Lezione 22

Teorema di De L'Hopital

Ip $f(x), g(x)$ DERIVABILI IN $I(c) =]c-\delta; c+\delta[$

$g'(x) \neq 0$ in I $\quad I(c) =]c-\delta, c[\cup]c, c+\delta[$

$\exists \lim_{x \to c} \frac{f(x)}{g(x)} \Rightarrow \left[\frac{0}{0}\right] \left[\frac{\infty}{\infty}\right]$

Ts $\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)} = \lim_{x \to c} \frac{f''(x)}{g''(x)}$

Dim $\left[\frac{0}{0}\right]$

PER IPOTESI $f(x), g(x)$ CONTINUE IN I

$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{0}{0} \Rightarrow \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)} = \frac{0}{0}$ $\quad \lim_{x \to 0} \frac{\sin x}{x}$ $\quad f(0) = 1$

$\lim_{x \to c} f(x) = \lim_{x \to c} g(x) = 0 \quad \not\exists\ f(c), g(c)$

DISC. ELIMINABILI DI 3ª SPECIE

$f_1(x) = \begin{cases} f(x) & \text{se } x \neq c \\ 0 & \text{se } x = c \end{cases}$ $\quad$ per $x \neq c$ $g(x) \neq 0$

$g_1(x) = \begin{cases} g(x) & \text{se } x \neq c \\ 0 & \text{se } x = c \end{cases}$

APPLICO LAGRANGE A g $\quad c_1 \in I$

$\frac{g(x) - g(c)}{x - c} = g'(c_1)$

se per assurdo $g(x) = 0 \Rightarrow g(c) = 0$

$\frac{0}{x-c} = g'(c_1) \Longrightarrow g'(c_1) = 0$

CONTRADDICE L'IPOTESI

$x \neq c \in I(c)$

$\frac{f(x)}{g(x)} = \frac{f(x) - 0}{g(x) - 0} = \frac{f(x) - f(c)}{g(x) - g(c)} = \frac{f'(c_1)}{g'(c_1)}$

TEO. DI CAUCHY

Ip $f(x), g(x)$ CONTINUE IN $[a,b]$, DERIVABILI IN $]a,b[$

$g'(x) \neq 0$

Ts $\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}$

$\frac{f(x)}{g(x)} = \frac{f'(c_1)}{g'(c_1)}$ $\quad c_1 \in]x, c[$

$x \to c$

$c_1 = c$

$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(c_1)}{g'(c_1)}$

$\Downarrow$

$$\boxed{\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}}$$

c.v.d.

ES.

$$\lim_{x\to 2}\frac{x^2-5x+6}{x^2+5x-14}=\frac{4-10+6}{4+10-14}=-\frac{6+6}{14-14}=\left[\frac{0}{0}\right]$$

$$\overset{H}{=}\lim_{x\to 2}\frac{2x-5}{2x+5}=\frac{4-5}{4+5}=\boxed{-\frac{1}{9}}$$

$$\lim_{x\to 2}\frac{\cancel{(x-2)}(x-3)}{\cancel{(x-2)}(x+7)}=\frac{2-3}{2+7}=\boxed{-\frac{1}{9}}$$

$p=6$
$s=-5$
$x^2-5x+6=(x-2)(x-3)$

$p=-14$
$s=5$
$x^2+5-14=(x-2)(x+7)$

$$\lim_{x\to+\infty}\frac{\overset{\to+\infty}{\log x}+\overset{\to+\infty}{x}}{x^2}=\left[\frac{+\infty}{+\infty}\right]\overset{H}{=}\lim_{x\to+\infty}\frac{\frac{1}{x}+1}{2x}=$$

$$\lim_{x\to+\infty}\frac{\frac{1+x}{x}}{2x}=\lim_{x\to+\infty}\frac{1+x}{2x^2}=\left[\frac{+\infty}{+\infty}\right]\overset{H}{=}$$

$$\lim_{x\to+\infty}\frac{1}{4x}=\frac{1}{+\infty}=0$$

$$\boxed{\lim_{x\to+\infty}\left(1+\frac{1}{x}\right)^x=e \quad 1^{\infty}}$$ NO!!