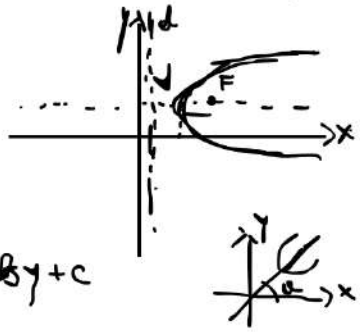
LEZIONE 19

Parabola con asse di simmetria parallelo all'asse x



$x = ay^2 + by + c$

$V\left(-\frac{\Delta}{4a}; -\frac{b}{2a}\right)$

$F\left(\frac{1-\Delta}{4a}; -\frac{b}{2a}\right)$

$a: \; y = -\frac{b}{2a}$

$d: \; x = -\frac{1+\Delta}{4a}$

$y = ax^2 + bx + c$

$F = \left(-\frac{b}{2a} ; \frac{1-\Delta}{4a}\right) = \left(3 ; -\frac{3}{4}\right)$

$F\left(3 ; -\frac{3}{4}\right)$

$P\ (4, 0)$

$\boxed{\Delta = b^2 - 4ac}$

$P(4;0) \Rightarrow \boxed{0 = 16a + 4b + c}$

$$\begin{cases} -\frac{b}{2a} = 3 \\ \frac{1-\Delta}{4a} = -\frac{3}{4} \\ 16a + 4b + c = 0 \end{cases} \Rightarrow \begin{cases} b = -6a \\ 16a + 4(-6a) + c = 0 \\ 16a - 24a + c = 0 \Rightarrow c = 8a \end{cases}$$

$$\frac{1-\Delta}{4a} = \frac{-3a}{4a}$$

$1 - \Delta = -3a \qquad 1 - (b^2 - 4ac) = -3a$

$1 - [36a^2 - 4a(8a)] = -3a$

$1 - (36a^2 - 32a^2) = -3a$

$1 - 4a^2 = -3a$

$-4a^2 + 3a + 1 = 0$

$4a^2 - 3a - 1 = 0$

$\Delta = 9 - 4(4)(-1) = 9 + 16 = 25$

$a = \frac{3 \pm 5}{8}$ → $-\frac{2}{8} = -\frac{1}{4}$; $\frac{8}{8} = 1$

$$\begin{cases} a = 1 \\ b = -6 \\ c = 8 \end{cases} \quad \boxed{y = x^2 - 6x + 8} \quad \checkmark$$

$$\begin{cases} a = -\frac{1}{4} \\ b = -6\left(-\frac{1}{4}\right) = \frac{3}{2} \\ c = 2\left(-\frac{1}{4}\right) = -2 \end{cases}$$

$$\boxed{y = -\frac{1}{4}x^2 + \frac{3}{2}x - 2}$$

$V(1;-2)$ $\quad F\left(1;-\frac{9}{4}\right)$ $\quad \boxed{\Delta = b^2-4ac}$

$V\left(-\frac{b}{2a};\frac{-\Delta}{4a}\right)$ $\quad F\left(-\frac{b}{2a};\frac{1-\Delta}{4a}\right)$

$$\begin{cases} -\frac{b}{2a}=1 \\ -\frac{\Delta}{4a}=-2 \\ \frac{1-\Delta}{4a}=-\frac{9}{4} \end{cases} \qquad \begin{cases} \boxed{b=-2a} \\ \boxed{\Delta=8a} \\ \boxed{1-\Delta=-9a} \end{cases}$$

$\frac{1-\Delta}{4a}=\frac{-9a}{4a}$

$1-8a=-9a$

$\boxed{a=-1}$

$b=-2a=-2(-1)=2$ $\quad \boxed{b=2}$

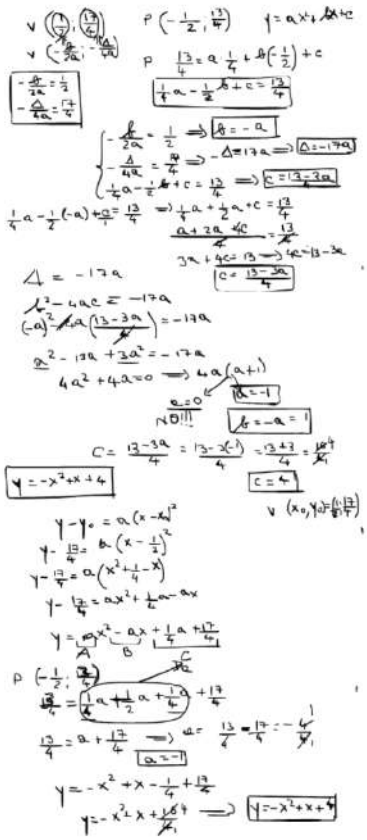
$b^2-4ac=8a$

$2^2-4\cdot(-1)\cdot c=8(-1)$

$4+4c=-8 \Rightarrow 4c=-12 \Rightarrow c=-\frac{12}{4}$

$\boxed{c=-3}$

$\boxed{y=-x^2+2x-3}$



$F(-1;-1)$

$F\left(-\frac{b}{2a};\frac{1-\Delta}{4a}\right)$

$d: y = +1$

$y = -\frac{1+\Delta}{4a}$

$\Delta = b^2 - 4ac$

$$\begin{cases} -\frac{b}{2a} = -1 \\ \frac{1-\Delta}{4a} = -1 \\ -\frac{1+\Delta}{4a} = +1 \end{cases} \Rightarrow \begin{cases} b = 2a \\ 1-\Delta = -4a \\ -1-\Delta = 4a \end{cases}$$

$R_1 \begin{cases} 1-\Delta = -4a \\ 1+\Delta = -4a \end{cases}$

$R_1 + R_2 \quad 2 = -8a$

$-8a = 2$

$a = -\frac{1}{4}$

$b = 2a = 2\left(-\frac{1}{4}\right) = -\frac{1}{2}$

$b = -\frac{1}{2}$

$R_1 - R_2 \quad -2\Delta = 0$

$\Delta = 0$

$b^2 - 4ac = 0$

$\left(-\frac{1}{2}\right)^2 - 4\left(-\frac{1}{4}\right)c = 0$

$\frac{1}{4} + c = 0 \Rightarrow c = -\frac{1}{4}$

$$y = -\frac{1}{4}x^2 - \frac{1}{2}x - \frac{1}{4}$$

$\overline{PF} = d(P,d)$

$P(x_1, y_1)$

$F(-1;-1)$

$d: y = 1$

$y - 1 = 0$

$ax + by + c = 0$

$a = 0 \quad b = 1 \quad c = -1$

$$\sqrt{(x_F - x_P)^2 + (y_F - y_P)^2} = \frac{|ax_P + by_P + c|}{\sqrt{a^2 + b^2}}$$

$$\sqrt{(x+1)^2 + (y+1)^2} = \frac{|1 \cdot y - 1|}{\sqrt{1}}$$

$$\sqrt{(x+1)^2 + (y+1)^2} = (y-1)^2$$

$$(x+1)^2 + (y+1)^2 = y^2 + 1 - 2y$$

$$x^2 + 1 + 2x + y^2 + 1 + 2y = y^2 + 1 - 2y$$

$$4y = -x^2 - 2x - 1$$

$$y = -\frac{1}{4}x^2 - \frac{1}{2}x - \frac{1}{4}$$

$\checkmark \left(\frac{1}{2}; \frac{13}{12}\right)$

$\checkmark \left(-\frac{b}{2a}; -\frac{\Delta}{4a}\right)$

$$\begin{cases} -\frac{b}{2a} = \frac{1}{2} \\ -\frac{\Delta}{4a} = \frac{13}{12} \\ -\frac{1+\Delta}{4a} = \frac{11}{6} \end{cases} \Longrightarrow \begin{cases} b = -a \\ \frac{-3\Delta}{12a} = \frac{13a}{12a} \Rightarrow \boxed{\Delta = -\frac{13}{3}a} \\ -\frac{3(1+\Delta)}{12a} = \frac{22a}{12a} \Rightarrow -3-3\Delta = 22a \\ -3\Delta = 22a+3 \\ \boxed{\Delta = -\frac{22a+3}{3}} \end{cases}$$

$\Delta = -\frac{13}{3}a = -\frac{22a+3}{3}$

$\cancel{3}\ \frac{-13a}{\cancel{3}} = -\frac{22a+3}{\cancel{3}}\ \cancel{3}$

$-13a = -(22a+3)$

$-13a + 22a = -3$

$9a = -3 \Rightarrow a = -\frac{3}{9} = -\frac{1}{3}$

$a = -\frac{1}{3}$

$b = +\frac{1}{3}$

$\Delta = \frac{13}{3}a$

$b^2 - 4ac = -\frac{13}{3}a$

$\frac{1}{9} + 4 \cdot \frac{1}{3}c = +\frac{13}{3} \cdot \frac{1}{3}$

$\frac{1}{9} + \frac{4}{3}c = +\frac{13}{9}$

$\cancel{9}\ \frac{1+12c}{\cancel{9}} = \frac{+13}{\cancel{9}}\ \cancel{9}$

$+12c = 12 \Rightarrow \boxed{c = 1}$

$\begin{cases} a = -\frac{1}{3} \\ b = \frac{1}{3} \\ c = 1 \end{cases}$

$\boxed{y = -\frac{1}{3}x^2 + \frac{1}{3}x + 1}$

$r: x = \frac{9}{28}$ $\quad d: y = \frac{109}{196}$

$r: x = -\frac{b}{2a} \Rightarrow -\frac{b}{2a} = \frac{9}{28} \Rightarrow b = -\frac{9}{28}a$

$d: y = -\frac{1+\Delta}{4a} \Rightarrow -\frac{1+\Delta}{4a} = \frac{109}{196} \Rightarrow \frac{-49(1+\Delta)}{196a} = \frac{109a}{196a}$

$-49 - 49\Delta = 109a$

$49\Delta + 49 = -109a$

$\Delta = -\frac{49 + 109a}{49}$

INDETERM.