$(a+b)^3=a^3+b^3+3a^2b+3ab^2$ LEZIONE 40

① $(a+b+c)^3-(b+c-a)^3-(a-b+c)^3-(a+b-c)^3=$

$=[\underbrace{(a+b)}_{A}+\underbrace{c}_{C}]^3-②[\underbrace{(-a+b)}_{A}+\underbrace{c}_{C}]^3-③[\underbrace{(a-b)}_{A}+\underbrace{c}_{C}]^3-④[\underbrace{(a+b)}_{A}-\underbrace{c}_{C}]^3=$

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① $[\underbrace{(a+b)}_{A}+\underbrace{c}_{C}]^3=$

$=(a+b)^3+c^3+3(a+b)^2\cdot c+3(a+b)\cdot c^2=$

$=a^3+b^3+3a^2b+3ab^2+c^3+3(a^2+b^2+2ab)\cdot c+(3a+3b)c^2$

$=a^3+b^3+3a^2b+3ab^2+c^3+(3a^2+3b^2+6ab)c+3ac^2+3bc^2$

$=\boxed{a^3+\cancel{b^3}+\cancel{3a^2b}+3ab^2+\cancel{c^3}+\cancel{3a^2c}+\cancel{3b^2c}+6abc+3ac^2+\cancel{3bc^2}}$

② $[\underbrace{(-a+b)}+\underbrace{c}]^3=$

$=(-a+b)^3+c^3+3(-a+b)^2\cdot c+3(-a+b)\cdot c^2=$

$=-a^3+b^3+3a^2b-3ab^2+c^3+3(a^2+b^2-2ab)c+3(-a+b)c^2$

$=\boxed{-a^3+\cancel{b^3}+\cancel{3a^2b}-3ab^2+\cancel{c^3}+\cancel{3a^2c}+\cancel{3b^2c}-6abc-3ac^2+\cancel{3bc^2}}$

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③ $[(a-b)+c]^3=$

$=(a-b)^3+c^3+3(a-b)^2c+3(a-b)c^2=$

$=a^3-b^3-3a^2b+3ab^2+c^3+3(a^2+b^2-2ab)c+3ac^2-3bc^2=$

$=\boxed{a^3-\cancel{b^3}-\cancel{3a^2b}+3ab^2+\cancel{c^3}+\cancel{3a^2c}+\cancel{3b^2c}-6abc+3ac^2-\cancel{3bc^2}}$

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④ $[(a+b)-c]^3=$

$(a+b)^3-c^3-3(a+b)^2c+3(a+b)c^2$

$=(a+b)^3-c^3-3(a+b)^2c+3(a+b)c^2$

$=a^3+b^3+3a^2b+3ab^2-c^3-3(a^2+b^2+2ab)c+3ac^2+3bc^2$

$=\boxed{a^3+\cancel{b^3}+\cancel{3a^2b}+3ab^2-\cancel{c^3}-\cancel{3a^2c}-\cancel{3b^2c}-6abc+3ac^2+\cancel{3bc^2}}$

① − ②

$\cancel{2a^3}+\cancel{6ab^2}+12abc+\cancel{6a^2c}$

③ − ④

$-\cancel{2a^3}-\cancel{6ab^2}+12abc-\cancel{6ac^2}$

$=\underline{24abc}$