

$$\frac{1}{6} \cdot \frac{9}{4} = \frac{9}{24}$$

$$\begin{aligned} & \frac{1}{2}(xy)^2 \cdot \left(\frac{3}{4}x^3 + \frac{1}{3}x + \frac{1}{2}\right) - \left(\frac{1}{2}xy + 3\right) \cdot \left(\frac{1}{2}xy - 3\right) + \frac{1}{6}x^2 \left(\frac{1}{2}x^2 - \frac{9}{4}x^3\right) \\ &= \frac{1}{2}x^2y^2 \left(\frac{3}{4}x^3 + \frac{1}{3}x + \frac{1}{2}\right) - \left(\frac{1}{4}x^2y^2 - \frac{3}{2}xy + \frac{3}{2}xy - 9\right) + \frac{1}{12}x^3y^2 - \frac{3}{8}x^5y^2 \\ &= \frac{3}{8}x^5y^2 + \frac{1}{6}x^3y^2 + \frac{1}{4}xy^2 - \frac{1}{4}x^2y^2 + 9 + \frac{1}{12}x^3y^2 - \frac{3}{8}x^5y^2 \\ &= \left(\frac{1}{6} + \frac{1}{12}\right)x^3y^2 + 9 = \left(\frac{2+1}{12}\right)x^3y^2 + 9 = \\ &= \frac{1}{4}x^3y^2 + 9 = \boxed{\frac{1}{4}x^3y^2 + 9} \end{aligned}$$

$$\begin{aligned}
& \left(\frac{1}{6}x^2 + xy\right) \cdot \left(-\frac{1}{9}x + \frac{2}{3}y\right) + \left(\frac{1}{7}xy - \frac{8}{3}y^2\right) \cdot (x - 3y) + \frac{1}{2} \cdot \left(\frac{1}{5}x\right)^3 - (2y)^3 = \\
& = \left(-\frac{1}{54}x^3 + \frac{2}{9}x^2y - \frac{1}{9}x^2y + \frac{2}{3}xy^2\right) + \left(\frac{1}{7}x^2y - \frac{3}{7}xy^2 - \frac{8}{3}xy^2 + \frac{24}{3}y^3\right) + \frac{1}{2} \cdot \frac{1}{125}x^3 - 8y^3 = \\
& = \left(-\frac{1}{54}x^3 + \frac{1}{9}x^2y - \frac{1}{9}x^2y + \frac{2}{3}xy^2\right) + \frac{1}{7}x^2y - \frac{3}{7}xy^2 - \frac{8}{3}xy^2 + 8y^3 + \frac{1}{250}x^3 - 8y^3 = \\
& = \left(-\frac{1}{54}x^3 + \frac{2}{3}xy^2 + \frac{1}{7}x^2y - \frac{3}{7}xy^2 - \frac{8}{3}xy^2 + 8y^3 + \frac{1}{250}x^3 - 8y^3\right) = \\
& = \left(\frac{2}{3} - \frac{3}{7} - \frac{8}{3}\right)xy^2 + \frac{1}{7}x^2y = \left(\frac{14 - 9 - 56}{21}\right)xy^2 + \frac{1}{7}x^2y = \\
& = -\frac{51}{21}xy^2 + \frac{1}{7}x^2y = \boxed{-\frac{17}{7}xy^2 + \frac{1}{7}x^2y}
\end{aligned}$$

$$\begin{aligned}
& \frac{12}{2} X^6 \cdot \left(\frac{1}{3} X^2 Y^4\right)^3 - \left[\frac{4}{3} X^4 Y^6 \cdot \left(\frac{1}{4} X^4 Y^3\right)^2 + \frac{1}{6} Y^6 \cdot \left(\frac{1}{2} X^4 Y^2\right)^3 \right] + (X^3 Y^2)^2 \\
&= \frac{9}{2} X^6 \cdot \left(\frac{1}{27} X^6 Y^{12}\right) - \left[\frac{4}{3} X^4 Y^6 \cdot \left(\frac{1}{16} X^8 Y^6\right) + \frac{1}{6} Y^6 \cdot \left(\frac{1}{8} X^{12} Y^6\right) \right] + X^6 Y^4 \\
&= \frac{9}{2} \cdot \frac{1}{27} X^{12} Y^{12} - \left[\frac{4}{48} X^{12} Y^{12} + \frac{1}{48} X^{12} Y^{12} \right] + X^6 Y^4 \\
&= \frac{1}{6} X^{12} Y^{12} - \frac{5}{48} X^{12} Y^{12} - \frac{1}{48} X^{12} Y^{12} + X^6 Y^4 \\
&= \left(\frac{1}{6} - \frac{1}{12} - \frac{1}{48}\right) X^{12} Y^{12} + X^6 Y^4 \\
&= \left(\frac{8-4-1}{48}\right) X^{12} Y^{12} + X^6 Y^4 = \frac{1}{16} X^{12} Y^{12} + X^6 Y^4 \\
&= \boxed{\frac{1}{16} X^{12} Y^{12} + X^6 Y^4}
\end{aligned}$$

$$\begin{aligned}
& (x+y)(x+y+1)(x+y-1) - (x+y)[x^2(x+2) - x^3 - x^2] - (x+y)[y^2(y+2) - y^2 - y^3] - (x+y)(2xy+1) = \\
& = (x^2+xy+x+xy+y^2+y)(x+y-1) - (x+y)[x^3+2x^2-x^3-x^2] - (x+y)[y^3+2y^2-y^2-y^3] - (2x^2y+x+2xy^2+y) = \\
& = \cancel{x^3+x^2y+x^2+x^2y+xy^2-xy+x^2+xy-x+x^2y+xy^2-xy+xy+y^3-xy^2-xy^2-y} + \\
& - (x+y)[x^2] - (x+y)[y^2] - 2x^2y^2 - x - 2xy^2 - y = \\
& = \cancel{x^3+x^2y+x^2y+xy^2} - \cancel{x+x^2y+x^2y+xy^2} - \cancel{2x^2y^2} - \cancel{x} - \cancel{2xy^2} - \cancel{y} = \\
& = \underline{-2x-2y}
\end{aligned}$$

$$\begin{aligned}
& \left[\left(\frac{1}{3}\right)^2 \cdot \left(-\frac{2}{3} + 1\right)^{-2} \cdot \left(1 - \frac{8}{9}\right)^2 \cdot \left(\frac{1}{27}\right) \right] : \left[\left(\frac{1}{3}\right)^6 : 27^{-2} \right] = \\
& = \left[\frac{1}{9} \cdot \left(\frac{-2+3}{3}\right)^{-2} \cdot \left(\frac{9-8}{9}\right)^2 \cdot \frac{1}{27} \right] : \left[\left(\frac{1}{3}\right)^6 : (3^3)^{-2} \right] = \\
& = \left[\frac{1}{9} \cdot \left(\frac{1}{3}\right)^{-2} \cdot \left(\frac{1}{9}\right)^2 \cdot \frac{1}{27} \right] : \left[3^{-6} : 3^{-6} \right] = \\
& = \left[\frac{1}{9} \cdot \cancel{3^2} \cdot \left(\frac{1}{\cancel{9}}\right)^2 \cdot \frac{1}{27} \right] : 1 = \frac{1}{9} \cdot \frac{1}{27} = \frac{1}{243} \\
& = \left[\left(\frac{1}{9} \cdot \frac{1}{9}\right) \cdot \frac{1}{27} \right] : 1 = \left[\frac{1}{81} \cdot \frac{1}{27} \right] = \frac{1}{2187} \\
& \quad \left(\frac{1}{3}\right)^{-2} = 3^2 = 9 \qquad \quad = \left(\frac{1}{3}\right)^6
\end{aligned}$$