Teorema di Rolle

Ip $y = f(x)$ CONTINUA IN $[a, b]$
DERIVABILE IN (a, b)

$f(a) = f(b)$

Ts $\exists\, c \in [a, b] \mid f'(c) = 0$

Dim

1° CASO $\boxed{m = M}$

$\underset{\text{MINIMO}}{\underline{\exists\, c, d \in [a, b] \quad f(c) = m}} \quad \underset{\text{MASSIMO}}{\underline{f(d) = M}}$

$\underline{m} = f(c) \le f(x) \le f(d) = \underline{M}$

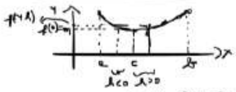
$m = f(c) = f(x) = f(d) = M$

f COSTANTE

$\exists\, c \quad f'(x) = 0$ c.v.d

2° CASO $\boxed{m < M}$

$h \in \mathbb{R}$



$f(c+h) \ge f(c) \Rightarrow \boxed{f(c+h) - f(c) \ge 0}$

DIVIDIAMO PER $h \ne 0$

se $h > 0 \quad \dfrac{f(c+h) - f(c)}{h} \ge 0$ se $h < 0 \quad \dfrac{f(c+h) - f(c)}{h} \le 0$

$\displaystyle\lim_{h \to 0^+} \underbrace{\frac{f(c+h) - f(c)}{h}}_{} \ge 0 \qquad \lim_{h \to 0^-} \underbrace{\frac{f(c+h) - f(c)}{h}}_{} \le 0$

$$\lim_{h \to 0} \frac{f(c+h) - f(c)}{h} = 0$$

$$\Downarrow$$

$$\boxed{f'(c) = 0}$$

Dom f $[-2, 2]$

ES $y = \sqrt{4 - x^2}$ in $[-2; 2]$ ROLLE

$y' = \dfrac{1}{\cancel{2}\sqrt{4 - x^2}} \cdot (-\cancel{2}x) = -\dfrac{x}{\sqrt{4 - x^2}}$ in $(-2; 2)$

Dom f' $(-2; 2)$

$f(a) = f(b) \Rightarrow f(-2) = f(2)$ ✓

$f(-2) = \sqrt{4 - 4} = 0 \quad f(2) = 0$

$y' = 0 \Rightarrow -\dfrac{x}{\sqrt{4 - x^2}} = 0 \Rightarrow -x = 0 \Rightarrow \boxed{x = 0}$

$x = 0 \in [-2; 2]$ VERIFICATO ROLLE

Ip $y = f(x)$ CONTINUA IN $[a, b]$
DERIVABILE IN (a, b)

Ts $\exists c \in (a, b) : \dfrac{f(b) - f(a)}{b - a} = f'(c)$

<u>Dim</u> $\boxed{\varphi(x) = f(x) - Kx}$ $K \in \mathbb{R}$

φ CONTINUA IN $[a, b]$
φ DERIVABILE IN (a, b)

<u>$K \in \mathbb{R}$</u> : $\varphi(a) = \varphi(b)$

K = ? AFFINCHÉ φ SODDISFI IL TEOREMA DI ROLLE

$\varphi(a) = f(a) - Ka$
$\varphi(b) = f(b) - Kb$

$f(a) - Ka = f(b) - K \cdot b$ DEVE ESSERE VERA PER ...
$f(a) - f(b) = Ka - K \cdot b$
$f(b) - f(a) = K \cdot b - K a$
$K(b - a) = f(b) - f(a)$

$$\boxed{K = \frac{f(b) - f(a)}{b - a}}$$

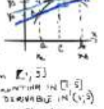
$$\varphi(x) = f(x) - \left[\frac{f(b) - f(a)}{b - a}\right] x$$

φ SODDISFA ROLLE $\Rightarrow \exists c : \varphi'(c) = 0$

$$\varphi'(c) = f'(c) - \left[\frac{f(b) - f(a)}{b - a}\right] = 0$$

$$\boxed{f'(c) = \frac{f(b) - f(a)}{b - a}}$$
m_{AB}

$$m_{AB} = \frac{y_B - y_A}{x_B - x_A} = \frac{f(b) - f(a)}{b - a}$$



<u>ES</u> $y = \sqrt{x - 1}$ in $[1, 5]$

$y' = \dfrac{1}{2\sqrt{x-1}}$

f CONTINUA IN $[1, 5]$
f DERIVABILE IN $(1, 5)$

$f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$f'(c) = \dfrac{f(5) - f(1)}{5 - 1} = \dfrac{f(5) - f(1)}{4}$

$f(5) = \sqrt{5 - 1} = \sqrt{4} = 2$
$f(1) = \sqrt{1 - 1} = 0$

$f'(c) = \dfrac{2 - 0}{4} = \dfrac{1}{2}$

$\boxed{f'(c) = \frac{1}{2}}$ $\boxed{f'(c) = \frac{1}{2\sqrt{c-1}}}$

$\dfrac{1}{\cancel{2}} = \dfrac{1}{\cancel{2}\sqrt{c-1}}$

$\sqrt{c - 1} = 1$ $c - 1 \geq 0$ $c \geq 1$

$c - 1 = 1$

in $\boxed{c = 2}$

$$f'(2) = \frac{f(5) - f(1)}{5 - 1} = \frac{f(5) - f(1)}{4}$$

SODDISFA IL TEOREMA DI LAGRANGE IN $[1, 5] \to$ CONTINUITÀ
$(1, 5) \to$ DERIVABILITÀ

Ip $\boxed{y = f(x)}$ CONTINUA IN $I = [a, b]$ $\to$ RG
DERIVABILE IN $I_{APERTO} = (a, b)$

$\boxed{\forall x \in (a, b) \Rightarrow f'(x) = 0}$

Ts $y = f(x) = \text{cost}$

<u>Dim</u> $x_1, x_2 \in I \Rightarrow$ f SODDISFA LAGRANGE $\exists c \in (a, b)$ $\boxed{f'(c) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}}$

PER IPOTESI $f'(c) = 0$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} = 0 \Rightarrow f(x_1) = f(x_2)$$

<u>c.v.d.</u>

Corollario 2 al teorema di Lagrange

Ip $f(x), g(x)$ CONTINUE IN I

DERIVABILI IN I_{APERTO}

$f'(x) = g'(x)$ in I_{APERTO}

Ts $f(x) - g(x) = \underline{\text{cost}}$

Dim $F(x) = f(x) - g(x)$

DERIVIAMO

$F'(x) = f'(x) - g'(x)$

$F'(x) = f'(x) - f'(x) = 0$

$f'(x) = g'(x)$ PER IPOTESI IN I APERTO

$F'(x) = 0 \implies F(x) = \text{cost}$

$\underline{f(x) - g(x)} = \underline{\text{cost}}$

Funzioni derivabili crescenti e decrescenti

REMEMBER

$$\forall x_1, x_2 \in I;\ x_1 < x_2 \Longrightarrow f(x_1) < f(x_2)$$

Funzione strettamente crescente

$$\forall x_1, x_2 \in I;\ x_1 < x_2 \Longrightarrow f(x_1) > f(x_2)$$

Funzione strettamente decrescente

Ip $y = f(x)$ CONTINUA IN I
DERIVABILE IN I APERTO

a) $f'(x) > 0$
b) $f'(x) < 0$

Ts
a) f CRESCENTE IN I
b) f DECRESCENTE

Dim

TESI

LAGRANGE

$$\forall x_1, x_2 \in I,\ \underline{x_1 < x_2} \Longrightarrow \boxed{\exists c \in (x_1, x_2) : f'(c) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}}$$

a) $f'(x) > 0 \ \forall x \in I \Longrightarrow f'(c) > 0$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} > 0 \Longrightarrow f(x_2) - f(x_1) > 0$$

$$f(x_2) > f(x_1)$$
$$\Downarrow$$
$$f(x_1) < f(x_2)$$

f STRETT CRESCENTE IN I

b) $f'(x) < 0 \ \forall x \in I \Rightarrow f'(c) < 0$
$x_1 < x_2$

$$\frac{f(x_2) - f(x_1)}{x_2 - x_1} < 0 \Longrightarrow f(x_2) - f(x_1) < 0$$

$$f(x_2) < f(x_1)$$
$$\Downarrow$$
$$f(x_1) > f(x_2)$$

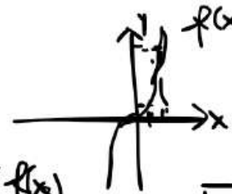
f STRETT. DECRESCENTE

E' ovvio che la stessa cosa vale anche per il senso lato ovvero:

$f'(x) \geq 0$ $\quad$ f CRESCENTE IN SENSO LATO

$f'(x) \leq 0$ $\quad$ f DECRESCENTE IN SENSO LATO

ES 8 $\boxed{f(x) = x^3}$



$f(x)=x^n$

$f'(x)=nx^{n-1}$

f STRETT. CRESCENTE IN $\mathbb{R}$

$\forall x_1, x_2 \,, x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$

ES $1, 2$ $\quad 1 < 2 \Rightarrow f(1) < f(2) \Rightarrow \boxed{1 < 8}$ ✓

$f'(x)=3x^2 \qquad f'(x) \geq 0$

$x^2 \geq 0 \quad \forall x \in \mathbb{R}$ $\quad$ f CRESCENTE IN $\mathbb{R}$

$$f(x) = \operatorname{arctg}(\ln x) + \ln \operatorname{arctg} x$$

$$f'(x) = \frac{1}{1+\ln^2 x} \cdot \frac{1}{x} + \frac{1}{\operatorname{arctg} x} \cdot \frac{1}{1+x^2} =$$

$$= \frac{1}{x(1+\ln^2 x)} + \frac{1}{(1+x^2)\operatorname{arctg} x}$$

$$f(x) = \boxed{\frac{\cos(\ln \operatorname{arcsen} 1)}{\sqrt{1-e^2}\ \operatorname{arcsen}(-1)}} - e^x(1-e^x)x$$

$$f'(x) = -\left[e^x(1-e^x)\cdot x + e^x(-e^x)\cdot x + e^x(1-e^x)\right]$$

$$f'(x) = -\left[(e^x - e^{2x})x - e^{2x}\cdot x + e^x - e^{2x}\right]$$

$$f'(x) = -\left[e^x\cdot x - e^{2x}\cdot x - e^{2x}\cdot x + e^x - e^{2x}\right]$$

$$f'(x) = -\left[e^x\cdot x - 2e^{2x}\cdot x + e^x - e^{2x}\right]$$

$$f'(x) = -e^x\left(x - 2e^x\cdot x + 1 + e^x\right)$$

$$\boxed{f'(x) = e^x\left(2xe^x - x - 1 - e^x\right)}$$