Lezione 17

Riepilogo risolutivo equazioni goniometriche

$$\frac{5}{2} = 2{,}5$$

$$2\cos x - 5 = 0 \Longrightarrow \frac{\cancel{2}}{\cancel{2}}\cos x = \frac{5}{2} \Longrightarrow \cos x = \frac{5}{2} \qquad 2{,}5 > 1$$

$$x = \arccos\left(\frac{5}{2}\right)$$

Equazione elementare in coseno

$$-1 \leq \cos x \leq 1$$

impossibile

$$\operatorname{sen}^2 x + (1-\sqrt{3}) \operatorname{sen}x \cos x - \sqrt{3}\cos^2 x = 0$$

Equazione omogenea di 2° grado in seno e coseno

DIVIDO TUTTO PER $\cos^2 x$

$\cos x \neq 0$

$x \neq \frac{\pi}{2} + k\pi \quad k \in \mathbb{Z}$

$\operatorname{tg}x = \frac{\operatorname{sen}x}{\cos x}$

$$\operatorname{tg}^2 x + (1-\sqrt{3})\operatorname{tg}x - \sqrt{3} = 0$$

$$\Delta = b^2 - 4ac = (1-\sqrt{3})^2 - 4(1)(-\sqrt{3}) =$$

$$= 1 + 3 - 2\sqrt{3} + 4\sqrt{3} = 4 + 2\sqrt{3}$$

$$(\operatorname{tg}x)_1, (\operatorname{tg}x)_2 = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{\sqrt{3} - 1 \pm \sqrt{4+2\sqrt{3}}}{2}$$

$$(\operatorname{tg}x)_1;\ (\operatorname{tg}x)_2 = \frac{\sqrt{3}-1 \pm \sqrt{4+2\sqrt{3}}}{2}$$

$$\sqrt{4+2\sqrt{3}} = \boxed{\sqrt{4+\sqrt{12}}} \qquad \sqrt{a+\sqrt{b}} = \sqrt{\frac{a+\sqrt{a^2-b}}{2}} + \sqrt{\frac{a-\sqrt{a^2-b}}{2}}$$

$$\sqrt{4+\sqrt{12}} = \sqrt{\frac{4+\sqrt{16-12}}{2}} + \sqrt{\frac{4-\sqrt{16-12}}{2}} = \sqrt{\frac{4+2}{2}} + \sqrt{\frac{4-2}{2}} =$$

$$= \sqrt{\frac{\cancel{6}^{3}}{\cancel{2}_{1}}} + \sqrt{\frac{\cancel{2}^{1}}{\cancel{2}_{1}}} = \underline{\sqrt{3}+1}$$

$$(tg x)_1 ; (tg x)_2 = \frac{\sqrt{3}-1 \pm (\sqrt{3}+1)}{2}$$

$$\frac{\cancel{\sqrt{3}}-1-\cancel{\sqrt{3}}-1}{2} = -\frac{\cancel{2}}{\cancel{2}} = -1$$

$$\frac{\sqrt{3}-\cancel{1}+\sqrt{3}+\cancel{1}}{2} = \frac{\cancel{2}\sqrt{3}}{\cancel{2}} = \sqrt{3}$$

$(tg x)_1 = -1$

$180° - 45° = 135°$

$45°$

$\frac{\cancel{135}}{\cancel{180}}$ 45 3 / 60 4

$x = arctg(-1) = \frac{3}{4}\pi + k\pi \quad k \in \mathbb{Z}$

$$\boxed{x = \frac{3}{4}\pi + k\pi,\ k \in \mathbb{Z}}$$

$-\frac{\pi}{4}$

$(tg x)_2 = \sqrt{3}$

$x = arctg(\sqrt{3})$

$$\boxed{x = \frac{\pi}{3} + k\pi,\ k \in \mathbb{Z}}$$

$$3\,\text{sen}^2 x + 7\,\text{sen}\, x = 0$$

Equazione di 2° grado solo in seno e quindi riconducibile ad equazione elementare

$\text{sen}\, x = t$

$$3t^2 + 7t = 0 \Rightarrow t(3t+7) = 0$$

$$\boxed{t = 0} \qquad 3t + 7 = 0 \qquad \boxed{t = -\frac{7}{3}}$$

$$\text{sen}\, x = 0 \Rightarrow x = \text{arcsen}\, 0^\circ \Rightarrow \boxed{x = k\pi,\ k \in \mathbb{Z}}$$

$$\text{sen}\, x = -\frac{7}{3} \Rightarrow \text{IMPOSSIBILE}$$

$\cos x = -\cos 2x$

$\boxed{\cos 2x = 2\cos^2 x - 1}$

Equazione lineare in coseno, ma con angoli differenti, con l'utilizzo della formula goniometrica di duplicazione del coseno.

$\frac{\pi}{3} + \frac{2}{3}k\pi$ $\quad k=1$

$\cos x = t$

$\cos x = -(2\cos^2 x - 1)$

$\cos x = -2\cos^2 x + 1 \Rightarrow 2\cos^2 x + \cos x - 1 = 0$

$2t^2 + t - 1 = 0$

$\Delta = 1 - 4(2)(-1) = 1 + 8 = 9$

$t_1, t_2 = \frac{-1 \pm \sqrt{9}}{4} = \frac{-1 \pm 3}{4}$ $\quad \frac{\cancel{2}}{\cancel{4}}\frac{1}{2}$ $\quad -\frac{\cancel{4}}{\cancel{4}}1$

$t_1 = -1 \quad t_2 = \frac{1}{2}$

$\cos x = -1 \Rightarrow x = \pi + 2k\pi$

$\cos x = \frac{1}{2} \Rightarrow x = \pm\frac{\pi}{3} + 2k\pi$

$$\sqrt{3}\,\text{sen}\,x + \cos x = 1$$

$$\boxed{\begin{array}{l} \text{sen}\,x = \dfrac{2t}{1+t^2} \\ \cos x = \dfrac{1-t^2}{1+t^2} \\ t = tg\,\dfrac{x}{2} \\ x \neq \pi + 2k\pi \end{array}}$$

Equazione lineare in seno e coseno

$$\sqrt{3}\cdot\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} = 1$$

$$\frac{2\sqrt{3}\,t + 1 - t^2}{\cancel{1+t^2}} = \frac{1+t^2}{\cancel{1+t^2}}$$

$$2\sqrt{3}\,t + \cancel{1} - t^2 = \cancel{1} + t^2$$

$$-2t^2 + 2\sqrt{3}\,t = 0 \Rightarrow 2t(-t+\sqrt{3}) = 0$$

$$\boxed{t = 0} \qquad -t+\sqrt{3} = 0 \quad \boxed{t = \sqrt{3}}$$

$$tg\,\frac{x}{2} = 0 \Rightarrow \frac{x}{2} = k\pi \Rightarrow \boxed{x = 2k\pi}$$

$$tg\,\frac{x}{2} = \sqrt{3} \Rightarrow \frac{x}{2} = \frac{\pi}{3} + k\pi \Rightarrow \boxed{x = \frac{2}{3}\pi + 2k\pi}$$

$$\sqrt{3}\,\mathrm{sen}x\cos x - \cos^2 x = 0$$

Equazione di 2° grado, omogenea, in seno e coseno

DIVIDO PER $\cos^2 x$

$\cos x \neq 0$

$x \neq \frac{\pi}{2} + k\pi$

$\frac{\mathrm{sen}x}{\cos x} = \mathrm{tg}x$

$$\sqrt{3}\,\mathrm{tg}x - 1 = 0$$

$$\frac{\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}}\,\mathrm{tg}x = \frac{1}{\sqrt{3}}$$

$$\mathrm{tg}x = \frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\boxed{\mathrm{tg}x = \frac{\sqrt{3}}{3}}$$

$$x = \mathrm{arctg}\frac{\sqrt{3}}{3}$$

$$\boxed{x = \frac{\pi}{6} + k\pi}$$

$$\cos x + \sqrt{3}\,\text{sen}\,x = -1$$

Equazione lineare in seno e coseno

$$\frac{1-t^2}{1+t^2} + \sqrt{3}\cdot\frac{2t}{1+t^2} = -1$$

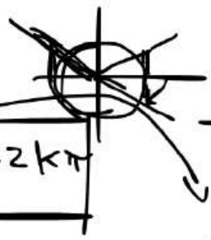
$$\frac{1-t^2+2\sqrt{3}\,t}{\cancel{1+t^2}} = \frac{-1-t^2}{\cancel{1+t^2}}$$

$$1-\cancel{t^2}+2\sqrt{3}\,t = -1-\cancel{t^2}$$

$$\cancel{2}\sqrt{3}\,t = -\cancel{2} \Rightarrow \sqrt{3}\,t = -1 \qquad t = -\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$\boxed{\begin{gathered}\text{sen}\,x = \frac{2t}{1+t^2} \\ \cos x = \frac{1-t^2}{1+t^2} \\ t = \text{tg}\,\frac{x}{2} \\ x \neq \pi + 2k\pi\end{gathered}}$$

$t = -\frac{\sqrt{3}}{3}$ $\qquad tg\frac{x}{2} = -\frac{\sqrt{3}}{3}$



$180° - 30°$
$= 150°$

$\frac{150}{180} = \frac{5}{6}$

$\frac{x}{2} = arctg\left(-\frac{\sqrt{3}}{3}\right)$

$\frac{x}{2} = \frac{5}{6}\pi + k\pi \Rightarrow \boxed{x = \frac{5}{3}\pi + 2k\pi}$

300

$-\frac{\pi}{6}$

$$5\,sen^2 x - 3\,sen x \cos x - 2\cos^2 x = 0$$

$$\boxed{\begin{array}{c}\text{DIVIDO PER}\\ \cos^2 x\\ \cos x \neq 0\\ x \neq \frac{\pi}{2} + k\pi\end{array}}$$

Equazione di secondo grado omogenea in seno e coseno

$$5\,tg^2 x - 3\,tg x - 2 = 0$$

$$\Delta = 9 - 4(5)(-2) = 9 + 40 = 49$$

$$(tg x)_1 ; (tg x)_2 = \frac{3 \pm \sqrt{49}}{10}$$

$$\frac{3-7}{10} = -\frac{\cancel{4}^{\,2}}{\cancel{10}_{\,5}}$$

$$\frac{3+7}{10} = \frac{\cancel{10}}{\cancel{10}} = 1$$

$$\boxed{\frac{sen x}{\cos x} = tg x}$$

$$tg x = -\frac{2}{5} \Rightarrow x = arctg\left(-\frac{2}{5}\right) = -arctg\left(\frac{2}{5}\right)$$

$$\boxed{x = -arctg\left(\frac{2}{5}\right) + k\pi}$$

$$tg x = 1 \Rightarrow \boxed{x = \frac{\pi}{4} + k\pi}$$

$\sin^2 x + \cos 2x = 1$ $\boxed{\cos 2x = 1 - 2\sin^2 x}$

$\sin^2 x + \cancel{1} - 2\sin^2 x = \cancel{1}$

$-\sin^2 x = 0 \Rightarrow \sin^2 x = 0$

$\sin x = 0 \Longrightarrow \boxed{x = k\pi}$

$$\underbrace{sen(3x)}_{\alpha} = \underbrace{sen\left(\frac{2\pi}{3}+x\right)}_{\beta}$$

$\alpha = 3x$

$\beta = \frac{2\pi}{3} + x$

Due seni sono uguali quando i due angoli sono uguali o supplementari.

$$\alpha = \beta$$

$$3x = \frac{2\pi}{3} + x$$

$$\cancel{2}x = \frac{\cancel{2}\pi}{3}$$

$$\boxed{x = \frac{\pi}{3} + 2k\pi}$$

$$\beta = \pi - \alpha$$

$$\frac{2\pi}{3} + x = \pi - 3x$$

$$4x = \pi - \frac{2}{3}\pi$$

$$4x = \frac{3\pi - 2\pi}{3}$$

$$4x = \frac{\pi}{3}$$

$$\boxed{x = \frac{\pi}{12} + 2k\pi}$$

$$\operatorname{sen} 3x = \operatorname{sen}\left(\frac{2}{3}\pi + x\right) \qquad \operatorname{sen} p - \operatorname{sen} q = 0$$

$$\operatorname{sen} 3x - \operatorname{sen}\left(\frac{2}{3}\pi + x\right) = 0$$

$$\boxed{\begin{array}{l} p = 3x \\ q = \frac{2}{3}\pi + x \end{array}}$$

$$\operatorname{sen} p - \operatorname{sen} q = 2 \operatorname{sen} \frac{p+q}{2} \cos \frac{p-q}{2}$$

$$2 \operatorname{sen}\left(\frac{3x + x + \frac{2}{3}\pi}{2}\right) \cos\left(\frac{3x - x - \frac{2}{3}\pi}{2}\right) = 0 \qquad \frac{\frac{12x + 2\pi}{3}}{2}$$

$$2 \operatorname{sen}\left(\frac{4x + \frac{2}{3}\pi}{2}\right) \cos\left(\frac{2x - \frac{2\pi}{3}}{2}\right) = 0 \qquad \frac{\frac{6x - 2\pi}{3}}{2}$$

$$2\cos\left(\frac{12x+2\pi}{6}\right)\operatorname{sen}\left(\frac{6x-2\pi}{6}\right)=0$$

$$\frac{\pi}{2}-\frac{\pi}{3}$$

$$\frac{3\pi-2\pi}{6}$$

$$2\cos\left(2x+\frac{\pi}{3}\right)\operatorname{sen}\left(x-\frac{\pi}{3}\right)=0$$

$$2x+\frac{\pi}{3}=\frac{\pi}{2}+k\pi$$

$$\cos\left(2x+\frac{\pi}{3}\right)=0 \qquad 2x=\frac{\pi}{6}+k\pi \qquad \boxed{x=\frac{\pi}{12}+k\frac{\pi}{2}}$$

$$\operatorname{sen}\left(x-\frac{\pi}{3}\right)=0 \qquad x-\frac{\pi}{3}=k\pi \qquad \boxed{x=\frac{\pi}{3}+k\pi}$$