Lezione 5

Vediamo come ottenere l'equazione di un fascio di rette a partire da due sue rette.
Le due rette che determinano o generano il fascio verranno definite rette generatrici.

$$\begin{cases} r: ax + by + c = 0 \\ s: a'x + b'y + c' = 0 \end{cases}$$

$a, b, c \in \mathbb{R}$

$a', b', c' \in \mathbb{R}$

FASCIO PROPRIO $\boxed{\exists \; P_0(x_0, y_0) = r \cap s}$

con $\lambda, \mu \in \mathbb{R}$

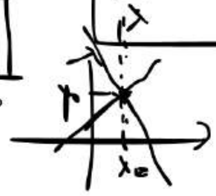
$$\lambda r + \mu s = 0 \Rightarrow \lambda(ax + by + c) + \mu(a'x + b'y + c) = 0$$

$$\frac{\lambda(ax+by+c)}{\lambda}+\frac{\mu}{\lambda}(a'x+b'y+c')=\frac{0}{\lambda}$$

$$\lambda \neq 0$$

$$\boxed{ax+by+c+k(a'x+b'y+c')=0}$$

$$\boxed{\frac{\mu}{\lambda}=K}$$



Equazione del fascio di rette generato dalle rette r ed s, espresso nella forma implicita.

$r: x - 3y + 5 = 0 \qquad s: 2x + y - 4 = 0$

$$\boxed{(x - 3y + 5) + k(2x + y - 4) = 0}$$

$$P_0(x_0; y_0) \Longrightarrow \begin{cases} x - 3y + 5 = 0 \\ 2x + y - 4 = 0 \end{cases} \Longrightarrow \begin{cases} x = 3y - 5 \\ 2(3y - 5) + y - 4 = 0 \end{cases}$$

$$6y - 10 + y - 4 = 0 \qquad 7y = 14 \qquad y = 2$$

$$x = 3 \cdot 2 - 5 = 1$$

$P_0(1; 2)$

$$x - 3y + 5 + 2kx + ky - 4k = 0$$

$$(k - 3)y = -(1 + 2k)x + 4k - 5 \qquad \boxed{k \neq 3}$$

$$(3 - k)y = (1 + 2k)x + 5 - 4k$$

$$y = \underbrace{\frac{1 + 2k}{3 - k}}_{m}x + \underbrace{\frac{5 - 4k}{3 - k}}_{q}$$

$k = 3 \qquad 0 = 7x + 5 - 12 \qquad 0 = 7x - 7$

$$7x = 7 \Longrightarrow \boxed{x = 1}$$

$\boxed{\text{RETTA VERTICALE ESCLUSA}}$

1° FASCIO $\quad r: \; 5x - 2y + 4 = 0$

$\qquad\qquad\quad s: \; 2x - 5y - 4 = 0$

RETTA COMUNE

2° FASCIO $\quad r': \; x + y + 3 = 0$

$\qquad\qquad\quad s': \; 2x - y = 0$

$$5x-2y+4+K(2x-5y-4)=0$$
$$5x-2y+4+2Kx-5Ky-4K=0$$
$$(5+2K)x-(2+5K)y+4-4K=0$$
$$-(2+5K)y=-(5+2K)x+4K-4$$
$$(2+5K)y=+(5+2K)x+4-4K$$
$$\boxed{y=+\frac{5+2K}{2+5K}x+\frac{4-4K}{2+5K}}$$

$$\boxed{K\neq-\frac{5}{2}}$$

$$x + y + 3 + K(2x - y) = 0$$

$$(1 + 2K)x + (1 - K)y + 3 = 0$$

$$(1 - K)y = -(1 + 2K)x - 3 \qquad K \neq 1$$

$$(K - 1)y = (1 + 2K)x + 3$$

$$\boxed{y = \left(\frac{1 + 2K}{K - 1}\right)x + \frac{3}{K - 1}}$$

$m \qquad q$

$$\begin{cases} \dfrac{5 + 2K}{2 + 5K} = \dfrac{1 + 2K}{K - 1} \\[2ex] \dfrac{4 - 4K}{2 + 5K} = \dfrac{3}{K - 1} \end{cases} \qquad A$$

$$(5 + 2K)(K - 1) = (1 + 2K)(2 + 5K)$$

$$5K - 5 + 2K^2 - 2K = 2 + 5K + 4K + 10K^2$$

$$-8K^2 - 6K - 7 = 0$$

$$8K^2 + 6K + 7 = 0$$

$$(4 - 4K)(K - 1) = 3(2 + 5K)$$

$$4K - 4 - 4K^2 + \cancel{4K} = 6 + 15K$$

$$-4K^2 - 7K - 10 = 0 \qquad \Delta < 0$$

$$4K^2 + 7K + 10 = 0$$