Lezioni 21

Ultimo modello di razionalizzazione

$$a^3+b^3=(a+b)(a^2-ab+b^2)$$

ES

$$\frac{6}{\sqrt[3]{4}+\sqrt[3]{2}}\cdot\frac{(\sqrt[3]{4^2}-\sqrt[3]{8}+\sqrt[3]{2^2})}{(\sqrt[3]{4^2}-\sqrt[3]{8}+\sqrt[3]{2^2})}=\frac{6\sqrt[3]{16}-12+6\sqrt[3]{4}}{4+2}=\frac{\cancel{6}(\sqrt[3]{16}-2+\sqrt[3]{4})}{\cancel{6}}$$

$$=\sqrt[3]{16}-2+\sqrt[3]{4}=\sqrt[3]{2^4}-2+\sqrt[3]{4}=$$

$$=\sqrt[3]{2^3\cdot 2}-2+\sqrt[3]{\ }=2\sqrt[3]{2}-2+\sqrt[3]{4}$$

$$\frac{1}{\sqrt[3]{2}-1}\cdot\frac{\sqrt[3]{2^2}+\sqrt[3]{2}+1}{\sqrt[3]{2^2}+\sqrt[3]{2}+1}=\frac{\sqrt[3]{4}+\sqrt[3]{2}+1}{2-1}\qquad a^3-b^3=(a-b)(a^2+ab+b^2)$$

$$=\sqrt[3]{4}+\sqrt[3]{2}+1$$

Radicale Doppio

(1)
$$\sqrt{a+\sqrt{b}} = \left(\sqrt{\frac{a+\sqrt{a^2-b}}{2}} + \sqrt{\frac{a-\sqrt{a^2-b}}{2}}\right)$$

(2)
$$\sqrt{a-\sqrt{b}} = \sqrt{\frac{a+\sqrt{a^2-b}}{2}} - \sqrt{\frac{a-\sqrt{a^2-b}}{2}}$$

Dim (1) ELEVO AL QUADRATO TUTTO

$$a+\sqrt{b} = \frac{a+\sqrt{a^2-b}}{2} + \frac{a-\sqrt{a^2-b}}{2} + 2\sqrt{\left(\frac{a+\sqrt{a^2-b}}{2}\right)\left(\frac{a-\sqrt{a^2-b}}{2}\right)}$$

$$a+\sqrt{b} = \frac{a+\cancel{\sqrt{a^2-b}}+a-\cancel{\sqrt{a^2-b}}}{2} + 2\sqrt{\frac{a^2-(a^2-b)}{4}}$$

$$a+\sqrt{b} = \frac{\cancel{2}a}{\cancel{2}} + \cancel{2}\cdot\frac{1}{\cancel{2}}\sqrt{\cancel{a^2}-\cancel{a^2}+b}$$

$$a+\sqrt{b} = a+\sqrt{b}$$

C.V.D.

ESEMPI RADICALE DOPPIO

$$\sqrt{4+\sqrt{7}} = \sqrt{\frac{4+\sqrt{16-7}}{2}} + \sqrt{\frac{4-\sqrt{16-7}}{2}} =$$

$$= \sqrt{\frac{4+\sqrt{9}}{2}} + \sqrt{\frac{4-\sqrt{9}}{2}} =$$

$$= \sqrt{\frac{4+3}{2}} + \sqrt{\frac{4-3}{2}} =$$

$$= \sqrt{\frac{7}{2}} + \sqrt{\frac{1}{2}} =$$

$$= \frac{\sqrt{7}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} + \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{14}}{2} + \frac{\sqrt{2}}{2}}$$

$$\frac{3\sqrt{3}}{2-\sqrt{2}+\sqrt{3}} = \frac{3\sqrt{3}}{\underbrace{(\sqrt{3}-\sqrt{2})}_{A}+\underbrace{2}_{B}} \cdot \frac{(\sqrt{3}-\sqrt{2}-2)}{\underbrace{(\sqrt{3}-\sqrt{2})}_{A}-\underbrace{2}_{B}} =$$

$(A+B)(A-B)$
$= A^2-B^2$

$$= \frac{9-3\sqrt{6}-6\sqrt{3}}{(\sqrt{3}-\sqrt{2})^2-4} = \frac{9-3\sqrt{6}-6\sqrt{3}}{3+2-2\sqrt{6}-4} = \frac{9-3\sqrt{6}-6\sqrt{3}}{1-2\sqrt{6}} =$$

$$= \frac{(9-3\sqrt{6}-6\sqrt{3})}{1-2\sqrt{6}} \cdot \frac{(1+2\sqrt{6})}{1+2\sqrt{6}} = \frac{9+18\sqrt{6}-3\sqrt{6}-6\cdot 6-6\sqrt{3}-12\sqrt{18}}{1-4\cdot 6} =$$

$$\begin{array}{r|l} 18 & 2 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array}$$

$18 = 2\cdot 3^2$

$$= \frac{9+15\sqrt{6}-36-6\sqrt{3}-12\sqrt{2\cdot 3^2}}{-23} =$$

$$= \frac{15\sqrt{6}-27-6\sqrt{3}-12\cdot 3\sqrt{2}}{-23} = \frac{-1}{-1}$$

$$= \frac{27-15\sqrt{6}+6\sqrt{3}+36\sqrt{2}}{23}$$

$\frac{1}{6}\cdot\frac{3}{1}$

$\boxed{(a+b)^3 = a^3+3a^2b+3ab^2+b^3}$

$\boxed{(a-b)(a+b)=a^2-b^2}$

$$\left(\frac{2}{5}a - b^2\right)\left(\frac{2}{5}a+b^2\right) - \left[\left(\frac{1}{2}a+\frac{2}{3}b\right)^3 - \frac{8}{27}b^3 - \frac{1}{6}ab(3a+4b)\right] : \frac{25}{32}a$$

$$=\left(\frac{2}{5}a\right)^2 - \left(b^2\right)^2 - \left[\left(\frac{1}{2}a\right)^3 + 3\cdot\left(\frac{1}{2}a\right)^2\cdot\left(\frac{2}{3}b\right) + 3\cdot\left(\frac{1}{2}a\right)\left(\frac{2}{3}b\right)^2 + \left(\frac{2}{3}b\right)^3 - \frac{8}{27}b^3 - \frac{\cancel{3}\,1}{\cancel{6}\,2}a^2b - \frac{\cancel{4}\,2}{\cancel{6}\,3}ab^2\right] : \frac{25}{32}a =$$

$$=\frac{4}{25}a^2 - b^4 - \left[\frac{1}{8}a^3 + \cancel{3}\cdot\frac{1}{\cancel{4}\,2}a^2\cdot\frac{\cancel{2}\,1}{\cancel{3}}b + \cancel{3}\cdot\frac{1}{8}a\cdot\frac{\cancel{4}\,2}{\cancel{9}\,3}b^2 + \cancel{\frac{8}{27}b^3} - \cancel{\frac{8}{27}b^3} - \frac{1}{2}a^2b - \frac{2}{3}ab^2\right] : \frac{25}{32}a =$$

$$=\frac{4}{25}a^2 - b^4 - \left[\frac{1}{8}a^3 + \cancel{\frac{1}{2}a^2b} + \cancel{\frac{2}{3}ab^2} - \cancel{\frac{1}{2}a^2b} - \cancel{\frac{2}{3}ab^2}\right] : \frac{25}{32}a =$$

$$=\frac{4}{25}a^2 - b^4 - \frac{1}{8}a^3 : \frac{25}{32}a =$$

$$=\frac{4}{25}a^2 - b^4 - \frac{1}{\cancel{8}}\cdot\frac{\cancel{32}\,4}{25}a^2 =$$

$$=\cancel{\frac{4}{25}a^2} - b^4 - \cancel{\frac{4}{25}a^2} = -b^4$$

$$\frac{\sqrt{2}}{1-\sqrt{5}} + \frac{\sqrt{2}-3}{\sqrt{5}} - \frac{3(\sqrt{5}-1)}{\sqrt{5}-5}$$

$$\text{A}\quad \frac{\sqrt{2}}{1-\sqrt{5}} \cdot \frac{(1+\sqrt{5})}{(1+\sqrt{5})} = \frac{\sqrt{2}+\sqrt{10}}{1-5} = \boxed{-\frac{(\sqrt{2}+\sqrt{10})}{4}}$$

$$\text{B}\quad \frac{\sqrt{2}-3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \boxed{\frac{\sqrt{10}-3\sqrt{5}}{5}}$$

$$\text{C}\quad \frac{3(\sqrt{5}-1)}{\sqrt{5}-5} \cdot \frac{(\sqrt{5}+5)}{\sqrt{5}+5} = \frac{\cancel{15}+15\sqrt{5}-3\sqrt{5}\cancel{(-15)}}{5-25} = \frac{\cancel{12}^{3}\sqrt{5}}{-\cancel{20}_{5}} =$$

$$= \boxed{-\frac{3\sqrt{5}}{5}}$$

$$-\frac{(\sqrt{2}+\sqrt{10})}{4} + \frac{\sqrt{10}-3\sqrt{5}}{5} - \left(-\frac{3\sqrt{5}}{5}\right) =$$

$$= -\frac{(\sqrt{2}+\sqrt{10})}{4} + \frac{\sqrt{10}-3\sqrt{5}}{5} + \frac{3\sqrt{5}}{5} =$$

$$= \frac{-5(\sqrt{2}+\sqrt{10})+4(\sqrt{10}-3\sqrt{5})+12\sqrt{5}}{20} =$$

$$= \frac{-5\sqrt{2}-5\sqrt{10}+4\sqrt{10}\cancel{-12\sqrt{5}}\cancel{+12\sqrt{5}}}{20} = \frac{-\sqrt{10}-5\sqrt{2}}{20} =$$

$$= \boxed{\frac{-\sqrt{2}(\sqrt{5}+5)}{20}}$$

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