Lezione 20

Ripresa equazioni esponenziali

$$\frac{1}{4} = \left(\frac{1}{2}\right)^2$$

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$$\left(\frac{1}{4}\right)^{x^2-2x} - \left(\frac{1}{2}\right)^{\frac{2x^2-4x-1}{2}} = 0$$

$$\left(\frac{1}{2}\right)^{2(x^2-2x)} = \left(\frac{1}{2}\right)^{\frac{2x^2-4x-1}{2}}$$

$$\frac{2x^2-4x}{1} = \frac{2x^2-4x-1}{2}$$

$$\cancel{2}\cdot\frac{4x^2-8x}{\cancel{2}} = \frac{2x^2-4x-1}{\cancel{2}}\cancel{2}$$

$$8 = 2^3$$

$$\sqrt{2^3} = \sqrt{2^2\cdot 2} = 2\sqrt{2}$$

$$2x^2-4x+1=0$$

$$\Delta = b^2-4ac = (-4)^2-4(2)(1) = 16-8 = 8$$

$$x_1,x_2 = \frac{-b\pm\sqrt{\Delta}}{2a} = \frac{4\pm\sqrt{8}}{4} = \frac{4\pm2\sqrt{2}}{4} = \frac{\cancel{2}(2\pm\sqrt{2})}{\cancel{4}2}$$

$$\boxed{x_1 = \frac{2-\sqrt{2}}{2};\quad x_2 = \frac{2+\sqrt{2}}{2}}$$

$$\boxed{4 = 2^2}$$

$$\boxed{2^{x+1} = 2^x \cdot 2^1}$$

$$\boxed{2^x = t}$$

$$\frac{4^x + 2^{x+1} + 11}{7} = 2^x + 1$$

$$4^x + 2^{x+1} + 11 = 7 \cdot 2^x + 7$$

$$2^{2x} + 2^x \cdot 2 + 11 = 7 \cdot 2^x + 7$$

$$t^2 + 2t + 11 = 7t + 7$$

$$t^2 - 5t + 4 = 0$$

$$\Delta = b^2 - 4ac = (-5)^2 - 4(1)(4) = 25 - 16 = 9$$

$$t_1, t_2 = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{5 \pm 3}{2} \begin{cases} \frac{\cancel{2}^1}{\cancel{2}_1} = 1 \\ \frac{\cancel{8}^4}{\cancel{2}_1} = 4 \end{cases}$$

$$t_1 = 1 \; ; \; t_2 = 4$$

$$\downarrow \qquad\qquad \downarrow$$

$$2^x = 1 \qquad 2^x = 4$$

$$2^x = 2^0 \qquad 2^x = 2^2$$

$$\boxed{x = 0} \qquad \boxed{x = 2}$$

$$4^{x+8} = \frac{1}{4^{2x-5}} \Rightarrow 4^{x+8} = \left(\frac{1}{4}\right)^{2x-5} \qquad \boxed{\frac{1}{4} = 4^{-1}}$$

$$\underline{4^{x+8}} = \underline{4^{-1(2x-5)}}$$

$$x + 8 = -2x + 5$$

$$3x = -3 \Rightarrow x = -\frac{\cancel{3}}{\cancel{3}} = -1 \Rightarrow \boxed{x = -1}$$

$$\frac{5^{2x}-125}{5^x-1}=0$$

$5^x=t$

<u>C.E.</u> $t-1\neq 0$

$t\neq 1$

$5^x\neq 1 \Rightarrow 5^x\neq 5^0$

$\boxed{x\neq 0}$

$$\cancel{(t-1)}\ \frac{t^2-125}{\cancel{t-1}}=0\cdot(t-1)$$

$t^2-125=0 \Rightarrow t^2=125$

$t=\pm\sqrt{125}=\pm\sqrt{5^2\cdot 5}=\pm 5\sqrt{5}$

$\boxed{t=\pm 5\sqrt{5}}$ $\quad t_1=-5\sqrt{5}$ No $\quad \boxed{t_2=5\sqrt{5}}$

$5^x=5\sqrt{5}$ $\qquad \sqrt[2]{5^1}=5^{\frac{1}{2}}$

$1+\frac{1}{2}=\frac{2+1}{2}=\frac{3}{2}$

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$5^x=5^1\cdot 5^{\frac{1}{2}}$

$5^x=5^{1+\frac{1}{2}} \Rightarrow 5^x=5^{\frac{3}{2}} \Rightarrow \boxed{x=\frac{3}{2}}$

Proprietà dei logaritmi

1)Logaritmo del prodotto

$$\boxed{\log_a(m \cdot n) = \log_a m + \log_a n}$$

$\log_2 8 = 3$

$2^x = 8$

$a \in \mathbb{R}^+$ $\quad m \in \mathbb{R}^+$

$a \neq 1$ $\quad n \in \mathbb{R}^+$

DIM

$\log_a m = x \qquad \log_a n = y$

$\Downarrow \qquad \Downarrow$

$a^x = m \qquad a^y = n$

$$m \cdot n = a^x \cdot a^y = a^{x+y} \Rightarrow \boxed{m \cdot n = a^{x+y}}$$

$$x + y = \log_a(m \cdot n)$$

$$\boxed{\log_a m + \log_a n = \log_a(m \cdot n)}$$ c.v.d

ES

$\log_2 4 \cdot 2 = \log_2 8 = 3$ (SENZA PROPRIETÀ)

$\log_2 4 + \log_2 2 = 2 + 1 = 3$ (CON PROPRIETÀ)

2)Logaritmo del rapporto

$$\boxed{\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n}$$

$a \in \mathbb{R}^+,\ a \neq 1$
$m \in \mathbb{R}^+;\ n \in \mathbb{R}^+$

<u>Dim</u>

$\log_a m = x \quad\quad \log_a n = y$

$\downarrow \quad\quad\quad\quad \downarrow$

$a^x = m \quad\quad a^y = n$

$$\frac{m}{n} = \frac{a^x}{a^y} = a^{x-y} \Rightarrow \boxed{\frac{m}{n} = a^{x-y}}$$

$$\boxed{x - y = \log_a \frac{m}{n}}$$

$$\boxed{\log_a m - \log_a n = \log_a \frac{m}{n}}$$ <u>C.V.d</u>

<u>Es</u> $\log_2 \frac{8}{2} = \log_2 4 = \underline{2}$ (SENZA PROPRIETÀ)

$\log_2 8 - \log_2 2 = 3 - 1 = \underline{2}$ (CON LA PROPRIETÀ)

Il logaritmo di un argomento elevato a potenza

$$\boxed{\log_a b^m = m \log_a b} \qquad a \in \mathbb{R}^+ \setminus \{1\} \quad m \in \mathbb{R};\ b \in \mathbb{R}^+$$

DIM

$$x = \log_a b$$

$$\downarrow$$

$$a^x = b \Longrightarrow \boxed{b = a^x}$$

ELEVO TUTTO A m

$$b^m = (a^x)^m \Longrightarrow b^m = a^{mx}$$

$$\Downarrow$$

$$mx = \log_a b^m$$

$$\downarrow$$

$$\boxed{m \log_a b = \log_a b^m}$$

C.V.D.

ES $\log_2 4^2 = \log_2 16 = 4$ (SENZA PROPRIETÀ)

$\log_2 4^2 = 2 \log_2 4 = 2 \cdot 2 = 4$ (CON LA PROPRIETÀ)

$\boxed{m = \frac{1}{n}}$ $\quad \log_a b^m = \log_a b^{\frac{1}{n}} = \boxed{\log_a \sqrt[n]{b} = \frac{1}{n} \log_a b}$

$\boxed{b^{\frac{1}{n}} = \sqrt[n]{b}}$ $\quad \boxed{\log_a \sqrt[n]{b} = \frac{1}{n} \log_a b}$

ES $\quad \boxed{\log_2 \sqrt[2]{3} = \log_2 3^{\frac{1}{2}} = \frac{1}{2} \log_2 3}$ ✓

$$\boxed{a^{\log_a b} = b}$$

$a \in \mathbb{R}^+ \setminus \{1\}$ $\quad (\sqrt{x})^2 = x$
$b > 0$

Proprietà del cambiamento di base

$$\boxed{\log_a b = \log_a c \cdot \log_c b}$$

$a, c \in \mathbb{R}^+ \setminus \{1\}$
$b \in \mathbb{R}^+ \setminus \{0\}$

<u>Dim</u> $\quad x = \log_a c \qquad y = \log_c b$

$\boxed{a^x = c} \qquad c^y = b$

ELEVATO A Y TUTTA

$$(a^x)^y = c^y \Longrightarrow a^{xy} = c^y = b \Longrightarrow a^{xy} = b$$

$$xy = \log_a b$$

$$\boxed{\log_a c \cdot \log_c b = \log_a b} \quad \underline{\text{c.v.d}}$$

se $a = b \qquad \log_a c \cdot \log_c a = \log_a a = 1$

$$\log_a c \cdot \log_c a = 1$$

$$\boxed{\log_a c = \frac{1}{\log_c a}}$$

$\sqrt[3]{8} = 2$

$8^{\frac{1}{3}} = 2$

ES. $\quad \boxed{\log_2 8 = \frac{1}{\log_8 2}} \quad \boxed{3 = \frac{1}{\frac{1}{3}}}$

a NUOVA BASE $\quad c$ VECCHIA BASE

b ARGOMENTO

LOGARITMO NELLA NUOVA BASE a $\quad \boxed{\log_a b} = \log_a c \cdot \boxed{\log_c b}$ $\quad$ LOGARITMO NELLA VECCHIA BASE C

FORMULA DEL CAMBIAMENTO DI BASE

$$\boxed{\log_c b = \frac{\log_a b}{\log_a c}}$$

LOGARITMO NELLA NUOVA BASE a DI ARGOMENTO LA VECCHIA BASE C

$$\boxed{\log_3 8 = \frac{\log_2 8}{\log_2 3} = \frac{3}{\log_2 3} = 3\log_3 2} \quad \frac{1}{\log_2 3} = \log_3 2$$

$\boxed{c = 3 \quad b = 8}$

$$\boxed{\log_3 \sqrt{3\sqrt{3}}} = \log_3 (3\sqrt{3})^{\frac{1}{2}} = \frac{1}{2}\log_3(3\sqrt{3}) =$$

$$= \frac{1}{2}\left[\log_3 3 + \log_3 \sqrt{3}\right] =$$

$$= \frac{1}{2}\left[1 + \log_3 3^{\frac{1}{2}}\right] =$$

$$= \frac{1}{2}\left[1 + \frac{1}{2}\left(\log_3 3\right)\right] = \frac{1}{2}\left[1 + \frac{1}{2}\right] = \frac{1}{2}\left[\frac{2+1}{2}\right] = \frac{1}{2}\cdot\frac{3}{2} = \boxed{\frac{3}{4}}$$

$$\boxed{\log_2 \left(4\sqrt[3]{2\sqrt{2}}\right)} =$$

$$= \log_2 4 + \log_2 \sqrt[3]{2\sqrt{2}}$$

$$= 2 + \log_2 (2\sqrt{2})^{\frac{1}{3}} =$$

$$= 2 + \frac{1}{3}\log_2(2\sqrt{2}) = 2 + \frac{1}{3}\left[\log_2 2 + \log_2 \sqrt{2}\right] =$$

$$= 2 + \frac{1}{3}\left[1 + \log_2 2^{\frac{1}{2}}\right] = 2 + \frac{1}{3}\left[1 + \frac{1}{2}\log_2 2\right] =$$

$$= 2 + \frac{1}{3}\left[1 + \frac{1}{2}\right] = \frac{1}{3}\left[\frac{2+1}{2}\right] = \frac{1}{\cancel{3}}\cdot\frac{\cancel{3}}{2} = \boxed{\frac{1}{2} + 2} = \frac{1+4}{2} = \boxed{\frac{5}{2}}$$

Equazioni esponenziali del 2° tipo

$2^{x+1} = 5^{1-x}$

APPLICO $\log_{10} = \log$

$\log_{10} 10 = 1$

$\log 2^{x+1} = \log 5^{1-x}$

$(x+1)\log 2 = (1-x)\log 5$

$x \cdot \log 2 + \log 2 = \log 5 - x\log 5$

$x\log 2 + x\log 5 = \log 5 - \log 2$

$(\log 2 + \log 5)\,x = \log 5 - \log 2$

$$x = \frac{\log 5 - \log 2}{\log 2 + \log 5} = \frac{\log\frac{5}{2}}{\log 2\cdot 5} = \frac{\log\frac{5}{2}}{\log 10} = \frac{\log\frac{5}{2}}{1} = \boxed{\log\frac{5}{2}}$$

$$\log_{10}\frac{5}{2} = \frac{\log_e\frac{5}{2}}{\log_e 10} = \frac{\log_e\frac{5}{2}}{\log_e 10}$$

BASE $a = 10$

BASE $a = e \simeq 2{,}718\ldots$

$$\frac{2^{x-1} \cdot 4^{1+x}}{3} = 6^{1-x}$$

$6 = 2 \cdot 3$

$$2^{x-1} \cdot 4^{1+x} = 3 \cdot 6^{1-x}$$

$3^1 \cdot 3^{1-x} = 3^{2-x}$

$$2^{x-1} \cdot 2^{2(1+x)} = 3 \cdot 6^{1-x}$$

$$2^{x-1} \cdot 2^{2(1+x)} = \underline{3} \cdot 2^{1-x} \cdot \underline{3^{1-x}}$$

$$2^{x-1} \cdot 2^{2+2x} = 2^{1-x} \cdot 3^{2-x}$$

$$\frac{2^{x-1} \cdot 2^{2+2x}}{2^{1-x}} = 3^{2-x}$$

Your paragraph text

$$\frac{2^{x-1+2+2x}}{2^{1-x}} = 3^{2-x}$$

$$2^{3x\cancel{+1}\cancel{-1}+x} = 3^{2-x}$$

$$2^{4x} = 3^{2-x}$$

PASSO AI LOGARITMI

$$\log 2^{4x} = \log 3^{2-x}$$

$$4x \log 2 = (2-x) \log 3$$

$$4x \log 2 + x \log 3 = 2 \log 3$$

$$(4 \log 2 + \log 3)x = 2 \log 3$$

$$x = \frac{2 \log 3}{4 \log 2 + \log 3} = \frac{\log 3^2}{\log 2^4 + \log 3} = \frac{\log 9}{\log 16 + \log 3} = \frac{\log 9}{\log(16 \cdot 3)}$$

$$\boxed{x = \frac{\log 9}{\log 48}}$$