |f(x) - l'| < \frac{\varepsilon}{2} + \frac{\varepsilon}{2}$

$|f(x) - l| + |f(x) - l'| < \varepsilon$

$l' - l = f(x) - l - f(x) + l' = [f(x) - l] - [f(x) - l']$

$|l' - l| = |[f(x) - l] - [f(x) - l']| = |f(x) - l| - |f(x) - l'|$

$|l' - l| < |f(x) - l| + |f(x) - l'| < \varepsilon$

A $|l' - l| < \varepsilon \Longrightarrow$

PER COSTRUZIONE

B $\varepsilon < |l' - l|$

$|l' - l| < \varepsilon$

$\varepsilon < |l' - l|$

ASSURDO!!

$\boxed{l' = l}$

$\exists! \lim_{x \to c} f(x) = l$

Teorema della permanenza del segno

Ip $\lim_{x \to c} f(x) = l \neq 0$

Ts $\exists I(c)$: $f(x) > 0 \Rightarrow l > 0$

$f(x) < 0 \Rightarrow l < 0$

Dim

$\lim_{x \to c} f(x) = l$ $\qquad$ $l \neq 0$

$\forall \varepsilon > 0 \ \exists I(c) : \forall x \in I(c) \Rightarrow \boxed{|f(x) - l| < \varepsilon}$

$l - \varepsilon < f(x) - \cancel{l} + \cancel{l} < \varepsilon + l$

$l - \varepsilon < f(x) < l + \varepsilon$

$\boxed{l - \frac{|l|}{2} < f(x) < l + \frac{|l|}{2}}$

$|f(x)| < K$

SCELGO $-K < f(x) < K$

$\boxed{\varepsilon = \frac{|l|}{2}}$

$$\ell - \frac{|\ell|}{2} < f(x) < \ell + \frac{|\ell|}{2}$$

$$\ell > 0$$
$$\Downarrow$$
$$f(x) > 0$$

$$\begin{cases} f(x) > \ell - \frac{|\ell|}{2} & (2) \\ f(x) < \ell + \frac{|\ell|}{2} & (1) \end{cases}$$

$\boxed{\ell > 0}$ la (1) NON HA SIGNIFICATO IMPORTANTE

(2) $f(x) > \ell - \frac{\ell}{2} = \frac{2\ell - \ell}{2} = \frac{\ell}{2}$ $\qquad \boxed{f(x) > \frac{\ell}{2}}$

$l < 0$ ℓ_a (1) ASSUME $|\ell| = -\ell$

$f(x) < \ell + \frac{|\ell|}{2} = \ell - \frac{\ell}{2} = \frac{\ell}{2}$

$f(x) < \frac{\ell}{2}$ $\boxed{\ell < 0 \Rightarrow f(x) < 0}$

ESEMPIO

$$\boxed{f(x) = \frac{1-x^2}{100}}$$

$$\boxed{\lim_{x \to 0} \frac{1-x^2}{100} = \frac{1}{100}}$$

$\forall\ \varepsilon > 0\ \exists\ I(0) : \forall x \in I(0) \Rightarrow \left| \frac{1-x^2}{100} - \frac{1}{100} \right| < \varepsilon$

$$\left| \frac{\cancel{1} - x^2 - \cancel{1}}{100} \right| < \varepsilon \Rightarrow \left| -\frac{x^2}{100} \right| < \varepsilon$$

$$\left| \frac{x^2}{100} \right| < \varepsilon \Longrightarrow |x^2| < 100\,\varepsilon$$

$$-100\varepsilon < x^2 < 100\,\varepsilon$$

ed $\begin{cases} x^2 > -100\varepsilon & \forall x \in \mathbb{R} \\ x^2 < 100\,\varepsilon \end{cases}$

$x = \pm 10\sqrt{\varepsilon}$

$$\boxed{-10\sqrt{\varepsilon} < x < 10\sqrt{\varepsilon}} \quad I(0) =]-10\sqrt{\varepsilon}; 10\sqrt{\varepsilon}[$$

$-3{,}16 \qquad 3{,}16$

$l = \frac{1}{100} > 0$

DAL TEOREMA DELLA PERM. DEL SEGNO

$f(x) > 0$

$$\frac{1-x^2}{100} > 0$$

$1 - x^2 > 0$

$x^2 - 1 < 0$

$x^2 < 1$

$$\boxed{-1 < x < 1}$$

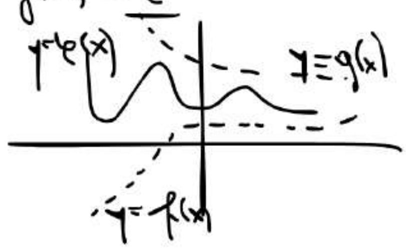
$I(0) =]-1; 1[$

VERIFICATO IL TEOR. DELLA PERM. DEL SEGNO IN $I(0) =]-1; 1[$

Primo teorema del confronto

Ip $\lim_{x \to c} f(x) = l \qquad \lim_{x \to c} g(x) = l$

$$f(x) \le \varphi(x) \le g(x)$$



Ts $\lim_{x \to c} \varphi(x) = l$

DIM $\quad f(x) \le \varphi(x) \le g(x) \quad I_3(c)$

DEF. L. P. (A) $\lim_{x \to c} f(x) = l \quad \forall \varepsilon > 0 \; \exists I_1(c) : \forall x \in I_1(c) \Rightarrow |f(x) - l| < \varepsilon$

(B) $\lim_{x \to c} g(x) = l \quad \forall \varepsilon > 0 \; \exists I_2(c) : \forall x \in I_2(c) \Rightarrow |g(x) - l| < \varepsilon$

(A) $l - \varepsilon < f(x) < l + \varepsilon \qquad I(c) = I_1(c) \cap I_2(c) \cap I_3(c)$

(B) $l - \varepsilon < g(x) < l + \varepsilon$

$$\begin{cases} l - \varepsilon < f(x) < l + \varepsilon \\ l - \varepsilon < g(x) < l + \varepsilon \\ f(x) \le \varphi(x) \le g(x) \end{cases}$$

$l - \varepsilon < \varphi(x) < l + \varepsilon$

$\lim_{x \to c} \varphi(x) = l$

$$\boxed{l - \varepsilon} < f(x) \le \varphi(x) \le g(x) \le \boxed{l + \varepsilon}$$

C.V.D