

Lezione 19

Espressioni con i prodotti notevoli

$$\begin{aligned}
 & \left(a + \frac{1}{3}x\right)^2 - \left(a - \frac{1}{2}x\right)\left(a + \frac{1}{2}x\right) + \frac{11}{36}x^2 = \\
 & = a^2 + 2a \cdot \frac{1}{3}x + \frac{1}{9}x^2 - \left(a^2 - \frac{1}{4}x^2\right) + \frac{11}{36}x^2 = \\
 & = \cancel{a^2} + \frac{2}{3}ax + \frac{1}{9}x^2 - \cancel{a^2} + \frac{1}{4}x^2 + \frac{11}{36}x^2 = \\
 & = \left(\frac{1}{9} + \frac{1}{4} + \frac{11}{36}\right)x^2 + \frac{2}{3}ax = \\
 & = \left(\frac{4 + 9 + 11}{36}\right)x^2 + \frac{2}{3}ax = \\
 & = \frac{24}{36}x^2 + \frac{2}{3}ax = \boxed{\frac{2}{3}x^2 + \frac{2}{3}ax}
 \end{aligned}$$

$$\boxed{2 \cdot 2x \cdot (-5) = -20x}$$

$$(2x-3)^3 + 2x(2x-5)^2 - 4(2x-1)^2(x-3) + 15 =$$

$$= \left[\underbrace{(2x)^3 + 3 \cdot (2x)^2 \cdot (-3)}_{\text{a}^3 + 3\text{a}^2\text{b}} + \underbrace{3 \cdot (2x) \cdot (-3)^2 + (-3)^3}_{\text{3a}^2\text{b} + \text{b}^3} \right] + 2x(4x^2 - 20x + 25) - 4(4x^2 - 4x + 1)(x-3) + 15 =$$

$$= 8x^3 - 36x^2 + 54x - 27 + 8x^3 - 40x^2 + 50x - 4(4x^3 - 12x^2 - 4x^2 + 12x + x - 3) + 15 =$$

$$= \cancel{8x^3} - 36x^2 + 54x - 27 + \cancel{8x^3} - 40x^2 + 50x - 16x^3 + 48x^2 + 16x^2 - 48x + 12 + 15 =$$

$$= (-36 - 40 + 48 + 16)x^2 + (104 - 52)x =$$

$$= -12x^2 + 52x$$

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$$\frac{76}{48} = -28$$

$$\begin{aligned}
&= \frac{\left(\frac{1}{3}x - y\right)^2 \left(\frac{1}{3}x + y\right) - \left(\frac{1}{3}x + y\right)^3}{\left(\frac{1}{3}x - y\right)\left(\frac{1}{3}x - y\right)\left(\frac{1}{3}x + y\right)} \\
&= \left[\left(\frac{1}{3}x\right)^2 + 2\left(\frac{1}{3}x\right)(-y) + (-y)^2 \right] \left(\frac{1}{3}x + y\right) - \left[\left(\frac{1}{3}x\right)^3 + 3\left(\frac{1}{3}x\right)^2 y + 3\left(\frac{1}{3}x\right)y^2 + y^3 \right] \\
&= \left(\frac{1}{9}x^2 - \frac{2}{3}xy + y^2 \right) \left(\frac{1}{3}x + y \right) - \left(\frac{1}{27}x^3 + \frac{1}{3}x^2y + xy^2 + y^3 \right) \\
&= \frac{1}{27}x^3 + \frac{1}{9}x^2y - \frac{2}{9}x^2y - \frac{2}{3}xy^2 + \frac{1}{3}xy^2 + y^3 - \frac{1}{27}x^3 - \frac{1}{3}x^2y - xy^2 - y^3 \\
&= \left(\frac{1}{9} - \frac{2}{9} - \frac{1}{3} \right) x^2y + \left(-\frac{2}{3} + \frac{1}{3} - 1 \right) xy^2 = \\
&= \left(\frac{1 - 2 - 3}{9} \right) x^2y + \left(\frac{-2 + 1 - 3}{3} \right) xy^2 = \\
&= \boxed{-\frac{4}{9}x^2y - \frac{4}{3}xy^2}
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{3} (x+b)^2 - \frac{4}{9} b^2 + \left(\frac{1}{6} x + \frac{1}{3} b \right) \left(\frac{1}{3} b - \frac{1}{6} x \right) = \\
& = \frac{1}{3} (x^2 + 2bx + b^2) - \frac{4}{9} b^2 + \frac{1}{9} b^2 - \frac{1}{36} x^2 = \\
& = \frac{1}{3} x^2 + \frac{2}{3} bx + \frac{1}{3} b^2 - \frac{4}{9} b^2 + \frac{1}{9} b^2 - \frac{1}{36} x^2 = \\
& = \left(\frac{1}{3} - \frac{1}{36} \right) x^2 + \frac{2}{3} bx + \left(\frac{1}{3} - \frac{4}{9} + \frac{1}{9} \right) b^2 = \\
& = \left(\frac{12-1}{36} \right) x^2 + \frac{2}{3} bx + \left(\frac{3-4+1}{9} \right) b^2 = \\
& = \frac{11}{36} x^2 + \frac{2}{3} bx
\end{aligned}$$