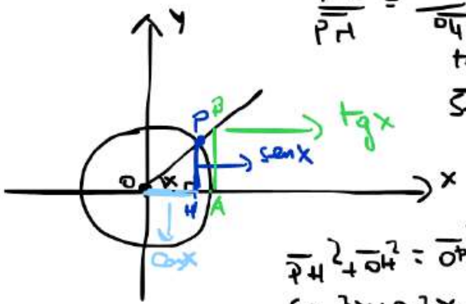
Lezione 16

Limiti notevoli goniometrici e in altre forme particolari.

$$\boxed{\lim_{x \to 0} \frac{\operatorname{sen} x}{x} = \left[\frac{0}{0}\right] = 1}$$

$$\lim_{f(x) \to 0} \frac{\operatorname{sen} f(x)}{f(x)} = 1$$

Dimostriamo questo attraverso la goniometria.



$$\frac{\overline{AB}}{\overline{PH}} = \frac{\overline{AO}}{\overline{OH}}$$

$A(1,0)$

$O(0;0)$

$R = 1$

$$\frac{tg\,x}{sen\,x} = \frac{1}{cos\,x}$$

$$\boxed{tg\,x = \frac{sen\,x}{cos\,x}}$$

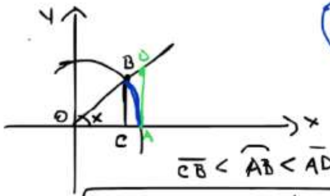
$\overline{PH} = sen\,x$

$\overline{OH} = cos\,x$

$\overline{AB} = tg\,x = \frac{sen\,x}{cos\,x}$

$\overline{PH}^2 + \overline{OH}^2 = \overline{OP}^2$

$sen^2x + cos^2x = 1 \Longrightarrow sen^2x = 1 - cos^2x$



$\overline{BC} = \operatorname{sen} x$

$\overline{AD} = \operatorname{tg} x$

$\overline{OB} = 1$

$\overline{AB}$ ARCO RETTIF

$\overline{CB} < \overset{\frown}{AB} < \overline{AD}$

GRAFICAMENTE LO VEDO

$$\boxed{\operatorname{sen} x < x < \operatorname{tg} x}$$

DIVIDO TUTTO PER $\operatorname{sen} x$

$\operatorname{sen} x \neq 0$

$\boxed{x \neq k\pi;\ k \in \mathbb{Z}}$

$k = \pm 1, \pm 2, \ldots$

$$\frac{\cancel{\operatorname{sen} x}}{\cancel{\operatorname{sen} x}} < \frac{x}{\operatorname{sen} x} < \frac{\operatorname{tg} x}{\operatorname{sen} x}$$

$\operatorname{tg} x = \dfrac{\operatorname{sen} x}{\cos x}$

$$1 < \frac{x}{\operatorname{sen} x} < \frac{\cancel{\operatorname{sen} x}}{\cos x} \frac{1}{\cancel{\operatorname{sen} x}}$$

$$\boxed{1 < \frac{x}{\operatorname{sen} x} < \frac{1}{\cos x}}$$

PASSO AGLI INVERSI TUTTA LA RELAZIONE

$$1 > \frac{\operatorname{sen} x}{x} > \cos x$$

$$\cos x < \frac{\operatorname{sen} x}{x} < 1$$

$\lim\limits_{x \to 0^+} \cos x = \cos 0 = 1$

$\lim\limits_{x \to 0^+} 1 = 1$

$$\lim_{x \to 0^+} \frac{\operatorname{sen} x}{x} = 1$$

PER IL TEOREMA DEL CONFRONTO

$$\boxed{\lim_{x \to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}} \qquad \left[\frac{0}{0}\right] \qquad \operatorname{sen}^2 x + \cos^2 x = 1$$

DIM

$$\lim_{x \to 0} \frac{1-\cos x}{x^2} \cdot \frac{(1+\cos x)}{1+\cos x} = \lim_{x \to 0} \frac{1-\cos^2 x}{x^2(1+\cos x)} =$$

$$= \lim_{x \to 0} \frac{\operatorname{sen}^2 x}{x^2} \cdot \frac{1}{(1+\cos x)} = \lim_{x \to 0} \frac{\operatorname{sen}^2 x}{x^2} \cdot \lim_{x \to 0} \frac{1}{1+\cos x} \to \frac{1}{1+1} = \frac{1}{2}$$

$$= \lim_{x \to 0} \left(\frac{\operatorname{sen} x}{x}\right)^2 \cdot \frac{1}{2} = 1^2 \cdot \frac{1}{2} = 1 \cdot \frac{1}{2} = \frac{1}{2}$$

C.V.D

$$\lim_{x\to 0} \frac{2x + \operatorname{sen} x}{x} = \lim_{x\to 0} \overset{\to 2}{\frac{2x}{x}} + \lim_{x\to 0} \overset{\to 1}{\frac{\operatorname{sen} x}{x}} = 2+1 = 3$$

$$\lim_{x\to 0} \frac{\operatorname{sen} 2x}{\operatorname{sen} 3x} = \lim_{x\to 0} \frac{\operatorname{sen} 2x}{2x} \cdot 2\cancel{x} \cdot \frac{3x}{\operatorname{sen} 3x} \cdot \frac{1}{3\cancel{x}} =$$

$$= \frac{2}{3} \lim_{x\to 0} \underset{\to 1}{\frac{\operatorname{sen} 2x}{2x}} \cdot \lim_{x\to 0} \overset{\to 1}{\frac{3x}{\operatorname{sen} 3x}} = \frac{2}{3} \cdot 1 \cdot 1 = \frac{2}{3}$$

$$\boxed{y = f(x)^{g(x)}} \Longrightarrow \boxed{y = e^{g(x)\log f(x)}} \qquad \boxed{0^0}$$

$$\lim_{x\to+\infty}\left(\frac{1}{x}\right)^{\frac{1}{\log x+3}} = \lim_{x\to+\infty} e^{\frac{1}{\log x+3}\log\left(\frac{1}{x}\right)} = \lim_{x\to+\infty} e^{\frac{1}{\log x+3}\log x^{-1}} = \lim_{x\to+\infty} e^{\frac{1}{\log x+3}\cdot(-\log x)} =$$

$$= e^{-\lim_{x\to+\infty}\frac{\log x}{\log x+3}} = e^{-\lim_{x\to+\infty}\frac{\cancel{\log x}}{\cancel{\log x}\left(1+\frac{1}{\log x}\right)}} = e^{-\frac{1}{1+0}} = e^{-1} = \frac{1}{e}$$

$$\lim_{x\to+\infty} (x+2)^{\frac{1}{\log(x+1)}} = \lim_{x\to+\infty} e^{\frac{1}{\log(x+1)}\log(x+2)} = \lim_{x\to+\infty} \frac{\log(x+2)}{\log(x+1)} =$$

$$\boxed{\log A\cdot B = \log A + \log B}$$

$$f(x) = x+2$$

$$g(x) = \frac{1}{\log(x+1)}$$

$$= \lim_{x\to+\infty} e^{\frac{\log(x+2)}{\log(x+1)}} = \lim_{x\to+\infty} \frac{\log x + \log\left(1+\frac{2}{x}\right)}{\log x + \log\left(1+\frac{1}{x}\right)} =$$

$$= e^{\lim\limits_{x\to+\infty} \frac{\log\left[x\cdot\left(1+\frac{2}{x}\right)\right]}{\log\left[x\left(1+\frac{1}{x}\right)\right]}} = e$$

$$= e^{\lim\limits_{x\to+\infty} \frac{1+\frac{\log\left(1+\frac{2}{x}\right)}{\log x}}{1+\frac{\log\left(1+\frac{1}{x}\right)}{\log x}}} = e^{1} = \underline{e}$$

$$\frac{0}{+\infty} = 0$$

$$\frac{0}{+\infty} = 0$$

$$\lim_{x \to +\infty} \left(\sqrt{x+1} - \sqrt{x}\right) = [+\infty - \infty]$$

$$= \lim_{x \to +\infty} \frac{\left(\sqrt{x+1} - \sqrt{x}\right)}{} \cdot \frac{\left(\sqrt{x+1} + \sqrt{x}\right)}{\left(\sqrt{x+1} + \sqrt{x}\right)} =$$

$$= \lim_{x \to +\infty} \frac{\cancel{x} + 1 - \cancel{x}}{\sqrt{x+1} + \sqrt{x}} = \lim_{x \to +\infty} \frac{1}{\underbrace{\sqrt{x+1} + \sqrt{x}}_{+\infty + \infty}} = \frac{1}{+\infty} = \underline{0}$$

$$\lim_{x \to \infty} \left(\sqrt{2x^2-1} - \sqrt{2x^2-x-1}\right) =$$

$$= \lim_{x \to \infty} \left(\sqrt{2x^2-1} - \sqrt{2x^2-x-1}\right) \frac{\left(\sqrt{2x^2-1} + \sqrt{2x^2-x-1}\right)}{\left(\sqrt{2x^2-1} + \sqrt{2x^2-x-1}\right)}$$

$$= \lim_{x \to \infty} \frac{2x^2-1-(2x^2-x-1)}{\sqrt{2x^2-1}+\sqrt{2x^2-x-1}} =$$

$$= \lim_{x \to \infty} \frac{\cancel{2x^2} \cancel{-1} \cancel{-2x^2} + x \cancel{+1}}{\sqrt{2x^2-1}+\sqrt{2x^2-x-1}} = \boxed{\lim_{x \to \infty} \frac{x}{\sqrt{2x^2-1}+\sqrt{2x^2-x-1}}}$$

$$\lim_{x\to\infty} \frac{x}{\sqrt{2x^2-1}+\sqrt{2x^2-x-1}} =$$

$$= \lim_{x\to\infty} \frac{x}{\sqrt{x^2\left(2-\frac{1}{x^2}\right)}+\sqrt{x^2\left(2-\frac{1}{x}-\frac{1}{x^2}\right)}} =$$

$$= \lim_{x\to\infty} \frac{x}{|x|\sqrt{2-\frac{1}{x^2}}+|x|\sqrt{2-\frac{1}{x}-\frac{1}{x^2}}} =$$

$$= \lim_{x\to\infty} \frac{x}{|x|\left[\sqrt{2-\frac{1}{x^2}}+\sqrt{2-\frac{1}{x}-\frac{1}{x^2}}\right]}$$

ne $x\to+\infty$ $|x|=x$

$$\lim_{x\to+\infty} \frac{\cancel{x}}{\cancel{x}\left[\sqrt{2-\cancel{\frac{1}{x^2}}^{\,0}}+\sqrt{2-\cancel{\frac{1}{x}}^{\,0}-\cancel{\frac{1}{x^2}}^{\,0}}\right]} = \frac{1}{2\sqrt{2}}$$ ne $x\to+\infty$

ne $x\to-\infty$

$$\lim_{x\to-\infty} \frac{\cancel{x}}{-\cancel{x}\left[\sqrt{2-\underset{0}{\frac{1}{x}}}+\sqrt{2-\underset{0}{\frac{1}{x}}-\frac{1}{x^2}}\right]} = -\frac{1}{2\sqrt{2}}$$

ne $x\to-\infty$