$$3 \cdot 7^x + 4 \cdot 3^x = 7^x + 10 \cdot 3^x$$

$$3 \cdot 7^x - 7^x = 10 \cdot 3^x - 4 \cdot 3^x$$

$$7^x (3-1) = 3^x (10-4)$$

$$\overset{1}{\cancel{2}} \cdot 7^x = \overset{3}{\cancel{6}} \cdot 3^x \Longrightarrow \boxed{7^x = 3 \cdot 3^x}$$

$\log_e = \ln$

Applico il logaritmo in base naturale a destra e a sinistra

$$\ln 7^x = \ln \underbrace{3}_{a} \cdot \underbrace{3^x}_{b}$$

$$x \ln 7 = \ln 3 + \ln 3^x$$

$\boxed{\ln a \cdot b = \ln a + \ln b}$

$$\boxed{x \ln 7 = \ln 3 + x \ln 3}$$

$\boxed{\ln a^b = b \cdot \ln a}$

$$x \ln 7 - x \ln 3 = \ln 3$$

$$\frac{\cancel{(\ln 7 - \ln 3)}\, x}{\cancel{\ln 7 - \ln 3}} = \frac{\ln 3}{\ln 7 - \ln 3}$$

$$\boxed{x = \frac{\ln 3}{\ln 7 - \ln 3}}$$

$$\boxed{\ln \underline{6} - \ln \underline{2} = \ln \frac{\cancel{6}^{3}}{\cancel{2}_{1}}}$$

$$\boxed{\ln a - \ln b = \ln \frac{a}{b}}$$

$$5^{x+1} + 2^{x+1} = 3 \cdot 5^x + 5 \cdot 2^x$$

$$5^x \cdot 5^1 + 2^x \cdot 2^1 = 3 \cdot 5^x + 5 \cdot 2^x$$

$$5 \cdot 5^x - 3 \cdot 5^x = 5 \cdot 2^x - 2 \cdot 2^x$$

$$2 \cdot 5^x = 3 \cdot 2^x$$

$$\ln(2 \cdot 5^x) = \ln(3 \cdot 2^x)$$

$$\ln 2 + \ln 5^x = \ln 3 + \ln 2^x$$

$$\ln 2 + x \ln 5 = \ln 3 + x \ln 2$$

$$x \ln 5 - x \ln 2 = \ln 3 - \ln 2$$

$$(\ln 5 - \ln 2) x = \ln 3 - \ln 2$$

$$\boxed{x = \frac{\ln 3 - \ln 2}{\ln 5 - \ln 2}}$$

$$3^{\frac{2}{x-1}} = 5^{3(x+1)}$$

C.E $x-1 \neq 0$ $\boxed{x \neq 1}$

$$\ln 3^{\frac{2}{x-1}} = \ln 5^{3(x+1)}$$

$$\frac{2}{x-1} \ln 3 = 3(x+1) \ln 5$$

$$\cancel{(x-1)} \frac{2 \ln 3}{\cancel{x-1}} = \frac{3(x^2-1) \ln 5}{\cancel{x-1}} \cancel{(x-1)}$$

$$2 \ln 3 = 3x^2 \ln 5 - 3 \ln 5$$

$$3x^2 \ln 5 = 2 \ln 3 + 3 \ln 5$$

$$x^2 = \frac{2 \ln 3 + 3 \ln 5}{3 \ln 5}$$

$$x = \pm \sqrt{\frac{2 \ln 3 + 3 \ln 5}{3 \ln 5}} = \pm \sqrt{\frac{\ln 3^2 + \ln 5^3}{3 \ln 5}}$$

$$x = \pm \sqrt{\frac{\ln 9 + \ln 125}{3 \ln 5}} = \pm \sqrt{\frac{\ln (9 \cdot 125)}{3 \ln 5}}$$

$$x = \pm \sqrt{\frac{\ln 1125}{3 \ln 5}} = \pm \sqrt{\frac{1}{3}} \cdot \sqrt{\frac{\ln 1125}{\ln 5}}$$

$$\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\boxed{x = \pm \frac{\sqrt{3}}{3} \cdot \sqrt{\frac{\ln 1125}{\ln 5}}}$$

$$4^{\frac{x}{2}} + 4^{2x} = 9^{x+1} + 2^{x}$$

$$2^{\frac{\cancel{2}x}{\cancel{2}}} + 2^{2\cdot 2x} = 3^{2(x+1)} + 2^{x}$$

$$\cancel{2^{x}} + 2^{4x} = 3^{2x+2} + \cancel{2^{x}}$$

$$2^{4x} = 3^{2x+2}$$

$$\ln 2^{4x} = \ln 3^{2x+2}$$

$$4x \ln 2 = (2x+2)\ln 3$$

$$4x \ln 2 = 2x \ln 3 + 2 \ln 3$$

$$4x \ln 2 - 2x \ln 3 = 2 \ln 3$$

$$\underline{2x(2\ln 2 - \ln 3)} = 2 \ln 3$$

$$x = \frac{\cancel{2}\cdot \ln 3}{\cancel{2}(2\ln 2 - \ln 3)} \Rightarrow \boxed{x = \frac{\ln 3}{\ln 4 - \ln 3}}$$

$$2 \ln 2 = \ln 2^{2} = \ln 4$$

$$x = \frac{2 \ln 3}{4 \ln 2 - 2 \ln 3}$$

$$x = \frac{\ln 3^{2}}{\ln 2^{4} - \ln 3^{2}}$$

$$\boxed{x = \frac{\ln 9}{\ln 16 - \ln 9}}$$

Disequazioni esponenziali

$$a^{f(x)} > a^{g(x)}$$

$$\boxed{\begin{array}{l} \text{se } a > 1 \\ f(x) > g(x) \\ \text{se } 0 < a < 1 \\ f(x) < g(x) \end{array}}$$

$$\left(\frac{1}{2}\right)^2 > \left(\frac{1}{2}\right)^3 \Rightarrow 2 < 3$$

Attenzione al cambio del del verso

$$2^x \geq 8$$

$$2^x \geq 2^3 \Longrightarrow \boxed{x \geq 3}$$

$2 > 1$? SI

$\frac{1}{5} \cancel{\geq} 1$? NO!!!

$$\left(\frac{1}{5}\right)^x \geq \left(\frac{1}{5}\right)^1 \Longrightarrow \boxed{x \leq 1}$$

$3^{2x} - 10 \cdot 3^{x} + 9 < 0$ $\qquad$ $3^{x} = t$

$t^{2} - 10t + 9 < 0$

$\Delta = 100 - 4(1)(9) = 100 - 36 = 64$

$t_{1,2} = \dfrac{10 \pm \sqrt{64}}{2}$

$\dfrac{10-8}{2} = \dfrac{\cancel{2}^{1}}{\cancel{2}_{1}} = 1$

$\dfrac{10+8}{2} = \dfrac{\cancel{18}^{9}}{\cancel{2}_{1}} = 9$

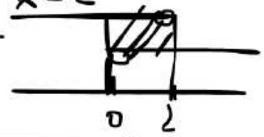
$t_1 < t < t_2$

$\boxed{1 < t < 9}$

$\boxed{1 < 3^{x} < 9}$

$\begin{cases} 3^{x} > 1 \\ 3^{x} < 9 \end{cases}$ $\qquad$ $\begin{cases} 3^{x} > 3^{0} \Rightarrow \boxed{x > 0} \\ 3^{x} < 3^{2} \Rightarrow \boxed{x < 2} \end{cases}$

$\begin{cases} x > 0 \\ x < 2 \end{cases}$



$\boxed{0 < x < 2}$

$$4^{x+1} - 17 \cdot 2^x + 4 > 0$$
$$2^{2(x+1)} - 17 \cdot 2^x + 4 > 0$$
$$2^{2x+2} - 17 \cdot 2^x + 4 > 0$$
$$2^{2x} \cdot 2^2 - 17 \cdot 2^x + 4 > 0$$
$$4 \cdot 2^{2x} - 17 \cdot 2^x + 4 > 0$$
$$4 \cdot t^2 - 17t + 4 > 0$$

$$\boxed{2^{2x+2} = 2^{2x} \cdot 2^2}$$

$$2^x = t$$

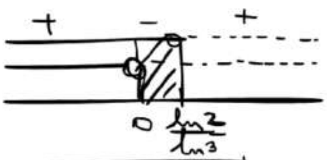
$$\Delta = b^2 - 4ac = (-17)^2 - 4(4)(4) = 289 - 64 = 225$$
$$t_1, t_2 = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{17 \pm \sqrt{225}}{8} = \frac{17 \pm 15}{8}$$

$$\frac{17-15}{8} = \frac{\cancel{2}^{1}}{\cancel{8}_{4}} \qquad \frac{17+15}{8} = \frac{\cancel{32}^{4}}{\cancel{8}_{1}}$$

$$t_1 = \frac{1}{4} \qquad t_2 = 4$$
$$t < t_1 \lor t > t_2$$
$$\boxed{t < \frac{1}{4} \lor t > 4}$$
$$2^x < \frac{1}{4} \lor 2^x > 4$$
$$2^x < 2^{-2} \lor 2^x > 2^2$$
$$\boxed{x < -2 \lor x > 2}$$

$$2 - 2^{x+1} < 3^x - 6^x$$
$$2 - 2^x \cdot 2^1 < 3^x - 2^x \cdot 3^x$$
$$2 - 2 \cdot 2^x < 3^x(1 - 2^x)$$
$$2(1 - 2^x) < 3^x(1 - 2^x)$$
$$2(1 - 2^x) - 3^x(1 - 2^x) < 0$$
$$\underbrace{(1 - 2^x)}_{N_1}\underbrace{(2 - 3^x)}_{N_2} < 0$$

$$2^{x+1} = 2^x \cdot 2^1$$
$$6 = 2 \cdot 3$$
$$6^x = (2 \cdot 3)^x$$
$$= 2^x \cdot 3^x$$



$$\boxed{0 < x < \frac{\ln 2}{\ln 3}}$$

N_1 $\quad 1 - 2^x > 0$
$$-2^x > -1$$
$$2^x < 1 \Longrightarrow 2^x < 2^0$$
$$\boxed{x < 0}$$

N_2 $\quad 2 - 3^x > 0$
$$-3^x > -2$$
$$3^x < 2$$
$$\ln 3^x < \ln 2$$
$$x \ln 3 < \ln 2$$
$$x < \frac{\ln 2}{\ln 3}$$
$$\simeq 0,63\ldots$$

$$10^{\sqrt{2x+4}} > 4^x$$

$$\ln 10^{\sqrt{2x+4}} > \ln 4^x$$

$$\boxed{\sqrt{2x+4}\ \ln 10 > x \ln 4}$$

$$\boxed{\sqrt{2x+4} > x\,\frac{\ln 4}{\ln 10}}$$

C.E.
$2x+4 \geq 0$
$2x \geq -4$
$x \geq -\frac{4}{2}$
$x \geq -2$

$$\sqrt{f(x)} > g(x)$$

$$\begin{cases} f(x) \geq 0 \\ g(x) < 0 \end{cases} \cup \begin{cases} g(x) \geq 0 \\ f(x) > g^2(x) \end{cases}$$

$$\begin{cases} 2x+4 \geq 0 \\ \frac{\ln 4}{\ln 10} \cdot x < 0 \end{cases} \cup \begin{cases} \frac{\ln 4}{\ln 10} x \geq 0 \\ 2x+4 > x^2 \frac{\ln^2 4}{\ln^2 10} \end{cases}$$

$\boxed{x \geq 0}$

$$\begin{cases} x \geq -2 \\ x < 0 \end{cases}$$

-2 $\quad 0$

$\boxed{-2 \leq x < 0}$

$$\boxed{2x + 4 > x\,\frac{\ln^2 4}{\ln^2 10}}$$

$$x = \frac{2 \quad 0 \quad 1 \qquad 1}{2 \ln 4}$$