$$\boxed{\frac{\operatorname{sen} x}{\cos x} = \operatorname{tg} x}$$

Lezione 18

Equazioni goniometriche in seno e coseno di 2° grado omogenee

$$\boxed{\begin{array}{l}\cos x \neq 0 \\ x \neq \frac{\pi}{2} + k\pi\end{array}}$$

$$\sqrt{3}\,\frac{\operatorname{sen}^2 x}{\cos^2 x} - 2\,\frac{\operatorname{sen} x \cancel{\cos x}}{\cos^2 x} - \sqrt{3}\,\frac{\cancel{\cos^2 x}}{\cancel{\cos^2 x}} = \frac{0}{\cos^2 x}$$

$$\boxed{\sqrt{3}\,\operatorname{tg}^2 x - 2\operatorname{tg} x - \sqrt{3} = 0}$$

$$\Delta = b^2 - 4ac = (-2)^2 - 4(\sqrt{3})(-\sqrt{3})$$

$$\Delta = 4 + 12 = 16$$

$$(\operatorname{tg} x)_1 ; (\operatorname{tg} x)_2 = -\frac{b \pm \sqrt{\Delta}}{2a} = \frac{2 \pm 4}{2\sqrt{3}}$$

$$\rightarrow -\frac{\cancel{2}}{\cancel{2}\sqrt{3}} = -\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

$$\rightarrow \frac{\cancel{6}\,3}{1\cancel{2}\sqrt{3}} = \frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\cancel{3}\sqrt{3}}{\cancel{3}} = \sqrt{3}$$

$$(\operatorname{tg} x)_1 = -\frac{\sqrt{3}}{3} \qquad (\operatorname{tg} x)_2 = \sqrt{3}$$

$$150° = \frac{5\pi}{6}$$

$$\frac{150°}{180} = \frac{\cancel{15}\,5}{\cancel{18}\,6}$$

$$x_1 = \operatorname{arctg}\left(-\frac{\sqrt{3}}{3}\right)$$

$$\boxed{x_1 = \frac{5}{6}\pi + k\pi}$$

ANCHE $-\frac{\pi}{6}$ FA PARTE

$$x_2 = \operatorname{arctg}(\sqrt{3})$$

$$\boxed{x_2 = \frac{\pi}{3} + k\pi}$$

$$3\,\mathrm{sen}^2x - 8\sqrt{3}\,\mathrm{sen}x\cos x + 15\cos^2 x = 0$$

$$3\,\mathrm{tg}^2x - 8\sqrt{3}\,\mathrm{tg}x + 15 = 0 \qquad \cos x \neq 0,\quad x \neq \frac{\pi}{2} + k\pi$$

$$\Delta = b^2 - 4ac = (-8\sqrt{3})^2 - 4(3)(15) =$$

$$= 192 - 180 = 12$$

$$(\mathrm{tg}x)_1 ; (\mathrm{tg}x)_2 = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{8\sqrt{3} \pm \sqrt{12}}{6} = \frac{8\sqrt{3} \pm 2\sqrt{3}}{6}$$

$$\begin{array}{r|l} 12 & 2 \\ 6 & 2 \\ 3 & 3 \\ 1 & \end{array} \qquad \sqrt{12} = \sqrt{2^2 \cdot 3} = 2\sqrt{3}$$

$$(\mathrm{tg}x)_1 = \frac{\cancel{6}\sqrt{3}}{\cancel{6}} \qquad (\mathrm{tg}x)_2 = \frac{\overset{5}{\cancel{10}}\sqrt{3}}{\cancel{6}\,3}$$

$$(\mathrm{tg}x)_1 = \sqrt{3} \qquad x_1 = \mathrm{arctg}\sqrt{3} = \frac{\pi}{3} + k\pi$$

$$(\mathrm{tg}x)_2 = \frac{5\sqrt{3}}{3} \qquad x_2 = \mathrm{arctg}\,\frac{5\sqrt{3}}{3} + k\pi$$

$$3\frac{\operatorname{sen}^4 x}{\cancel{\cos^4 x}} - \frac{4\operatorname{sen}^2 x\,\cancel{\cos^2 x}}{\cos^4 x} + \frac{\cancel{\cos^4 x}}{\cancel{\cos^4 x}} = 0$$

$$\boxed{\cos x \neq \frac{\pi}{2} + k\pi}$$

$$\boxed{tg\,x = \frac{\operatorname{sen} x}{\cos x}}$$

$$\boxed{tg^2 x = t}$$

$$3\,tg^4 x - 4\,tg^2 x + 1 = 0$$

$$3t^2 - 4t + 1 = 0$$

$$\Delta = b^2 - 4ac = (-4)^2 - 4(3)(1) = 16 - 12 = 4$$

$$t_1, t_2 = -\frac{b \pm \sqrt{\Delta}}{2a} = \frac{4 \pm 2}{6}$$

$$\frac{\cancel{2}}{\cancel{6}} = \frac{1}{3}$$

$$\frac{\cancel{6}}{\cancel{6}} = 1$$

$$tg^2 x = \frac{1}{3}$$

$$tg\,x = \pm\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$$

$$\boxed{tg\,x = \pm\frac{\sqrt{3}}{3}}$$

$$tg^2 x = 1$$

$$tg\,x = \pm\sqrt{1} = \pm 1$$

$$\boxed{tg\,x = \pm 1}$$

$$x_1 = \operatorname{arctg}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6} + k\pi$$

$$x_2 = \operatorname{arctg}\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6} + k\pi$$

$$x_3 = \operatorname{arctg}(1) = \frac{\pi}{4} + k\pi$$

$$x_4 = \operatorname{arctg}(-1) = -\frac{\pi}{4} + k\pi$$

$$\boxed{x = \pm\frac{\pi}{6} + k\pi \quad ; \quad x = \pm\frac{\pi}{4} + k\pi}$$

$4\,\text{sen}^2 x \cos^2 x - 4\cos^4 x = 0$

$\boxed{\cos^2 x = 1 - \text{sen}^2 x}$

$\cos^2 x\,(4\,\text{sen}^2 x - 4\cos^2 x) = 0$

$\cos^2 x = 0$

$\cos^2 x = 0$

$\cos x = 0$

$\boxed{x = \frac{\pi}{2} + k\pi}$

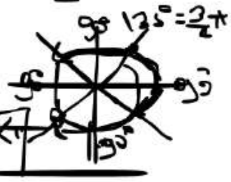
$\cancel{4}(\text{sen}^2 x - \cos^2 x) = \frac{0}{4}$

$\text{sen}^2 x - (1 - \text{sen}^2 x) = 0$

$2\,\text{sen}^2 x - 1 = 0 \Rightarrow 2\,\text{sen}^2 x = 1$

$\text{sen}^2 x = \frac{1}{2} \Longrightarrow \text{sen}\, x = \pm\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}$

$\text{sen}\, x = \pm\frac{\sqrt{2}}{2}$



$315 + (-1)\,5$

$= 1\,30$

$x = \text{arc sen}\left(\frac{\sqrt{2}}{2}\right)$

$\boxed{x = \frac{\pi}{4} + 2k\pi \qquad x = \frac{3}{4}\pi + 2k\pi}$

$x = \text{arc sen}\left(-\frac{\sqrt{2}}{2}\right)$

$\boxed{x = -\frac{\pi}{4} + 2k\pi \qquad x = -\frac{3}{4}\pi + 2k\pi}$

$x = \frac{\pi}{4} + k\frac{\pi}{2}$

$$\text{sen}\,x + \cos x + 2\,\text{sen}\,x\cos x + 1 = 0$$

$\boxed{x = 45° + z}$ CAMBIAMENTO DI VARIABILE

$$\text{sen}(45° + z) + \cos(45° + z) + 2\,\text{sen}(45° + z)\cos(45° + z) + 1 = 0$$

$$\text{sen}\,45° \cos z + \text{sen}\,z \cos 45° + \cos 45° \cos z - \text{sen}\,45° \,\text{sen}\,z + 2\left[\text{sen}\,45° \cos z + \text{sen}\,z \cos 45°\right]\left[\cos 45° \cos z - \text{sen}\,45° \,\text{sen}\,z\right]$$

$$\frac{\sqrt{2}}{2}\cos z + \cancel{\frac{\sqrt{2}}{2}\,\text{sen}\,z} + \frac{\sqrt{2}}{2}\cos z - \cancel{\frac{\sqrt{2}}{2}\,\text{sen}\,z} + 2\left[\frac{\sqrt{2}}{2}\cos z + \frac{\sqrt{2}}{2}\,\text{sen}\,z\right]\left[\frac{\sqrt{2}}{2}\cos z - \frac{\sqrt{2}}{2}\,\text{sen}\,z\right] + 1 = 0$$

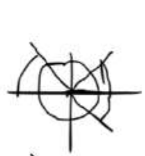
$$\sqrt{2}\cos z + 2\left[\frac{1}{2}\cos^2 z - \frac{1}{2}\,\text{sen}^2 z\right] + 1 = 0 \qquad \boxed{\text{sen}^2 z = 1 - \cos^2 z}$$

$$\sqrt{2}\cos z + \cos^2 z - \text{sen}^2 z + 1 = 0$$

$$\sqrt{2}\cos z + \cos^2 z \cancel{-1} + \cos^2 z \cancel{+1} = 0$$

$$2\cos^2 z + \sqrt{2}\cos z = 0$$

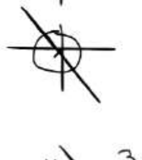
$$\cos z\left(2\cos z + \sqrt{2}\right) = 0$$



$$\boxed{\cos z = 0} \qquad \boxed{\cos z = -\frac{\sqrt{2}}{2}}$$

$$\boxed{z = \frac{\pi}{2} + 2k\pi} \qquad z = \arccos\left(-\frac{\sqrt{2}}{2}\right)$$

$$\boxed{z = \frac{3}{4}\pi + 2k\pi}$$



$$x = \frac{\pi}{4} + z = \frac{\pi}{4} + \frac{\pi}{2} + 2k\pi = \frac{\pi + 2\pi}{4} + 2k\pi = \frac{3}{4}\pi + 2k\pi$$

$$x = \frac{\pi}{4} + z = \frac{\pi}{4} + \frac{3}{4}\pi + 2k = \pi + 2k\pi$$