Lezione 16

$$\frac{2\ \text{sen}\,x\,\cancel{\cos x}}{\cos^2 x} = \frac{\overset{1}{\cancel{\cos^2 x}}}{\underset{1}{\cancel{\cos^2 x}}} - \frac{\text{sen}^2 x}{\cos^2 x}$$

$$2\,\text{tg}\,x = 1 - \text{tg}^2 x$$

$$\text{tg}^2 x + 2\,\text{tg}\,x - 1 = 0$$

$$\Delta = b^2 - 4ac = 2^2 - 4(1)(-1) = 4+4 = 8$$

$$\frac{\text{sen}\,x}{\cos x} = \text{tg}\,x$$

$$\cos x \neq 0$$

$$x \neq \frac{\pi}{2} + k\pi$$

$$(tg\,x)_1 ; (tg\,x)_2 = -\frac{\cancel{b} \pm \sqrt{\Delta}}{\cancel{2}a} = -\frac{2 \pm \sqrt{8}}{2} =$$

$$= -\frac{2 \pm \sqrt{2^3}}{2} = \frac{-2 \pm \sqrt{2^2 \cdot 2}}{2} = -\frac{2 \pm 2\sqrt{2}}{2} =$$

$$\begin{array}{r|l} 8 & 2 \\ 4 & 2 \\ 2 & 2 \\ 1 & \end{array}$$

$$= \frac{\cancel{2}(-1 \pm \sqrt{2})}{\cancel{2}}$$

$$(tg\,x)_1 = -1 - \sqrt{2}$$

$$(tg\,x)_2 = -1 + \sqrt{2}$$

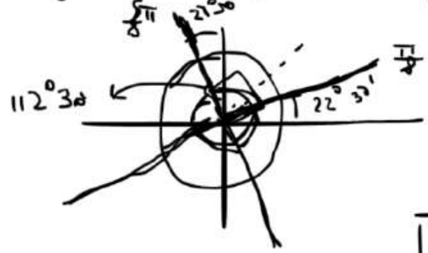
$$\pi - \frac{3}{8}\pi = \frac{8\pi - 3\pi}{8} = \frac{5}{8}\pi$$

$$x_1 = arctg(-1-\sqrt{2}) = arctg[-(1+\sqrt{2})]$$

$$\boxed{x_1 = \frac{5}{8}\pi + k\pi}$$

$$x_2 = arctg(-1+\sqrt{2}) = \frac{\pi}{8} + k\pi$$

$$\boxed{x_2 = \frac{\pi}{8} + k\pi}$$



$112°30'$

$$\boxed{x = \frac{\pi}{8} + k\frac{\pi}{2}}$$

RADIANTI

$$\boxed{x = 22°30' + k \cdot 90°}$$

GRADI

$$3\,\mathrm{sen}^2 x - 2\sqrt{3}\,\mathrm{sen}\,x\cos x - 3\cos^2 x = -2\sqrt{3}$$

$$3\,\mathrm{sen}^2 x - 2\sqrt{3}\,\mathrm{sen}\,x\cos x - 3\cos^2 x = -2\sqrt{3}\,(\underbrace{\mathrm{sen}^2 x + \cos^2 x}_{=1})$$

$$3\,\mathrm{sen}^2 x - 2\sqrt{3}\,\mathrm{sen}\,x\cos x - 3\cos^2 x + 2\sqrt{3}\,\mathrm{sen}^2 x + 2\sqrt{3}\cos^2 x = 0$$

$$(3+2\sqrt{3})\frac{\mathrm{sen}^2 x}{\cos^2 x} - 2\sqrt{3}\,\frac{\mathrm{sen}\,x\cancel{\cos x}}{\cos^2 x} - (3-2\sqrt{3})\frac{\cancel{\cos^2 x}}{\cancel{\cos^2 x}} = 0$$

$$\frac{\mathrm{sen}\,x}{\cos x} = \mathrm{tg}\,x \qquad \cos x \neq 0$$

$$\boxed{x \neq \frac{\pi}{2} + k\pi}$$

$$(3+2\sqrt{3})\,tg^2x - 2\sqrt{3}\,tgx - (3-2\sqrt{3}) = 0$$

$$\Delta = b^2 - 4ac = (-2\sqrt{3})^2 + 4(3+2\sqrt{3})(3-2\sqrt{3}) =$$

$$= 12 + 4(9-12) = 12 + 4(-3) = 12 - 12 = 0$$

$$(tgx)_1 = (tgx)_2 = -\frac{b}{2a} = \frac{\cancel{2}\sqrt{3}}{\cancel{2}(3+2\sqrt{3})} = \frac{\sqrt{3}}{3+2\sqrt{3}}$$

$$\frac{\sqrt{3}}{3+2\sqrt{3}} \cdot \frac{3-2\sqrt{3}}{3-2\sqrt{3}} = \frac{3\sqrt{3}-6}{9-12} = \frac{3\sqrt{3}-6}{-3} = \frac{6-3\sqrt{3}}{3} = \frac{\cancel{3}(2-\sqrt{3})}{\cancel{3}}$$

$$tgx = 2-\sqrt{3} \qquad x = arctg(2-\sqrt{3}) \qquad 15° = \frac{\pi}{12}$$

$$\boxed{x = \frac{\pi}{12} + k\pi} \quad k \in \mathbb{Z}$$

$$\operatorname{sen}^2 x + \frac{3}{4}\cos^2 x + \frac{1}{4}\operatorname{sen}^2 x = \frac{\sqrt{3}}{2}\operatorname{sen} x \cos x + \frac{1}{2}$$

$$\left(1 + \frac{1}{4}\right)\operatorname{sen}^2 x + \frac{3}{4}\cos^2 x - \frac{\sqrt{3}}{2}\operatorname{sen} x \cos x = \frac{1}{2}\left(\operatorname{sen}^2 x + \cos^2 x\right)$$

$$\frac{5}{4}\operatorname{sen}^2 x - \frac{\sqrt{3}}{2}\operatorname{sen} x \cos x + \frac{3}{4}\cos^2 x - \frac{1}{2}\operatorname{sen}^2 x - \frac{1}{2}\cos^2 x = 0$$

$$\left(\frac{5}{4} - \frac{1}{2}\right)\operatorname{sen}^2 x - \frac{\sqrt{3}}{2}\operatorname{sen} x \cos x + \left(\frac{3}{4} + \frac{1}{2}\right)\cos^2 x = 0$$

$$\left(\frac{5-2}{4}\right)\operatorname{sen}^2 x - \frac{\sqrt{3}}{2}\operatorname{sen}x\cos x + \left(\frac{3-2}{4}\right)\cos^2 x = 0$$

$$\frac{3}{4}\operatorname{sen}^2 x - \frac{\sqrt{3}}{2}\operatorname{sen}x\cos x + \frac{1}{4}\cos^2 x = 0$$

$$3\operatorname{sen}^2 x - 2\sqrt{3}\operatorname{sen}x\cos x + \cos^2 x = 0$$

$\cos x \neq 0$

$x \neq \frac{\pi}{2} + k\pi$

$$3\operatorname{tg}^2 x - 2\sqrt{3}\operatorname{tg}x + 1 = 0$$

$$\Delta = b^2 - 4ac = (-2\sqrt{3})^2 - 4(3)(1) = 12 - 12 = 0$$

$$(\operatorname{tg}x)_1 = (\operatorname{tg}x)_2 = -\frac{b}{2a} = \frac{\cancel{2}\sqrt{3}}{\cancel{6}\,3} = \frac{\sqrt{3}}{3}$$

$$\operatorname{tg}x = \frac{\sqrt{3}}{3} \qquad x = \operatorname{arctg}\frac{\sqrt{3}}{3} \Longrightarrow \boxed{x = \frac{\pi}{6} + k\pi}$$

$k \in \mathbb{Z}$

Equazioni simmetriche

$$\underline{\operatorname{sen} x + \cos x + \operatorname{sen} x \cos x = 1} \qquad x = 45^\circ + z$$

$$\operatorname{sen} x = \operatorname{sen}(45^\circ + z) = \operatorname{sen} 45^\circ \cos z + \operatorname{sen} z \cos 45^\circ =$$

$$= \frac{\sqrt{2}}{2} \cos z + \frac{\sqrt{2}}{2} \operatorname{sen} z = \boxed{\frac{\sqrt{2}}{2} (\operatorname{sen} z + \cos z)}$$

$$\cos x = \cos(45^\circ + z) = \cos 45^\circ \cos z - \operatorname{sen} 45^\circ \operatorname{sen} z$$

$$= \frac{\sqrt{2}}{2} \cos z - \frac{\sqrt{2}}{2} \operatorname{sen} z = \boxed{\frac{\sqrt{2}}{2} (\cos z - \operatorname{sen} z)}$$

$$\operatorname{sen} x \cos x = \frac{1}{2} (\cos^2 z - \operatorname{sen}^2 z) = \frac{1}{2}(1 + \cos^2 z - \cos^2 z)$$

$$= \underline{\frac{1}{2}(-1 + 2\cos^2 z)}$$

$$\frac{\sqrt{2}}{2}(\cancel{\operatorname{sen} z} + \cos z) + \frac{\sqrt{2}}{2}(\cos z - \cancel{\operatorname{sen} z}) + \frac{1}{2}(1 - 2\cos^2 z) = 0$$

$$\sqrt{2}\cos z + \frac{1}{2} + \cos^2 z = 1$$

$$2\sqrt{2}\cos z + 1 - 2\cos^2 z = 2$$

$$-2\cos^2 z + 2\sqrt{2}\cos z - 3 = 0$$

$$\boxed{2\cos^2 z + 2\sqrt{2}\cos z - 3 = 0}$$

$$\Delta = 8 - 4(2)(-3) = 8 + 24 = 32$$

$$\sqrt{32} = 4\sqrt{2}$$

$$\cos z_1, \cos z_2 = \frac{-2\sqrt{2} \pm 4\sqrt{2}}{4}$$

$$\frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2}$$

$$-\frac{6\sqrt{2}}{4} = -\frac{3\sqrt{2}}{2}$$

imposible

$$\cancel{z} = \frac{\pi}{4} + 2k\pi$$

$$x = \frac{\pi}{2} + \cancel{k}$$

$$\operatorname{sen} x + \cos x = 1$$

$$\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = 1$$

$$\frac{2t + 1 - t^2}{\cancel{1+t^2}} = \frac{1+t^2}{\cancel{1+t^2}}$$

$$2t + \cancel{1} - t^2 = \cancel{1} + t^2$$

$$2t - 2t^2 = 0 \qquad 2t(1-t) = 0$$

$$t = 0 \qquad t = 1$$

$$\operatorname{sen} x = \frac{2t}{1+t^2}$$

$$\cos = \frac{1-t^2}{1+t^2}$$

$$t = \operatorname{tg}\frac{x}{2}$$

$$x \neq \pi + 2k\pi$$

$$t = tg\frac{x}{2} = 0 \Longrightarrow \frac{x}{2} = k\pi \Longrightarrow \boxed{x = 2k\pi}$$

$$t = tg\frac{x}{2} = 1 \Longrightarrow \frac{x}{2} = \frac{\pi}{4} + k\pi \Longrightarrow \boxed{x = \frac{\pi}{2} + 2k\pi}$$

RISOLTO CON LE FORMULE PARAMETRICHE

$x = 45^\circ + z$

$\operatorname{sen} x + \cos x = 1$

$\operatorname{sen} x = \operatorname{sen}(45^\circ + z)$

$= \frac{\sqrt{2}}{2}(\cos z + \operatorname{sen} z)$

$\cos x = \cos(45^\circ + z) =$

$= \frac{\sqrt{2}}{2}(\cos z - \operatorname{sen} z)$

$\frac{\sqrt{2}}{2}(\cos z + \cancel{\operatorname{sen} z}) + \frac{\sqrt{2}}{2}(\cos z - \cancel{\operatorname{sen} z}) = 1$

$\cancel{2}\,\frac{\sqrt{2}}{\cancel{2}}\cos z = 1$

$\sqrt{2}\cos z = 1$

$\cos z = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\cos z = \frac{\sqrt{2}}{2}$

$z = \arccos\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4} + 2k\pi$

$z_1 = -\frac{\pi}{4} + 2k\pi$

$z = \pm\frac{\pi}{4} + 2k\pi$

$x_1 = \frac{\pi}{4} + \frac{\pi}{4} + 2k\pi = \boxed{\frac{\pi}{2} + 2k\pi}$

$x_1 = \cancel{\frac{\pi}{4}} - \cancel{\frac{\pi}{4}} + 2k\pi \Rightarrow \boxed{x_2 = 2k\pi}$

$\operatorname{sen} x + \cos x + 2 \operatorname{sen} x \cos x + 1 = 0$

$\frac{\sqrt{2}}{2}(\cos z + \operatorname{sen} z) + \frac{\sqrt{2}}{2}(\cos z - \operatorname{sen} z) + \frac{1}{2}(\cos^2 z - \operatorname{sen}^2 z) + 1 = 0$

$\sqrt{2}\cos z + \cos^2 z + \operatorname{sen}^2 z + 1 = 0$

$\sqrt{2}\cos z + \cos^2 z + (1 - \cos^2 z) + 1 = 0$

$\sqrt{2}\cos z + \cos^2 z - \cancel{1} + \cos^2 z + \cancel{1} = 0$

$2\cos^2 z + \sqrt{2}\cos z = 0$

$\cos z\,(2\cos z + \sqrt{2}) = 0$

$\cos z = 0$ $\quad z = k\pi$

$\cos z = -\frac{\sqrt{2}}{2}$ $\quad z = \frac{3}{4}\pi + 2k\pi$

$\frac{5}{4}\pi + 2k\pi$

$x = 45° + z =$

$= \frac{\pi}{4} + z$

$\operatorname{sen} x = \frac{\sqrt{2}}{2}(\cos z + \operatorname{sen} z)$

$\cos x = \frac{\sqrt{2}}{2}(\cos z - \operatorname{sen} z)$

$\operatorname{sen}^2 z = 1 - \cos^2 z$

$x = \frac{\pi}{4} + k\pi$

$x = \frac{\pi}{4} + \frac{3}{4}\pi + 2k\pi = \pi + 2k\pi$

$x = \frac{3}{2}\pi + 2k\pi$

$-\frac{\pi}{2}$