$\boxed{(a-b)(a+b)=a^2-b^2}$

Lezione 20

$\boxed{(a-b)^2 = a^2-2ab+b^2}$

$$(x^2-x^4)(x^2+x^4)-(x^2-1)^2+\left[(x+1)(x-1)(x^2+1)+2\right]^2=$$

$$=(x^2)^2-(x^4)^2-\left[(x^2)^2-2x^2+1\right]+\left[(x^2-1)(x^2+1)+2\right]^2=$$

$$=x^4-x^8-\left[x^4-2x^2+1\right]+\left[(x^2)^2-1+2\right]^2=$$

$\boxed{(a+b)^2=a^2+2ab+b^2}$

$$=x^4-x^8-x^4+2x^2-1+(x^4+1)^2=$$

$$=\cancel{x^4}-x^8\cancel{-x^4}+2x^2\cancel{-1}+(x^4)^2+2x^4\cancel{+1}=$$

$$=\cancel{-x^8}+2x^2+\cancel{x^8}+2x^4 = 2x^2+2x^4$$

$$\boxed{(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3}$$

$$\left(\frac{2}{5}a - b^2\right)\left(\frac{2}{5}a + b^2\right) - \left[\left(\frac{1}{2}a + \frac{2}{3}b\right)^3 - \frac{8}{27}b^3 - \frac{1}{6}ab(3a+4b)\right] : \frac{25}{32}a =$$

$$= \frac{4}{25}a^2 - b^4 - \left[\left(\frac{1}{2}a\right)^3 + 3\cdot\left(\frac{1}{2}a\right)^2\cdot\frac{2}{3}b + 3\cdot\frac{1}{2}a\cdot\left(\frac{2}{3}b\right)^2 + \left(\frac{2}{3}b\right)^3 - \frac{8}{27}b^3 - \frac{1}{2}a^2b - \frac{2}{3}ab^2\right] : \frac{25}{32}a =$$

$$= \frac{4}{25}a^2 - b^4 - \left[\frac{1}{8}a^3 + 3\cdot\frac{1}{4}a^2\cdot\frac{2}{3}b + 3\cdot\frac{1}{2}a\cdot\frac{4}{9}b^2 + \cancel{\frac{8}{27}b^3} - \cancel{\frac{8}{27}b^3} - \frac{1}{2}a^2b - \frac{2}{3}ab^2\right] : \frac{25}{32}a =$$

$$= \frac{4}{25}a^2 - b^4 - \left[\frac{1}{8}a^3 + \cancel{\frac{1}{2}a^2b} + \cancel{\frac{2}{3}ab^2} - \cancel{\frac{1}{2}a^2b} - \cancel{\frac{2}{3}ab^2}\right] : \frac{25}{32}a$$

$$= \frac{4}{25}a^2 - b^4 - \frac{1}{8}a^3 : \frac{25}{32}a =$$

$$= \frac{4}{25}a^2 - b^4 - \left(\frac{1}{8}\cdot\frac{32}{25}\right)a^2 = \cancel{\frac{4}{25}a^2} - b^4 - \cancel{\frac{4}{25}a^2} = -b^4$$

$$\boxed{(a+b)(a-b)=a^2-b^2} \qquad (a+b)^2=a^2+2ab+b^2$$

$$\left(3+\frac{4}{3}ab\right)\left(3-\frac{4}{3}ab\right)+a\left(\frac{4}{3}a+\frac{2}{3}b^2\right)^2-(4a+6)\left(\frac{4}{9}a^2+1-\frac{2}{3}a\right)+\frac{5}{9}ab^4=$$

$$=9-\left(\frac{4}{3}ab\right)^2+a\left[\left(\frac{4}{3}a\right)^2+2\cdot\left(\frac{4}{3}a\right)\left(\frac{2}{3}b^2\right)+\left(\frac{2}{3}b^2\right)^2\right]-\left(\frac{16}{9}a^3+4a-\frac{8}{3}a^2+\frac{24}{3}a^2+6-\frac{12}{3}a\right)+\frac{5}{9}ab^4=$$

$$=9-\frac{16}{9}a^2b^2+a\left[\frac{16}{9}a^2+\frac{16}{9}ab^2+\frac{4}{9}b^4\right]-\frac{16}{9}a^3-4a+\frac{8}{3}a^2-\frac{8}{3}a^2-6+4a+\frac{5}{9}ab^4=$$

$$=9-\cancel{\frac{16}{9}a^2b^2}+\cancel{\frac{16}{9}a^3}+\cancel{\frac{16}{9}a^2b^2}+\frac{4}{9}ab^4-\cancel{\frac{16}{9}a^3}-\cancel{4a}+\cancel{\frac{8}{3}a^2}-\cancel{\frac{8}{3}a^2}-6+\cancel{4a}+\frac{5}{9}ab^4$$

$$=3+\left(\frac{4}{9}+\frac{5}{9}\right)ab^4=3+\frac{9}{9}ab^4=\boxed{3+ab^4}$$

$$\left[-9xy(3y^2+1)+(-x^2y-1)^2-(3x+9xy^2)(-3y)-(1+x^2y)^2\right]:x^2=$$

$$=\left[-27xy^3-9xy+(x^4y^2+2x^2y+1)-(-9xy-27xy^3)-(1+2x^2y+x^4y^2)\right]:x^2=$$

$$=\left[\cancel{-27xy^3}\cancel{-9xy}\cancel{+x^4y^2}\cancel{+2x^2y}\cancel{+1}\cancel{+9xy}\cancel{+27xy^3}\cancel{-1}\cancel{-2x^2y}\cancel{-x^4y^2}\right]:x^2=$$

$$=0:x^2=\boxed{0}$$