Lezione 11

$25x^2+25y^2-20x+10y-4=0$

$x^2+y^2+\alpha x+\beta y+\gamma=0$

$x^2+y^2-\frac{20}{25}x+\frac{10}{25}y-\frac{4}{25}=0$

$x^2+y^2-\frac{4}{5}x+\frac{2}{5}y-\frac{4}{25}=0$

$x_0=-\frac{\alpha}{2}=-\frac{\left(-\frac{4}{5}\right)}{2}=\frac{4}{10}=\frac{2}{5}$

$y_0=-\frac{\beta}{2}=-\frac{\frac{2}{5}}{2}=-\frac{2}{10}=-\frac{1}{5}$

$C\left(\frac{2}{5};-\frac{1}{5}\right)$

$R=\frac{3}{5}$

Dall'equazione alle coordinate del centro e al raggio.

$\alpha=-\frac{4}{5}$

$\beta=\frac{2}{5}$

$\gamma=-\frac{4}{25}$

$x_0=-\frac{\alpha}{2}$

$y_0=-\frac{\beta}{2}$

$R=\sqrt{x_0+y_0^2-\gamma}$

$R=\sqrt{\frac{4}{25}+\frac{1}{25}-\left(-\frac{4}{25}\right)}=\sqrt{\frac{8}{25}+\frac{1}{25}}=$

$=\sqrt{\frac{9}{25}}=\frac{3}{5}$

$C\left(-3;\frac{5}{2}\right)$ $R=\sqrt{5}$

$P(x,y)$

$\overline{PC} = R$

$$\sqrt{(x-(-3))^2+\left(y-\frac{5}{2}\right)^2} = \sqrt{5}$$

$$(x+3)^2+\left(y-\frac{5}{2}\right)^2 = 5$$

$$x^2+9+6x+y^2+\frac{25}{4}-5y = 5$$

$$x^2+y^2+6x-5y+9+\frac{25}{4}-5 = 0$$

$$\frac{4x^2+4y^2+24x-20y+36+25-20}{4} = \frac{0}{4}$$

$$\boxed{4x^2+4y^2+24x-20y+41 = 0}$$

Dal centro e raggio abbiamo ricavato l'equazione

Circonferenza passante per tre punti

$P(-3;3)$ $Q(3;1)$ $R(1;-5)$

$x^2+y^2+\alpha x+\beta y+\gamma=0$

$P(-3;3)$ $\quad 9+9-3\alpha+3\beta+\gamma=0$

$Q(3;1)$ $\quad 9+1+3\alpha+\beta+\gamma=0$

$R(1;-5)$ $\quad 1+25+\alpha-5\beta+\gamma=0$

$$\begin{cases} -3\alpha+3\beta+\gamma=-18 & R_1 \\ 3\alpha+\beta+\gamma=-10 & R_2 \\ \alpha-5\beta+\gamma=-26 & R_3 \end{cases}$$

R_1-R_2 $\quad -6\alpha+2\beta=-8$

R_2 $\quad 3\alpha+\beta+\gamma=-10$

R_3 $\quad \alpha-5\beta+\gamma=-26$

$$\begin{cases} -6\alpha+2\beta=-8 & R_1' = R_1-R_2 \\ 2\alpha+6\beta=16 & R_2' = R_2-R_3 \\ \alpha-5\beta+\gamma=-26 & R_3 \end{cases}$$

$$\begin{cases} -6\alpha+2\beta=-8 & R_1' \\ 2\alpha+6\beta=16 & R_2' \end{cases}$$

$$\Downarrow$$

$$\begin{cases} -6\alpha+2\beta=-8 & R_1' \\ 6\alpha+18\beta=48 & 3R_2' \end{cases}$$

$R_1'+3R_2'$ $\quad // \quad 20\beta=40$

$\beta=\frac{40}{20}=2$

$\beta=2$

$2\alpha+6\beta=16$

$2\alpha+12=16$

$2\alpha=4 \Rightarrow \alpha=\frac{4}{2}=2$

$\alpha-5\beta+\gamma=-26$

$2-10+\gamma=-26$

$\gamma=-26+8=-18$

$$\begin{cases} \alpha=2 \\ \beta=2 \\ \gamma=-18 \end{cases}$$

$$x^2+y^2+2x+2y-18=0$$

$C\left(-\frac{\alpha}{2};-\frac{\beta}{2}\right)=\left(-\frac{2}{2};-\frac{2}{2}\right)=(-1;-1)$

$R=\sqrt{x_c^2+y_c^2-\gamma}=\sqrt{1+1-(-18)}=\sqrt{2+18}$

$=\sqrt{20}=\sqrt{2^2\cdot 5}=2\sqrt{5}$

$20 \mid 2$

$10 \mid 2$

$5 \mid 5$

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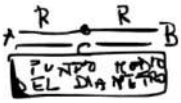
$\overline{AB} = d$ $\quad R = \frac{d}{2}$



$A\,(-2;4)$

$B\,(3;2)$

$\overline{AB} = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} = \sqrt{(-2-3)^2 + (4-2)^2} = \sqrt{25+4} = \sqrt{29}$

$R = \frac{d}{2} = \frac{\sqrt{29}}{2}$

$C\left(\frac{x_A + x_B}{2}; \frac{y_A + y_B}{2}\right) = \left(\frac{-2+3}{2}; \frac{4+2}{2}\right) = \left(\frac{1}{2}; \frac{6}{2}\right) = \left(\frac{1}{2}; 3\right)$

$C\left(\frac{1}{2}; 3\right) \qquad R = \frac{\sqrt{29}}{2}$

$(x - x_c)^2 + (y - y_c)^2 = R^2$

$\left(x - \frac{1}{2}\right)^2 + (y-3)^2 = \frac{29}{4}$

$x^2 + \frac{1}{4} - x + y^2 + 9 - 6y = \frac{29}{4}$

$x^2 + y^2 - x - 6y + \frac{1}{4} + \frac{9}{1} - \frac{29}{4} = 0$

$\frac{4x^2 + 4y^2 - 4x - 24y + 1 + 36 - 29}{4} = \frac{0}{4}$

$4x^2 + 4y^2 - 4x - 24y + 8 = 0$

$\boxed{x^2 + y^2 - x - 6y + 2 = 0}$