LEZIONE 21
Derivate notevoli

$$\boxed{\begin{aligned} y &= x^{\alpha} \\ y &= e^{\log_e x^{\alpha}} = e^{\alpha \log_e x} \end{aligned}}$$

$\alpha \in \mathbb{R}$

$y = x^n$, $n \in \mathbb{N}$, $y' = n x^{n-1}$

$$y' = e^{\alpha \log_e x} \cdot \left(\frac{d(\alpha \log_e x)}{dx}\right) = \underbrace{e^{\alpha \log_e x}}_{x^{\alpha}} \cdot \alpha \cdot \frac{1}{x} = \alpha \cdot \frac{x^{\alpha}}{x} = \boxed{\alpha x^{\alpha - 1}}$$

$$\boxed{y' = \alpha x^{\alpha - 1}} \quad \alpha \in \mathbb{R}$$

$$\boxed{\alpha = \frac{1}{n}} \qquad \boxed{y = x^{\frac{1}{n}} = \sqrt[n]{x}} \quad y' = \frac{1}{n} x^{\frac{1}{n} - 1} = \frac{1}{n} x^{\frac{1-n}{n}} =$$

$$= \frac{1}{n} x^{\frac{1-n}{n}} = \frac{1}{n} x^{-\frac{n-1}{n}} = \frac{1}{n} \frac{1}{x^{\frac{n-1}{n}}} = \frac{1}{n} \cdot \frac{1}{\sqrt[n]{x^{n-1}}}$$

$$\boxed{y = \sqrt[n]{x} \implies y' = \frac{1}{n\sqrt[n]{x^{n-1}}}}$$

$n \in \mathbb{N} \setminus \{0\}$ $\quad x > 0$

$$y = f(x)^{g(x)} \qquad y = (x+1)^{x+2}$$

$$\cancel{y = e^{\log_e f(x)^{g(x)}}} = e^{g(x)\log_e f(x)}$$

$$\boxed{y = e^{g(x)\cdot \log_e f(x)}}$$

$$\boxed{y' = \underbrace{e^{g(x)\log_e f(x)}} \cdot \left[g'(x)\log_e f(x) + g(x)\cdot \frac{1}{f(x)} f'(x) \right]}$$

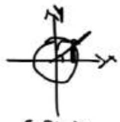
$$\boxed{y' = f(x)^{g(x)} \cdot \left[g'(x)\log_e f(x) + g(x)\frac{f'(x)}{f(x)} \right]}$$

$$y = x^x \implies y = e^{\log_e x^x} = e^{x\log_e x}$$

$$y' = x^x \cdot \left[1\cdot \log_e x + \cancel{x}\cdot \frac{1}{\cancel{x}} \right]$$

$$\boxed{y' = x^x \cdot (\log_e x + 1)}$$

Derivate funzioni inverse delle funzioni goniometriche



1) $y = \text{arcsen}\, x$ $\quad \boxed{-1 \leq x \leq 1}$

$y = \text{sen}\, x$ $\quad \boxed{f : \left[-\frac{\pi}{2}; \frac{\pi}{2}\right] \to [-1; 1]}$

$y = \text{arcsen}\, x$ $\quad \boxed{f : [-1; 1] \to \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]}$

$$\boxed{y' = \frac{d\,(\text{arcsen}\, x)}{dx}} \qquad \frac{d\, f(x)}{dx}$$

$$\text{sen}\,(\text{arcsen}\, x) = x$$

$$f \circ (f^{-1}(x)) = x$$

$-1 \leq 1 \leq 1$

$$y = \text{sen}\,(\text{arcsen}(x)) \qquad y = x$$

sono la stessa funzione in $[-1; 1]$

$$\frac{d\, \text{sen}\,(\text{arcsen}(x))}{dx} = \frac{dx}{dx}$$

$$1 = \cos(\text{arcsen}(x)) \cdot \frac{d\,(\text{arcsen}\, x)}{dx}$$

$$1 = \cos y \cdot \frac{d\,(\text{arcsen}\, x)}{dx}$$

$\cos^2 y + \text{sen}^2 y = 1$

$\cos^2 y = 1 - \text{sen}^2 y$

$\cos y = \sqrt{1 - \text{sen}^2 y}$

$$1 = \sqrt{1 - \text{sen}^2 y}\; \frac{d\,(\text{arcsen}\, x)}{dx}$$

$x = \text{sen}\, y$

$$1 = \sqrt{1 - x^2}\; \boxed{\frac{d\,(\text{arcsen}\, x)}{dx}}$$

$$\boxed{\frac{d\,(\text{arcsen}\, x)}{dx} = \frac{1}{\sqrt{1 - x^2}}}$$

$y = \arccos x$

$y = \cos x$

$f: [0;\pi] \to [-1;1]$

$y = \cos(\arccos x) \qquad y = x \quad in\ [-1;1]$

$y = \arccos x$

$f^{-1}: [-1;1] \to [0;\pi]$

$1 = \dfrac{d \cos(\arccos x)}{dx}$

$1 = -\operatorname{sen}(\arccos x) \cdot \dfrac{d \arccos x}{dx}$

$\operatorname{sen} y = \sqrt{1-\cos^2 y}$

$1 = -\operatorname{sen} y \cdot \dfrac{d \arccos x}{dx}$

$\cos y = x$

$1 = -\sqrt{1-\cos^2 y} \cdot \dfrac{d \arccos x}{dx}$

$\dfrac{d \arccos x}{dx} = -\dfrac{1}{\sqrt{1-x^2}}$

$\boxed{y' = -\dfrac{1}{\sqrt{1-x^2}}}$

$y = \operatorname{arctg} x$

$y = \operatorname{tg} x$

$f:]-\frac{\pi}{2};\frac{\pi}{2}[\to \mathbb{R}$

$\operatorname{tg} \operatorname{arctg} x = x \quad in\ \mathbb{R}$

$y = \operatorname{arctg} x$

$f^{-1}: \mathbb{R} \to]-\frac{\pi}{2};\frac{\pi}{2}[$

$\dfrac{d \operatorname{tg} \operatorname{arctg} x}{dx} = \dfrac{dx}{dx}$

$\dfrac{d \operatorname{tg} x}{dx} = 1 + \operatorname{tg}^2 x$

$1 = (1 + \underbrace{\operatorname{tg}^2(\operatorname{arctg} x)}_{x^2}) \cdot \dfrac{d \operatorname{arctg} x}{dx}$

$1 = (1+x^2) \cdot \dfrac{d \operatorname{arctg} x}{dx}$

$\boxed{\dfrac{d \operatorname{arctg} x}{dx} = \dfrac{1}{1+x^2}}$

$\boxed{y' = \dfrac{1}{1+x^2}}$

$$y = \frac{\operatorname{arctg} x}{x^2}$$

$$y' = \frac{\frac{1}{1+x^2} \cdot x^2 - \operatorname{arctg} x \cdot 2x}{x^4} = \frac{\cancel{x}\left[\frac{x}{1+x^2} - 2 \operatorname{arctg} x\right]}{x^{\cancel{4}\,3}} =$$

$$= \frac{\frac{x}{1+x^2} - 2 \operatorname{arctg} x}{x^3} = \frac{x - 2(1+x^2) \operatorname{arctg} x}{1+x^2} \cdot \frac{1}{x^3} =$$

$$= \frac{x - 2(1+x^2) \operatorname{arctg} x}{x^3 (1+x^2)} =$$

$$= \frac{\cancel{x}}{x^{\cancel{3}\,2}(1+x^2)} - \frac{2 \cancel{(1+x^2)} \operatorname{arctg} x}{x^3 \cancel{(1+x^2)}} =$$

$$= \boxed{\frac{1}{x^2(1+x^2)} - \frac{2 \operatorname{arctg} x}{x^3}}$$

Derivata di una funzione inversa (REGOLA GENERALE)

$$y = f(x)$$

$$\boxed{\frac{\Delta y}{\Delta x} = \frac{f(x+\Delta x) - f(x)}{\Delta x}}$$

Rapporto incrementale della f(x)

$$y = f^{-1}(x) = F(x)$$

$$\boxed{\frac{\Delta x}{\Delta y} = \frac{F(y+\Delta y) - F(y)}{\Delta y}}$$

$$f \qquad f^{-1} = F \qquad F \qquad \frac{1}{f}$$

$$\frac{\Delta y}{\Delta x} \qquad \frac{\Delta x}{\Delta y} \qquad \frac{\Delta x}{\Delta y} = \frac{1}{\frac{\Delta y}{\Delta x}}$$

$$F'(y)$$

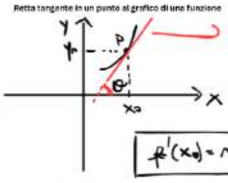
$$\lim_{\Delta y \to 0} \frac{\Delta x}{\Delta y} = \lim_{\Delta x \to 0} \frac{1}{\frac{\Delta y}{\Delta x}} = \frac{1}{\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}} = \frac{1}{f'(x)}$$

$$\boxed{F'(y) = \frac{1}{f'(x)}}$$

Retta tangente in un punto al grafico di una funzione

$P(x_0, y_0)$

$(x_0, f(x_0))$



RETTA TANGENTE IN P AL GRAFICO DI $y=f(x)$

$\boxed{f'(x_0) = m}$ $\boxed{m = tg\,\theta}$

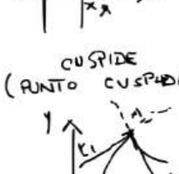
La derivata prima nel punto di ascissa di P, x0 rappresenta il coefficiente angolare della retta tangente in P al grafico della funzione.

$\boxed{y - y_0 = m\,(x - x_0)}$ Equazione della retta passante per il punto P con coefficiente angolare m

$\boxed{y - f(x_0) = f'(x_0)(x - x_0)}$

$\boxed{f(x) - f(x_0) = f'(x_0)(x - x_0)}$

$y_0 = f(x_0)$
$m = f'(x_0)$



TANGENTE VERTICALI

$m = tg\,90°$

$\nexists\, tg\,90°$

$\nexists\, f'(x_A)$

f NON È DERIVABILE IN x_A

CUSPIDE
(PUNTO CUSPIDALE)

$\exists\, t_1, t_2$ DUE RETTE TANGENTI

$\boxed{f'_-(x) \neq f'_+(x)}$

PUNTO ANGOLOSO NON DERIVABILITÀ

$m_1 \neq m_2$

Punto in cui la funzione ammette due derivate, una da destra e una da sinistra e quindi due coefficienti angolari e quindi due rette tangenti nello stesso punto.

ES. $\boxed{y = x^3 + 2x - 1}$

$P\,(1, f(1))$

$f'(x) = 3x^2 + 2$

$f(1) = \cancel{1} + 2 - \cancel{1} = 2$

$m = f'(1) = 3 \cdot 1^2 + 2 = 5 \Longrightarrow \underline{m = 5}$

$P\,(1; 2)$

$\underline{y_0 = 2}$

$y - y_0 = m(x - x_0)$

$y - 2 = 5\,(x - 1) \Longrightarrow y = 5x - 5 + 2 \Longrightarrow \boxed{y = 5x - 3}$