$y = ax^2 + bx + c$

$A(-2;1)$

$1 = 4a - 2b + c$

$B(3;2)$

$2 = 9a + 3b + c$

$C\left(0;\frac{1}{5}\right)$

$\frac{1}{5} = c$

$A(-2;1)$
$B(3;2)$
$C\left(0;\frac{1}{5}\right)$

$$\begin{cases} 4a - 2b + c = 1 \\ 9a + 3b + c = 2 \\ c = \frac{1}{5} \end{cases}$$

$$\begin{cases} 4a - 2b + \frac{1}{5} = 1 \\ 9a + 3b + \frac{1}{5} = 2 \end{cases}$$

$$\begin{cases} \frac{20a - 10b + 1}{5} = \frac{5}{5} \\ \frac{45a + 15b + 1}{5} = \frac{10}{5} \end{cases}$$

$$\begin{cases} 20a - 10b = 4 \\ 45a + 15b = 9 \end{cases} \quad \begin{matrix} R_1 \\ R_2 \end{matrix} \begin{cases} 10a - 5b = 2 \\ 15a + 5b = 3 \end{cases}$$

$R_1 + R_2 \quad 25a = 5$

$25a = 5$

$a = \frac{5}{25} = \frac{1}{5}$

$a = \frac{1}{5}$

$2 - 5b = 2$

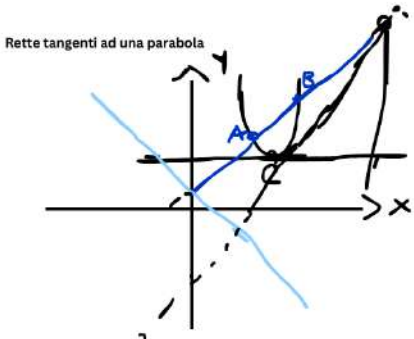
$-5b = 0$

$b = 0$

$a = \frac{1}{5}$

$c = \frac{1}{5}$

$$y = \frac{1}{5}x^2 + \frac{1}{5}$$



RETTA SECANTE

Tocca la parabola in due punti distinti

RETTA TANGENTE

Tocca la parabola in un solo punto, oppure in due punti coincidenti

RETTA ESTERNA

Non ha punti in comune con la parabola

$$\begin{cases} y = ax^2 + bx + c \\ y = mx + q \end{cases}$$

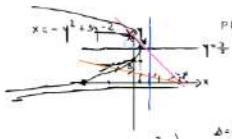
$ax^2 + bx + c = mx + q$ $\quad a, b, c \in \mathbb{R}$, $a \neq 0$, $m, q \in \mathbb{R}$

$\Delta = \Delta(m) > 0$ 2 sol. distinte — RETTE SECANTI

$\Delta = \Delta(m) = 0$ 2 sol. coincidenti — RETTE TANGENTI

$\Delta < 0$ — RETTE ESTERNE

$\begin{cases} x = -y^2 + 3y - 2 \\ y = 0 \end{cases}$ $(-2, 0)$



$\Delta = b^2 - 4ac = 9 - 4(-1)(-2)$
$= 9 - 8 = 1$

$V\left(-\frac{\Delta}{4a}; -\frac{b}{2a}\right) = \left(-\frac{1}{4(-1)}; -\frac{3}{2(-1)}\right)$

$\boxed{V\left(\frac{1}{4}; \frac{3}{2}\right)}$

$F\left(\frac{1-\Delta}{4a}; -\frac{b}{2a}\right) = \left(0; \frac{3}{2}\right)$

$d: y = \frac{3}{2}$

$x_F - \frac{1+\Delta}{4a} = -\frac{2}{4(-1)} = \frac{1}{2} \Rightarrow x = \frac{1}{2}$

$\begin{cases} x = -y^2 + 3y - 2 \\ x = 0 \end{cases}$

$-y^2 + 3y - 2 = 0$
$\Delta = 1$
$y_{1;2} = \frac{-3 \pm \sqrt{1}}{-2} = \frac{-3 \pm 1}{-2}$
$y_1 = 2 \quad y_2 = 1$

A (0; 2)
B (0; 1)

P (2, 0)

$y - y_P = m(x - x_P)$
$y - 0 = m(x - 2) \Rightarrow \boxed{y = mx - 2m}$

$\begin{cases} x = -y^2 + 3y - 2 \\ y = mx - 2m \end{cases}$

$x = -(mx - 2m)^2 + 3(mx - 2m) - 2$
$x = -(m^2x^2 + 4m^2 - 4m^2x) + 3mx - 6m - 2$
$x = -m^2x^2 - 4m^2 + 4m^2x + 3mx - 6m - 2$
$m^2x^2 + x - 4m^2x - 3mx + 4m^2 + 6m + 2 = 0$
$m^2x^2 + (-4m^2 - 3m + 1)x + 4m^2 + 6m + 2 = 0$
$\Delta = B^2 - 4AC$ $\boxed{\Delta = 0}$

$\Delta = (-4m^2 - 3m + 1)^2 - 4m^2(4m^2 + 6m + 2) = 0$
$16m^4 + 9m^2 + 1 + 24m^3 - 8m^2 - 6m - 16m^4 - 24m^3 - 8m^2 = 0$
$-7m^2 - 6m + 1 = 0$
$7m^2 + 6m - 1 = 0$
$\Delta = 36 - 4(7)(-1) = 36 + 28 = 64$
$m_{1;2} = \frac{-6 \pm \sqrt{64}}{14}$
$m_1 = \frac{-6-8}{14} = -\frac{14}{14} = -1$
$m_2 = \frac{-6+8}{14} = \frac{2}{14} = \frac{1}{7}$

$m_1 = -1$
$y = -x + 2$
$\boxed{x + y - 2 = 0}$

$m_2 = \frac{1}{7}$
$y = \frac{1}{7}x - \frac{2}{7}$
$7y = x + 2 = 0$
$\boxed{x - 7y - 2 = 0}$

$y = -x^2 + 2x + 4$ $\quad P\left(\frac{1}{2}; 7\right)$

$y - y_P = m(x - x_0)$

$y - 7 = m\left(x - \frac{1}{2}\right)$

$\boxed{y = mx - \frac{1}{2}m + 7}$ ER. DEL FASCIO DI RETTE CONDOTTE DA P

$$\begin{cases} y = -x^2 + 2x + 4 \\ y = mx - \frac{1}{2}m + 7 \end{cases}$$

$-x^2 + 2x + 4 = mx - \frac{1}{2}m + 7$

$\frac{-2x^2 + 4x + 8}{2} = \frac{2mx - m + 14}{2}$

$-2x^2 + 4x + 8 - 2mx + m - 14 = 0$

$-2x^2 + (6 - 2m)x + m - 6 = 0$

$2x^2 + (2m - 4)x + 6 - m = 0$ $\quad \Delta = 0$ COND. DI TANGENZA

$\Delta = (2m - 4)^2 - 4(2)(6 - m) = 0$

$4m^2 + 16 - 16m - 48 + 8m = 0$

$4m^2 - 8m - 32 = 0$

$m^2 - 2m - 8 = 0$

$\Delta = (-2)^2 - 4(1)(-8) = 4 + 32 = 36$ $\quad -\frac{\sqrt{\Delta}}{2a} = -2$

$m_1, m_2 = \frac{2 \pm \sqrt{36}}{2} = \frac{2 \pm 6}{2}$ $\quad \frac{\sqrt{\Delta}}{2a} = 4$

$y = mx - \frac{1}{2}m + 7$

$m = -2$

$y = -2x - \frac{1}{2}(-2) + 7$

$\boxed{y = -2x + 8}$

$2x + y - 8 = 0$

$m = 4$

$y = 4x - \frac{1}{2}(4) + 7$

$\boxed{y = 4x + 5}$

$-4x + y - 5 = 0$

$4x - y + 5 = 0$

$$\begin{cases} y = -x^2 + 2x + 4 \\ y = -2x + 8 \end{cases} \Rightarrow -x^2 + 2x + 4 = -2x + 8$$

$-x^2 + 4x - 4 = 0$

$x^2 - 4x + 4 = 0$

$(x - 2)^2 = 0$

$\boxed{x = 2}$

$y = -2 \cdot 2 + 8 = -4 + 8 = 4$

$\boxed{A(2; 4)}$

$$\begin{cases} y = -x^2 + 2x + 4 \\ y = 4x + 5 \end{cases}$$

$-x^2 + 2x + 4 = 4x + 5$

$-x^2 - 2x - 1 = 0$

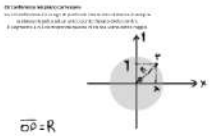
$x^2 + 2x + 1 = 0$

$(x + 1)^2 = 0$

$x = -1$

$y = 4(-1) + 5 = -4 + 5 = 1$

$\boxed{B(-1; 1)}$

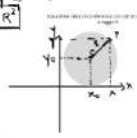


$\overline{OP} = R$

$\sqrt{(x_P - x_O)^2 + (y_P - y_O)^2} = R$

$\sqrt{(x-0)^2 + (y-0)^2} = R$ ELEVIAMO TUTTO AL QUADRATO

$\boxed{x^2 + y^2 = R^2}$



$\overline{PC} = R$

$\sqrt{(x_P - x_C)^2 + (y_P - y_C)^2} = R$

$\sqrt{(x-x_0)^2 + (y-y_0)^2} = R$ ELEVO AL QUADRATO

$\boxed{(x-x_0)^2 + (y-y_0)^2 = R^2}$ EQUAZIONE IN FORMA SEMPLICE

$x^2 + x_0^2 - 2x_0x + y^2 - y_0^2 - 2y_0y = R^2$

$x^2 + y^2 - 2x_0x - 2y_0y + x_0^2 + y_0^2 - R^2 = 0$

$\boxed{x^2 + y^2 + \alpha x + \beta y + \gamma = 0}$

$\begin{cases} -2x_0 = \alpha \\ -2y_0 = \beta \\ x_0^2 + y_0^2 - R^2 = \gamma \end{cases}$

$\begin{cases} \boxed{x_0 = -\frac{\alpha}{2}} \\ \boxed{y_0 = -\frac{\beta}{2}} \\ -R^2 = \gamma - x_0^2 - y_0^2 \end{cases}$

$R^2 = x_0^2 + y_0^2 - \gamma \Rightarrow R = \sqrt{x_0^2 + y_0^2 - \gamma}$

$\boxed{R = \sqrt{\left(-\frac{\alpha}{2}\right)^2 + \left(-\frac{\beta}{2}\right)^2 - \gamma}}$

ES. $2x^2 + 2y^2 + 4x - 3y = 18$

$x^2 + y^2 + 2x - \frac{3}{2}y - 9 = 0$

$\alpha = 2$

$\beta = -\frac{3}{2}$

$\gamma = -9$

$x_0 = -\frac{\alpha}{2} = -\frac{2}{2} = -1$

$y_0 = -\frac{\beta}{2} = -\left(-\frac{3}{4}\right) = \frac{3}{4}$

$\boxed{C\left(-1; \frac{3}{4}\right)}$

$R = \sqrt{\left(-\frac{\alpha}{2}\right)^2 + \left(-\frac{\beta}{2}\right)^2 - \gamma} = \sqrt{1 + \frac{9}{16} + 9} = \sqrt{\frac{16+9+144}{16}}$

$= \sqrt{\frac{169}{16}} = \sqrt{\frac{169}{\sqrt{16}}} = \frac{13}{4}$

$\boxed{R\left(\frac{13}{4}\right) = 3{,}25}$

$(x+1)^2 + \left(y - \frac{3}{4}\right)^2 = \frac{169}{16}$

$(x-x_0)^2 + (y-y_0)^2 = R^2$