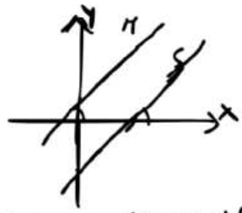
Lezione 3

Condizione di parallelismo e perpendicolarità

Condizione di parallelismo

Due rette sono parallele nel momento in cui il loro coefficiente angolare è lo stesso e viceversa

$$\boxed{r \parallel s \iff m_r = m_s}$$



Dim

$$\begin{cases} r: y = m_r x + q_r \\ s: y = m_s x + q_s \end{cases}$$

$$m_r x + q_r = m_s x + q_s$$

$$m_r x - m_s x = q_s - q_r$$

$$(m_r - m_s) x = q_s - q_r$$

$$x = \frac{q_s - q_r}{m_r - m_s} \qquad m_r - m_s \neq 0 \quad \text{se } m_r \neq m_s$$

$\exists (x,y)$ INTERS. DI r E s

$\nexists (x,y)$ INTERS $\Rightarrow m_r - m_s = 0 \Rightarrow \boxed{m_r = m_s}$

C.V.D

Connessione fra forma esplicita e implicita

$$r: ax + by + c = 0 \qquad b \neq 0$$

$$by = -ax - c$$

$$\boxed{y = -\frac{a}{b} x - \frac{c}{b}}$$

$$\boxed{y = mx + q}$$

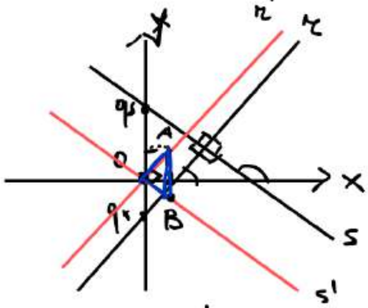
$$\boxed{\begin{aligned} m &= -\frac{a}{b} \\ q &= -\frac{c}{b} \end{aligned}}$$

Condizione di perpendicolarità

Due rette perpendicolari se e solo se i due coefficienti angolari delle due rette sono l'uno l'antireciproco (opposto e inverso) rispetto all'altro.

$$\boxed{r \perp s \iff m_r = -\frac{1}{m_s}}$$

DIM



$r: y = m_r x + q_r$

$s: y = m_s x + q_s$

$r' \parallel r$

$r': y = m_r x$

$s' \parallel s$

$s': y = m_s x$

$r' \perp s'$

$O(0,0)$

$A(1, m_r) \in r'$ $\quad$ $B(1, m_s) \in s'$

$\boxed{AOB \text{ TRIANG. RETTANGOLO}}$

$$\overline{AO}^2 + \overline{OB}^2 = \overline{AB}^2$$

$$\overline{AB} = |y_B - y_A| = |m_s - m_r|$$

$$\overline{AO} = \sqrt{(x_A - x_O)^2 + (y_A - y_O)^2} = \sqrt{(1-0)^2 + (m_r - 0)^2} = \sqrt{1 + m_r^2}$$

$$\overline{OB} = \sqrt{(x_B - x_O)^2 + (y_B - y_O)^2} = \sqrt{(1-0)^2 + (m_s - 0)^2} = \sqrt{1 + m_s^2}$$

$$\left(\sqrt{1 + m_r^2}\right)^2 + \left(\sqrt{1 + m_s^2}\right)^2 = |m_s - m_r|^2$$

c.v.d

$$1 + \cancel{m_r^2} + 1 + \cancel{m_s^2} = \cancel{m_s^2} + \cancel{m_r^2} - 2m_r m_s$$

$$\cancel{2} = -\cancel{2} m_r m_s$$

$$-m_r m_s = 1$$

$$\boxed{m_r = -\frac{1}{m_s}}$$

$$m_1 = -\frac{a}{b} \qquad m_r = -\frac{a}{b} \qquad m_s = -\frac{a'}{b'}$$

$$\boxed{m_r = -\frac{1}{m_s}} \implies -\frac{a}{b} = -\frac{1}{-\frac{a'}{b'}}$$

$$-\frac{a}{b} = \frac{b'}{a'} \implies -\frac{a}{b} - \frac{b'}{a'} = 0 \qquad \frac{a}{b} + \frac{b'}{a'} = 0$$

$$\cancel{a'b}\ \frac{aa' + bb'}{\cancel{a'b}} = 0\ \cancel{a'b} \qquad \boxed{a \cdot a' + b b' = 0}$$

$r: \ y = (\underbrace{K+1}) x + (\underbrace{K-1})$

$\boxed{\begin{array}{l} m_r = K+1 \\ q_r = K-1 \end{array}}$

$s: \ y = \underbrace{4} x + \underbrace{1}$

$\boxed{\begin{array}{l} m_s = 4 \\ q_s = 1 \end{array}}$

$K \in \mathbb{R} \Rightarrow r \parallel s$ A

$K \in \mathbb{R} \Rightarrow r \perp s$ B

A $\quad m_r = m_s \Rightarrow K+1 = 4 \Rightarrow \boxed{K = 3}$

B $\quad m_r = -\frac{1}{m_s} \Rightarrow K+1 = -\frac{1}{4} \Rightarrow 4\frac{4K+4}{\cancel{4}} = \frac{-1}{\cancel{4}}\cancel{4}$

$4K = -5$

$\boxed{K = -\frac{5}{4}}$

$r: y = (p+1)x \qquad s: y = (p-1)x \qquad \boxed{y = mx}$

$p \in \mathbb{R} \Rightarrow \begin{cases} r \parallel s \\ r \perp s \end{cases}$

$m_r = p+1$

$m_s = p-1$

$m_r = m_s \Longrightarrow \cancel{p}+1 = \cancel{p}-1 \Longrightarrow \boxed{0 = -2}$

$\nexists\, p \in \mathbb{R} \qquad r \parallel s$

$m_r = -\dfrac{1}{m_s} \qquad p+1 = -\dfrac{1}{p-1} \qquad p-1 \neq 0 \quad p \neq 1$

$\dfrac{(p+1)(p-1)}{\cancel{p-1}} = \dfrac{-1}{\cancel{p-1}} \qquad p^2 \cancel{-1} = \cancel{-1} \Longrightarrow p^2 = 0 \Longrightarrow \boxed{p = 0}$

$r: \boxed{y = x} \qquad s: \boxed{y = -x}$

$r: hx + (h-1)y + h + 2 = 0$

$s: (h-1)x + (h+2)y + h - 3 = 0$

$\boxed{m_r = -\frac{a}{b} = -\frac{h}{h-1}}$

$\boxed{m_s = -\frac{a'}{b'} = -\frac{(h-1)}{h+2}}$

$h \in \mathbb{R} \Rightarrow r \parallel s$ $\quad \boxed{m_r = m_s}$

$\cancel{-}\frac{h}{h-1} = \cancel{-}\frac{(h-1)}{h+2} \Rightarrow \frac{h}{h-1} = \frac{h-1}{h+2}$

C.E.
$h-1 \neq 0$
$\boxed{h \neq 1}$
$h+2 \neq 0$
$\boxed{h \neq 2}$

$\frac{h(h+2)}{\cancel{(h-1)(h+2)}} = \frac{(h-1)^2}{\cancel{(h-1)(h+2)}}$

$\cancel{h^2} + 2h = \cancel{h^2} + 1 - 2h$

$4h = 1 \Rightarrow \boxed{h = \frac{1}{4}} \Longrightarrow r \parallel s$

$h \in \mathbb{R} \Rightarrow r \perp s$ $\quad m_r = -\frac{1}{m_s}$

$-\frac{h}{h-1} = \frac{h+2}{h-1}$

C.E.
$h-1 \neq 0$
$h \neq 1$

$-h = h + 2$

$-2h = 2 \Rightarrow h = -\frac{\cancel{2}}{\cancel{2}} \Rightarrow \boxed{h = -1}$

$h = 1$ $\quad r: \boxed{x = -3}$

$s: 3y - 2 = 0 \Rightarrow \boxed{y = \frac{2}{3}}$ $\quad r \perp s$

CASO PARTICOLARE!